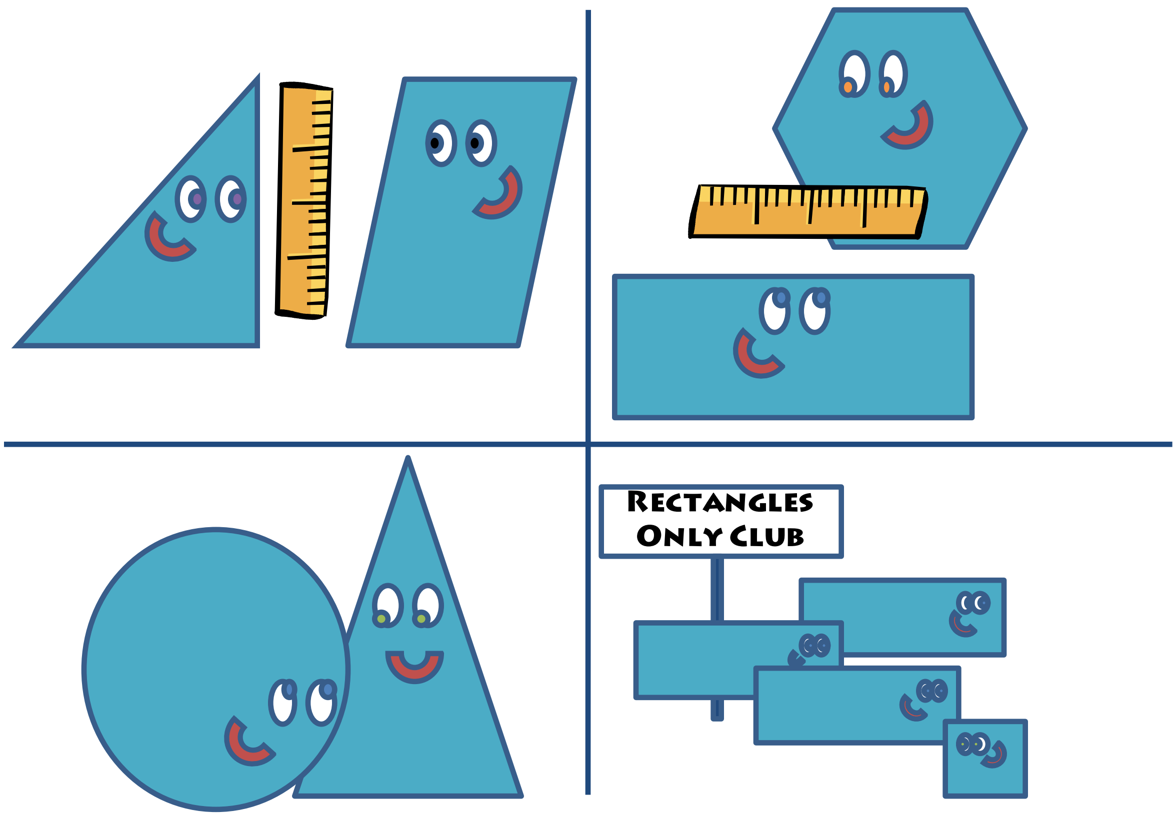
GEOMETRY & MEASUREMENT FOR ALL SHAPES & SIZES

Third Edition



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The National Research Center on the Gifted and Talented
University of Connecticut

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THE NATIONAL RESEARCH CENTER ON THE GIFTED AND TALENTED

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The University of Connecticut and the University of Virginia are collaborating on a 5 Year research study of identification, curriculum interventions, and assessment to determine What Works in Gifted Education: Excellence and Equity in Educating Gifted Students.

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Mathematical language is a thinking tool that helps us use reason to create and communicate ideas about concepts, numbers, and shapes to solve problems in the real world.

- NRC/GT Research Team, University of Connecticut, 2008

Geometry & Measurement for All Shapes & Sizes

Introduction

The Merriam-Webster dictionary defines geometry as “a branch of mathematics that deals with the measurement, properties, and relationships of points, lines, angles, surfaces, and solids” (Merriam-Webster, n.d.). Since geometric shapes and solids constitute the world in which we live, the study of geometry at the elementary level can include drawings, manipulatives, and relevant problems based on students’ past and present experiences both with mathematics and with the world. Using students’ surroundings as a springboard for geometry instruction may provide comfort as new concepts are introduced with respect to what they already know.

According to the National Council of Teachers of Mathematics (NCTM), students in grades 3-5 should learn to classify and explore interrelationships among different geometric figures by “thinking *and* doing” (NCTM, 2000). This study of geometric figures inevitably leads to the need for methods of accurately measuring the distance around and surface covered by such figures. Although geometry and measurement are two distinct content strands identified by NCTM’s *Principles and Standards for School Mathematics* (2000), it is important for students to see the interconnections between the two, especially as they move from classifying geometric shapes to investigating problems related to perimeter and area. This unit, therefore, integrates measurement concepts as they relate to the study of geometry.

In this unit students are encouraged to discover the relationships between geometric figures through hands-on, engaging activities and discussions. They are asked to think like practicing professionals in mathematics and related fields as they “invent” methods for making perfect circles, and grapple with the ideas of “estimated” versus “exact” as they relate to the measurement of geometric shapes. Similar to the way many groups in democratic societies operate, students are asked to agree on answers to mathematics problems that lend themselves to multiple responses.

The geometry unit is infused with interdisciplinary approaches to learning mathematics. In the example stated previously, students are assigned to "committees" and asked to come to agreements, which can be related to the social studies goal of creating active citizens. Literature and the arts can be seen throughout as students take the stage as actors and read and write stories and poems about geometry. Students are often creating their own drawings and models as they explore characteristics of shapes and investigate congruence. This type of interdisciplinary learning seeks to have students make connections between mathematics and their world as well as mathematics and other content areas.

The *Geometry & Measurement for All Shapes & Sizes* unit is divided into two sections: *Time to Shape Up* and *Living on the Edge*. In *Time to Shape Up*, students examine and classify shapes according to their properties with emphasis on common misconceptions such as identifying distinguishing characteristics of rectangles and squares. They develop methods for creating congruent polygons and perfect circles using student-centered exploration activities. In *Living on the Edge,* students explore perimeter and area of regular and irregular polygons and shapes. Students are challenged to use both estimation and accuracy as they perform both one and two-dimensional measurements on geometric figures.

This unit differs from traditional mathematics units in both its interdisciplinary approach and its embedded differentiation and enrichment strategies. Students are asked throughout the unit to relate the geometric concepts to their world by pulling in examples from their surroundings. In addition, the unit allows for in-depth exploration of important geometric and measurement concepts. By exposing students to these techniques embedded in a real-world meaningful context, we hope that students become excited, engaged, and enthusiastic about using mathematics as a tool in the everyday world around them.

Rationale

Meeting the needs of individual students creates instructional challenges, whereas whole group instruction can be accomplished quite easily. Nevertheless, teachers recognize that students have a continuum of skills influenced heavily by prior knowledge, and thus the need for differentiation exists in any diverse classroom (Tomlinson, 2005). The National Council of Teachers of Mathematics (NCTM) has outlined the need for classrooms that promote in student discourse, set high expectations, and emphasize learning tasks that focus on student understanding of mathematics in place of the typical learning of procedures that has dominated classroom practice since the 1930's (NCTM, 2000). Still, reform visions are seldom transferred into mathematics classrooms, as whole group, non-differentiated instruction dominates current and past teaching practice (Stigler & Hiebert, 2004). National standards are thrown by the wayside as the "dumbed down textbooks" become the "defacto" mathematics program of a school (Reis & Renzulli, 1992; Reys, 2004). Internationally, the United States ranks low in mathematics topic difficulty and 20% of eighth graders attend schools where basic arithmetic is the most challenging mathematics class offered (Cogan, Schmidt, & Wiley 2001). A report published in 2003 by the U.S. Department of Education (USDE) and The National Center for Education Statistics (NCES) reveals that the United States was outperformed in mathematics by all of the other six countries in both the 1995 and 1999 Trends in International Math and Science Study (TIMSS) video studies, suggesting a diminishing ability to compete at an international level in mathematics and related fields.

In response to the issue of global competitiveness in mathematics, President Bush created the National Mathematics Advisory Panel in 2006. The panel's final report issued in March 2008 contains recommendations for improving mathematics achievement for all students in the United States. The table that follows indicates how The National Research Center on the Gifted and Talented curriculum units address some of the recommendations of the National Math Panel.

NRC/GT Mathematics' Units

NCTM Recommendation	How NRC/GT Units Address Recommendation
Approaches that revisit the same topics year after year without bringing them to closure should be avoided.	The content of each unit aligns with NCTM Standards and Focal Points that are outlined by grade to avoid repetition of content. The embedded differentiation provides an additional safeguard to ensure that individual students are given content that extends prior knowledge rather than repeating it.
"…the curriculum must simultaneously develop conceptual understanding, computational fluency, and problem solving skills. These capabilities are mutually supportive, each facilitating learning of the others." (p. xix)	The three units integrate these types of understanding into the content and instructional strategies provided rather than developing each in isolation.
There is a relationship between teachers' mathematical content knowledge and students' achievement.	The units provide teachers with text that includes potential student difficulties and ideas for handling such problem areas. In addition, there are sample dialogues throughout units designed to help teachers initiate discourse among students.
Research supports the idea that developmentally appropriate curriculum depends on students' prior knowledge rather than their chronological ages.	NCTM has outlined standards for each of the three strands addressed by these units at the grade 3 level. In addition, lessons have been differentiated to address readiness levels of students.
Mathematically gifted students are often capable of learning at accelerated rates and should be encouraged and supported in doing so.	The NCTM focal points, which break the standards down by individual grade, were used in the creation of the units to ensure that extensions provided depth of content beyond the grade 3 level.
International research has revealed that the excessive length of textbooks in the United States is unnecessary as higher achieving nations use much smaller books. Publishers in the United States should work on focusing the content of their textbooks.	The Student Mathematician Journal that accompanies the curriculum units is designed to take the content in depth in a much more focused manner. Since the units have been broken down by content strand, they are much less overwhelming than traditional textbooks cited in the National Math Panel's research summary.
Further scientific research is needed to determine effective strategies for teaching and learning mathematics.	Application of The National Research Center on the Gifted and Talented units will provide current data on the effectiveness of differentiating algebra, geometry and measurement, and graphing and data analysis to develop the mathematical knowledge and skills of all students.

NCTM Standards and Focal Points

NCTM's (2000) *Principles and Standards for School Mathematics* provides guidance for educational decision makers in grades Pre-K through 12. The geometry unit assumes that students in grade 3 possess the prior knowledge indicated by the standards for grades Pre-K-2 and extends this knowledge by focusing on the standards for grades 3-5. In addition, the authors of this unit relied on the NCTM focal points, which provide additional specificity of content for grade 3. Connections to NCTM's geometry and measurement standards and focal points for each lesson within the unit have been delineated in the tables.

NCTM Curriculum Standards and Focal Points	Lesson 1: Classifying Two-Dimensional Shapes	Lesson 2: What's the Right Angle?	Lesson 3: *The Greedy Triangle*	Lesson 4: The Rectangles Only Club!	Lesson 5: Flipping, Turning, & Sliding	Lesson 6: Your Height Can Change Your Life	Lesson 7: Speaking Fractional Language	Lesson 8: A World Without Congruence	Lesson 9: Same Size, Same Shape	Lesson 10: Going in Circles	Lesson 11: Infinite Lines of Symmetry	Lesson 12: The Ants Go Marching	Lesson 13: Ruler of the Ruler	Lesson 14: The ?-Inch Ruler	Lesson 15: Same Perimeter, Different Shape	Lesson 16: A Fair Way to Shade	Lesson 17: Square Units in an Unsquare World
GEOMETRY																	
Students describe, analyze, compare, and classify two-dimensional shapes by sides and angles and connect attributes to definitions of shapes.	X	X	X	X						X							
Students investigate, describe, and reason about polygons.	X		X	X							X				X		
Students understand attributes and properties of two-dimensional space and use this knowledge in solving problems including applications involving congruence.					X			X	X		X						
Students extend their understanding of properties of two-dimensional shapes as they find the areas of polygons.																X	X
Students should make conjectures about geometric properties and relationships.									X		X			X			
Students should recognize geometric ideas and relationships and apply them to other disciplines and to problems that arise in the classroom or in everyday life.		X						X									

NCTM Curriculum Standards and Focal Points	Lesson 1: Classifying Two-Dimensional Shapes	Lesson 2: What's the Right Angle?	Lesson 3: *The Greedy Triangle*	Lesson 4: The Rectangles Only Club!	Lesson 5: Flipping, Turning, & Sliding	Lesson 6: Your Height Can Change Your Life	Lesson 7: Speaking Fractional Language	Lesson 8: A World Without Congruence	Lesson 9: Same Size, Same Shape	Lesson 10: Going in Circles	Lesson 11: Infinite Lines of Symmetry	Lesson 12: The Ants Go Marching	Lesson 13: Ruler of the Ruler	Lesson 14: The ?-Inch Ruler	Lesson 15: Same Perimeter, Different Shape	Lesson 16: A Fair Way to Shade	Lesson 17: Square Units in an Unsquare World
MEASUREMENT																	
Students strengthen their understanding of fractions as they confront problems in linear measurement that call for more precision than the whole unit.						X	X						X				
Students develop measurement concepts and skills through experiences in analyzing attributes and properties of two-dimensional objects.												X	X	X	X	X	X
Students form an understanding of perimeter as a measurable attribute and select appropriate units, strategies, and tools to solve problems involving perimeter.												X		X			
Students recognize area as an attribute of two-dimensional regions.																X	X
Students learn that they can quantify area by finding the total number of same-sized units of area that cover the shape without gaps or overlaps.																X	X
Students understand that a square that is 1 unit on a side is the standard unit for measuring area.																X	X

Geometry: A Model-based Unit

This unit was designed with specific modifications and differentiation for high ability or gifted learners. It was designed to be responsive to the academic diversity of the talent pool and all other students in general education classrooms. Elements of three well-known curricular models in the field of gifted and talented education were combined and utilized to develop this unit: The Differentiation of Instruction Model from Carol A. Tomlinson (2001), the Depth and Complexity Model from Sandra N. Kaplan, and the Schoolwide Enrichment Model from Joseph S. Renzulli and Sally M. Reis. These three research-based models support and promote qualitative differentiation of learning and the ways it is pursued, the nature and extent of student engagement, the active and investigative roles assumed by students, and the quality of student products. The Differentiation of Instruction Model is designed to provide rich and engaging curriculum matched to the diverse interests, readiness levels, and learning profiles of individual students. The model assumes that there is no distinct, single curriculum appropriate for gifted learners, but rather that all students, including the gifted, require educational experiences suited to their individual needs (Tomlinson, 1996).

The Depth and Complexity Model emphasizes the importance of rich, deep, and complex content in appropriately serving gifted learners (Kaplan, 1998). This model emphasizes the benefit of higher level thinking skills, elaborate product development, and more advanced resources in a curriculum for the gifted, but the crux of the model's curriculum equation is the redefinition of the nature of the content. It is based on the premise that appropriate high-level content is synonymous with the dimensions of depth, complexity, novelty, and acceleration (Kaplan, 1998).

The Schoolwide Enrichment Model identifies a talent pool of 15 to 20% of above average ability and/or high ability potential students who will be served through a variety of options, including learning experiences geared toward students' interests and learning styles, curriculum compacting, and enrichment experiences (Renzulli & Reis, 1985; 1997). Over 20 years, the Schoolwide Enrichment Model research demonstrates its effectiveness with a broad range of school socioeconomic levels, program organization patterns, and gifted learners (Baum, 1985, 1988; Burns, 1987; Delcourt, 1988; Gubbins, 1982; Imbeau, 1991; Olenchak, 1988; Olenchak & Renzulli, 1989; Reis, 1981; Reis & Renzulli, 2003; Schack, 1986; Starko, 1986).

This curriculum unit reflects the commonalities of these well-known models in the field of gifted and talented education. This unit tailors essential content, process, and products to the academic needs of students in academically diverse classrooms; emphasizes conceptual thinking, real-world disciplinary inquiry, and problem solving; assesses specific and developing learning needs of talent pool and all other students in general education classrooms; helps students acquire increasing levels of expertise; and encourages student involvement with problem solving and product development with real-world utility.

Geometry & Measurement for All Shapes & Sizes Pacing Chart

	Monday	Tuesday	Wednesday	Thursday	Friday
Week 1 Time to Shape Up! (Lessons 1-5)	**Pretest** **Lesson 1:** Classifying Two-Dimensional Shapes **Tiered by Activity** -Pythagoras' Shapes -Hypatia's Shapes	**Lesson 2:** What's the Right Angle? **Tiered Student Pages** -Pythagoras -Euclid	**Lesson 3:** *The Greedy Triangle* *(Marilyn Burns' book The Greedy Triangle provided.)* **Interdisciplinary** Students use creative abilities to write quadrilateral stories.	**Lesson 4:** The Rectangles Only Club! **Tiered Student Pages** -Euclid -Hypatia	**Lesson 5:** Flipping, Turning, and Sliding **Extensions** Additional prompts and Student Pages to increase the level of challenge
Week 2 Time to Shape Up! (Lessons 6-10)	**Lesson 6:** Your Height Can Change Your Life **Tiered Student Pages** -Pythagoras -Hypatia	**Lesson 7:** Speaking Fractional Language **Students Grouped by Readiness**	**Lesson 8:** A World Without Congruence **Non-Differentiated Lesson**	**Lesson 9:** Same Size, Same Shape **Tiered by Activity** -ACUTE (Pythagoras) -OBTUSE (Hypatia)	**Lesson 10:** Going in Circles (optional lesson) **Scaffolded Version** Extension questions to challenge students who finish early and a scaffolded task page
Week 3 Time to Shape Up! (Lesson 11) Living on the Edge (Lessons 12-15)	**Lesson 11:** Infinite Lines of Symmetry **Tiered by Activity** Pythagoras' Shapes Hypatia & Euclid Shapes	**Lesson 12:** The Ants Go Marching **Tiered Student Pages** -Hypatia -Euclid	**Lesson 13:** Ruler of the Ruler **Non-Differentiated Lesson**	**Lesson 14:** The ?-Inch Ruler **Tiered by Activity** Measurement lengths determined by mathematician name	**Lesson 15:** Same Perimeter, Different Shape **Non-Differentiated Lesson**
Week 4 Living on the Edge (Lessons 16-17)	**Lesson 16:** A Fair Way to Shade **Tiered by Activity** Pythagoras, Hypatia & Euclid activities	**Lesson 17:** Square Units in an Unsquare World **Non-Differentiated Lesson**	**Unit Project (Option 1):** A Geometry Scavenger Hunt **Unit Project (Option 2):** A Shapely Living Room	**Unit Project (Option 1):** A Geometry Scavenger Hunt **Unit Project (Option 2):** A Shapely Living Room	**Unit Project (Option 1):** A Geometry Scavenger Hunt **Unit Project (Option 2):** A Shapely Living Room **Posttest**

Organization of the Unit

Lesson Layout

Each lesson is written in the following format to allow for easy implementation:

Planning Phase

- **Big Ideas**
- **Lesson Objectives**
- **Materials**
- **Mathematical Terms**
- **Lesson Preview**

Teaching Phase

- **Initiate**
  NCTM (2000) highlights the importance of choosing tasks that "pique students' curiosity" (p. 18). The "Initiate" phase of the lesson is designed to accomplish this task by getting students to think about the direction of the lesson in relation to their prior knowledge and/or experiences. Wallas (1926) identifies four steps in the creative process: preparation, incubation, illumination, and revision. According to Torrance (1979), traditional instruction rarely capitalizes on opportunities to develop incubation processes thereby limiting students' abilities to make new connections, accurately depict the future, and use these skills to solve current and potential problems. Torrance's (1979) Instructional Model for Enhancing Incubation describes a warm-up process designed to heighten anticipation prior to the presentation of new information. The importance of the initiation phase that mimics this "warm-up process" and is defined by "how the teacher introduces new content topics" has been neglected with respect to classroom practice (Torrance, 1979; Zahorik, 1970).
- **Investigate**
  The "Investigate" section of the lesson is designed to have students pursue the ideas and curiosities that arise during the initiation. Activities are often student-centered and exploratory in nature, allowing students to draw their own conclusions on mathematical ideas and discuss them with classmates.
- **Conclude**
  Students solidify new knowledge from the lesson, communicating what they have learned in a variety of ways.
- **Look Ahead**
  Teacher provides a preview of the purpose of the next lesson.
- **Assess**
  Teachers are provided opportunities to assess, both formally and informally, student progress throughout the unit. Forms of assessment include:

- **Student Pages**
  The Student Pages that accompany the explorations offer opportunities for students to think critically about important geometric and measurement concepts. The title before the student's name gives each student a hint about what he/she is investigating in the lesson and allows him/her to take on the role of a professional in a mathematics-related field. After the questions related to the lessons, many of these pages include practice problems, which extend the ideas of geometry and measurement with relation to number and operations. Teachers can decide whether to formally assess students on these sections of the student pages, or to simply use them as a guide in determining which students need assistance in particular areas. All of these pages are accompanied by answer pages, which include possible student responses and possible student difficulties. Many of these pages have been tiered for different levels of readiness.
- **Check Ups**
  At the end of some lessons a brief assessment is included that can be used to check students' understanding of unit concepts and serves as an ongoing review of regular curriculum content.
- **Group Discussion**
  This involves student communication of mathematics either during the Conclude section of the lesson or while answering questions on the Student Pages.
- **Direct Inquiry**
  This occurs when teachers directly ask students comprehension questions or ask students to demonstrate understanding.

Unit Breakdown

The unit is designed to develop students' mathematical thinking in the areas of geometry and measurement. The unit is divided into two main segments: *Time to Shape Up* and *Living on the Edge*. The first section focuses on attributes of shapes and their interrelationships. The second section ties together the measurement and geometry standards by having students perform one- and two-dimensional measurements. At the end of the unit, there are two options for unit projects. These provide an opportunity for students to apply and demonstrate their understanding of geometry and measurement. The first project involves a scavenger hunt where students go out and use their understanding of geometry to find out how concepts exist in the world. In the second project, students create a "Shapely Living Room." They are asked to design and measure furniture for a miniature model of a room using the shapes they have studied in the unit. In addition, they apply measurement concepts of perimeter and area as they tile the floor and apply a border to the edge of the room. Teachers can choose to do one or both projects depending on timing and the needs of students. The following is a list of the topical sections throughout the unit that lead up to this culminating project. Each highlights a different focus within the unit:

- **Creating** allows students to apply their individual creativity to mathematics. In Lesson 1, students create their own shape puppets. They are asked to create quadrilateral stories in Lesson 3. Students also create drawings such as houses with no right angles and non-congruent humans. Empowering students as creators allows the application of important mathematical concepts to meaningful products. The unit project capitalizes on this creativity by placing students in the roles of interior designers.
- **Estimating** is an essential skill emphasized throughout this unit. Students are asked to make estimates either because an exact answer is not possible or because an exact answer is not necessary. This skill allows students to become better judges of the sensibility of their responses to problems of mathematics. In the unit project, this skill is emphasized as students calculate the area of a room that requires tiling.
- **Evaluating** the responses and creative work of classmates allows students to reflect on their own work and provides opportunities for them to agree and disagree on mathematical ideas and discuss common misconceptions. The Conclude section of the lesson often provides an opportunity for such discussions to occur. Students are asked to share the work they have done and receive feedback from other students and the teacher. Allowing students opportunities to evaluate one another fosters conceptual understanding of ideas that may have been previously unknown or unclear.
- **Comparing and contrasting** is emphasized as students learn to categorize and classify shapes from general to specific. Students gain understanding of the importance of examples and non-examples in geometry as they explore its interrelationships with the world. In addition to comparing and contrasting the attributes of different shapes, students are also asked to compare and contrast methodologies in problem solving. In Lesson 10, students create their own ways of making circles using everyday items. This lesson is designed for a compare/contrast discussion as students evaluate the different ways groups approach the challenging task. As students evaluate methodologies, teachers should look for opportunities to explore these similarities and differences to help students make decisions about what makes sense to them mathematically.

Student Mathematician Journal

The Student Mathematician Journal (SMJ) allows students to experience thinking and working like young mathematicians. The SMJ consists of all the Student Pages to promote reflection and practice with “big mathematical ideas.” The activities included in the Student Pages are designed to challenge the talents and abilities of all students in academically diverse classrooms.

Throughout the manual, teachers are guided in how to use the Student Pages. The teachers' manual includes a duplicate set of Student Pages. You will note an inset indicating the corresponding page numbers for the Student Mathematician Journal.

Periodically, we have included Check Up Pages that serve as review and practice.

Communication by Questioning

In his 1998 work, Wood categorized teachers' patterns of questioning as either "funneling" or "focusing." The major difference between the two types of questioning lies in who is responsible for the mathematical thinking. In funneling, this duty is the responsibility of the teacher, where a student is guided through a series of questions built around how the teacher would solve the problem. Focusing, on the other hand, is a type of questioning that enables the students to justify responses based on their own thinking. The geometry unit provides teachers with "focusing" questions designed to elicit multiple responses from students to promote productive whole class discussions. Since many problems throughout the unit lend themselves to multiple solution strategies, this type of questioning accommodates students' different learning styles and allows students to publicize ideas for consideration and evaluation by peers.

Differentiating Instruction

Lessons lend themselves to different types of differentiation. For example, Lesson 1 contains information for grouping students with similar background knowledge about shapes. It is essential that students who are well-versed in the attributes and names of common two-dimensional shapes be offered opportunities to explore unfamiliar shapes and apply their prior knowledge in meaningful ways to new mathematics. Students with less prior knowledge on the topic work with more traditional shapes. All students are working on identifying, classifying, and comparing and contrasting as they create and discuss different two-dimensional shapes. Many lessons throughout the unit have been differentiated by grouping to account for differences in prior knowledge.

Other lessons have tiered Student Pages. These lessons provide similar Student Pages with slight scaffolding adjustments for students with less prior knowledge. For example, Lesson 2 is designed to provide students with an understanding of right angles and the ability to identify them in the world around them. Students who have worked previously with right angles and have already grasped the concepts, are asked to take this task to the next level and estimate the measurements of different acute, obtuse, and right angles. Teachers can assess students' levels of prior knowledge very informally with a brief discussion about right angles and their purpose in the world.

The aforementioned focal point of the unit that emphasizes “creating” provides an additional type of differentiation. Since students are offered opportunities to explore their creative sides in mathematics, they are able to differentiate the products they create in this unit according to their personal interests. As students write stories, design borders, and develop their own rooms, they are adding personal touches to their products that distinguish their creations from those of their classmates. These opportunities allow students to bring in knowledge and skills of other disciplines and capitalize on their strengths, while applying their new and prior knowledge of geometry and measurement.

Implementing the Pretest and Posttest

The pretest and posttest for this unit are the same document with the same questions—in this manual they are simply referred to as the Unit Test. This test measures students’ abilities to identify and classify shapes, create congruent figures, measure segments to the nearest quarter inch, and apply the concepts of perimeter and area to geometric figures. The Unit Test should be utilized as a pretest to determine student readiness for the investigations of the unit. The research team will collect pretest and posttest data from each student for analysis purposes. Students’ performances on the pretest should be used as one indicator of their readiness for different tasks and grouping levels throughout the differentiated lessons in this unit. Teachers should also acknowledge that levels of knowledge related to specific concepts change for students as the unit progresses, and formal and informal assessments should be used throughout the unit as additional indicators of student placements and task selections. Using the same assessment tool as both a pretest and posttest allows the research team to assess the impact of the unit content and allows the teacher to determine individual student growth.

Lessons Tiered by Mathematician Name

Throughout the unit are tiered activities or tiered groups, with different levels of challenge. We used the names of famous mathematicians who made significant contributions to the field. The selected groups and their descriptions are below, followed by brief mathematician biographies. You should share these biographies with the students or have the students research the mathematicians. For additional information about and activities related to these famous mathematicians, refer to the Resources section of your CD-ROM.

- **Pythagoras—**Least challenging of the 3 levels. Designed for students with little to no prior knowledge of the topic.
- **Hypatia—**Medium level of challenge. Designed for students with some understanding of the topic.
- **Euclid—**Most challenging. Designed for students who already grasp the topic and are ready for more challenging applications with less scaffolding.

Famous Mathematicians

Pythagoras [pronounced py-THAG-or-us]
Pythagoras was a Greek mathematician. He is known as the “father of numbers.” Pythagoras loved thinking and came up with many ideas, or theories. One of his most famous theories is about the sides of a right triangle. His theory still proves true today, over 2500 years later! Pythagoras’ father was a merchant who sailed all over and sold goods. He was a good man who brought food to many starving people. Pythagoras traveled everywhere with his father. On his travels, Pythagoras learned many different things from the many scholars he met everywhere he went. He learned geometry and fell in love with it. When he was older, Pythagoras went to Egypt just to study mathematics and astronomy. At the time, Egypt was where the experts were on these subjects. War broke out in Egypt and Pythagoras was taken prisoner. After three years, he was released, and he returned to the little island in Greece where he was born. The Greeks did not like his mathematics ideas, so Pythagoras went to Italy, where he founded a school and became very popular. People followed Pythagoras and wanted to live like him and be like him. Pythagoras taught that all things in the world were connected to numbers, and his ideas live on to this day.

Hypatia [pronounced hy-PAY-shuh]
Hypatia of Alexandria is known as the first female mathematician. She had a great passion for knowledge. She was born in Greece 2300 years ago. Her father was one of the most educated men in Egypt and a professor at the university in Alexandria. He helped her travel all over to study mathematics, and he taught her everything he knew. Because of her knowledge, she was looked up to by powerful and intelligent men and women alike. Hypatia became the head of one of the biggest schools in Alexandria. She invented a tool that measured the thickness of liquids. Hypatia also drew charts that showed all of the stars in the sky. Because her ideas survived many dark periods in history, she is one of the reasons we have mathematics today.

Euclid [pronounced YOO-klid]
Euclid is known as the “father of geometry.” He was born in Alexandria, Egypt, and he went to school in Athens, Greece. He wrote the most popular textbook for mathematics that was used for over 2000 years. Even Abraham Lincoln studied from Euclid’s textbook. One of Abraham Lincoln’s most famous speeches was inspired by Euclid. Albert Einstein also looked up to Euclid. Euclid is one of the leading mathematics teachers of all time. He created one of the biggest mathematics schools in the world in Alexandria. Some of his ideas and inventions are about line segments, circles, and right angles.

References

Baum, S. M. (1985). *Learning disabled students with superior cognitive abilities: A validation study of descriptive behaviors* (Unpublished doctoral dissertation). University of Connecticut, Storrs.

Baum, S. M. (1988). An enrichment program for gifted learning disabled students. *Gifted Child Quarterly*, *32*, 226-230.

Burns, D. E. (1987). *The effectiveness of group training activities on students' creative productivity* (Unpublished doctoral dissertation). University of Connecticut, Storrs.

Cogan, L. S., Schmidt, W. S., & Wiley, D. E. (2001). Who takes what math and in which track? Using TIMSS to characterize U.S. students' eighth-grade mathematics learning opportunities. *Educational Evaluation and Policy Analysis, 23*(4), 323-341.

Delcourt, M. A. B. (1988). *Characteristics related to high levels of creative productive behavior in secondary school students: A multi-case study* (Unpublished doctoral dissertation). University of Connecticut, Storrs.

Hiebert, J., Gallimore, R., Garnier, H., Givvin, K. B., Hollingsworth, H., Jacobs, J., et al. (2003). *Teaching mathematics in seven countries: Results from the TIMSS 1999 video study,* (NCES 2003-013 Revised). U.S. Department of Education, National Center for Education Statistics: Washington, DC.

Gubbins, E. J. (1982). *Revolving door identification model: Characteristics of talent pool students* (Unpublished doctoral dissertation). University of Connecticut, Storrs.

Imbeau, M. B. (1991). *Teacher's attitudes toward curriculum compacting: A comparison of different inservice strategies* (Unpublished doctoral dissertation). University of Connecticut, Storrs.

Kaplan, S. (1998). *Project Curriculum T.W.O. Progress Status: Project Summary (II)*. Javits Grant R206A7006. Washington, DC: U.S. Department of Education.

Merriam-Webster. (n.d.). *Geometry.* Retrieved from http://www.merriam-webster.com/dictionary/geometry

National Mathematics Advisory Panel. (2008). *Foundations for success: The final report of the National Mathematics Advisory Panel.* Washington, DC: U.S. Department of Education.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics.* Reston, VA: Author.

Olenchak, F. R., & Renzulli, J. S. (1989). The effectiveness of the schoolwide enrichment model on selected aspects of elementary school change. *Gifted Child Quarterly*, *33*, 36-46.

Reis, S. M. (1981). *An analysis of the productivity of gifted students participating in programs using the revolving door identification model* (Unpublished doctoral dissertation). University of Connecticut, Storrs.

Reis, S. M., & Renzulli, J. S. (1992). Using curriculum compacting to challenge the above-average. *Educational Leadership, 50*(2), 51-57.

Reis, S. M., & Renzulli, J. S. (2003). Research related to the schoolwide enrichment triad model. *Gifted Education International*, *18*(1), 15-39.

Renzulli, J. S., & Reis, S. M. (1985). *The schoolwide enrichment model: A comprehensive plan for educational excellence*. Mansfield Center, CT: Creative Learning Press.

Renzulli, J. S., & Reis, S. M. (1997). *The schoolwide enrichment model: A comprehensive plan for educational excellence* (2nd ed.). Mansfield Center, CT: Creative Learning Press.

Reys, B. J., Reys, R. E., & Chavez, O. (2004). Why mathematics textbooks matter. *Educational Leadership, 61*(5), 61-66.

Schack, G. D. (1986). *Creative productivity and self-efficacy in children* (Unpublished doctoral dissertation). University of Connecticut, Storrs.

Starko, A. J. (1986). *The effects of the revolving door identification model on creative productivity and self-efficacy* (Unpublished doctoral dissertation). University of Connecticut, Storrs.

Tomlinson, C. A. (1996). Good teaching for one and all: Does gifted education have an instructional identity? *Journal for the Education of the Gifted*, *20*(2), 155-174.

Tomlinson, C. A. (2001). *How to differentiate instruction in mixed-ability classrooms* (2nd ed.). Alexandria, VA: Association for Supervision and Curriculum Development.

Tomlinson, C. A. (2005). Quality curriculum and instruction for highly able students. *Theory Into Practice*, *44*(2), 160-166.

Torrance, E. P. (1979). An instructional model for enhancing incubation. *The Journal of Creative Behavior, 13*(1), 23-35.

Wallas, G. (1926). *The art of thought.* London, England: Watts.

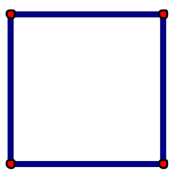
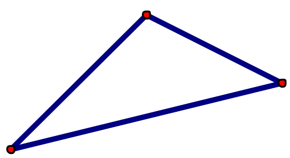
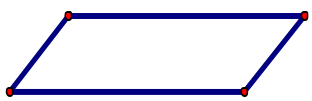
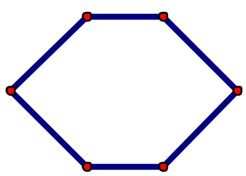
Wood, T. (1998) Alternative patterns of communication in mathematics classes: Funneling or focusing? In H. Steinbring, M. G. B. Bussi, and A. Sierpinska (Eds.), *Language and Communication in the Mathematics Classroom* (pp. 167-178). Reston, VA: National Council for Teachers of Mathematics.

Zahorik, J. A. (1970). The effect of planning on teaching. *The Elementary School Journal, 71*(3), 143-151.

Name: ______________________________ Date: ____________________

Shaping Up and Getting Around!

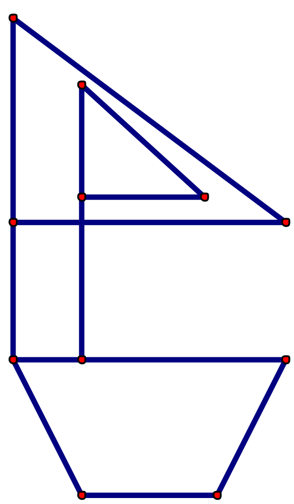
1. Complete the table below.

Shape	Name of Shape	Number of Sides	Number of Angles	Number of Right Angles
				
				
				
				
	Octagon			
		5		3

2. Measure each line segment in inches.

Line Segment	Measurement in inches
1. 	
2. 	
3. 	
4. 	

3. Using only a ruler and a pencil, draw a boat that is exactly like the boat in the picture.



Explain how you made your boat look exactly the same. Are there any parts that might not be “exact”? Why or why not?

4. Use the following information to help you find the perimeter and area of each figure.

    - Each box is 1 unit long.
    - Each box is 1 square unit.
    - Perimeter is the distance around a figure measured in units.
    - Area is the number of square units in a figure.

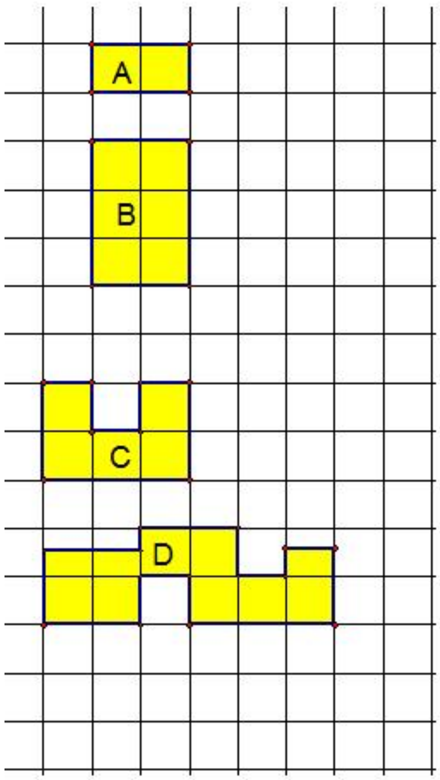


Figure A Perimeter: ______________

Figure A Area: __________________

Figure B Perimeter: ______________

Figure B Area: __________________

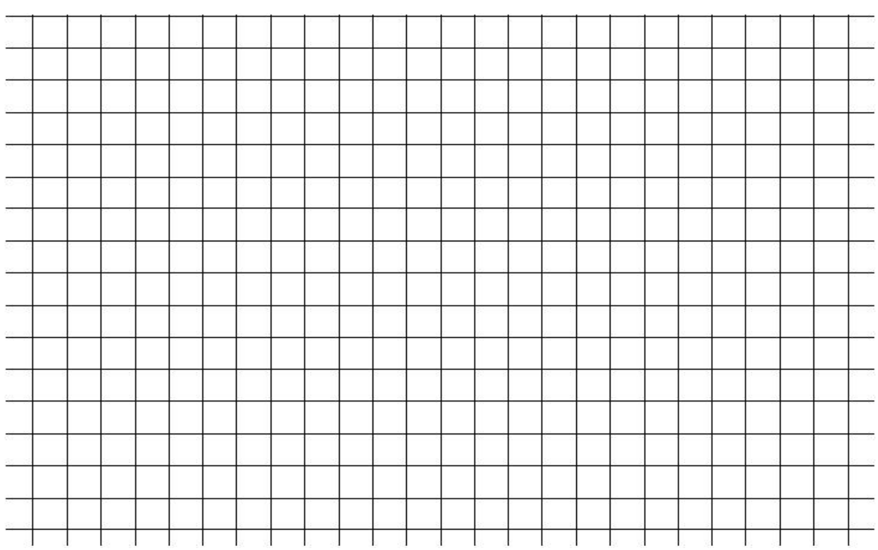
Figure C Perimeter: ______________

Figure C Area: __________________

Figure D Perimeter: ______________

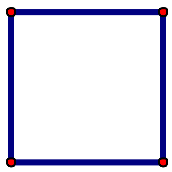
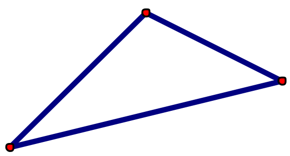
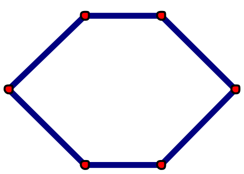
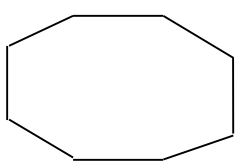
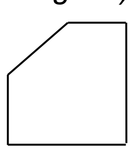
Figure D Area: __________________

5. Draw two objects that have the same perimeter but different areas.



Shaping Up and Getting Around!
ANSWER KEY

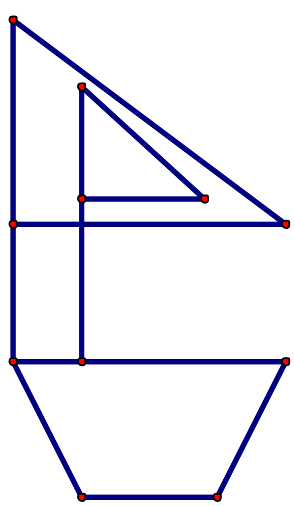
1. Complete the table below.

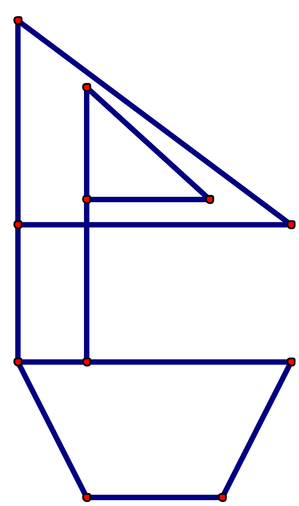
Shape	Name of Shape	Number of Sides	Number of Angles	Number of Right Angles
	*Any one of the following answers: Quadrilateral, Square, Rectangle, Parallelogram, Rhombus*	*4*	*4*	*4*
	*Triangle*	*3*	*3*	*0*
	*Any one of the following answers: Parallelogram, Quadrilateral*	*4*	*4*	*0*
	*Hexagon*	*6*	*6*	*0*
	Octagon	*8*	*8*	*This can be from 0 to 6 depending on drawing.*
(Drawing must have 3 right angles.) 	*Pentagon*	5	5	3

2. Measure each line segment in inches and centimeters.

Line Segment	Measurement in inches
1. 	*2 inches*
2. 	*1 inch*
3. 	*2 1/2 inches*
4. 	*1/2 inch*

3. Using only a ruler and a pencil, draw a boat that is exactly like the boat in the picture.





Explain how you made your boat look exactly the same. Are there any parts that might not be "exact"? Why or why not?

I measured each line segment in the first picture and drew the corresponding length in my picture. The angles may not be exact because there was no measurement tool used to ensure accuracy.

4. Use the following information to help you find the perimeter and area of each figure.

   - Each box is 1 unit long.
   - Each box is 1 square unit.
   - Perimeter is the distance around a figure measured in units.
   - Area is the number of square units in a figure.

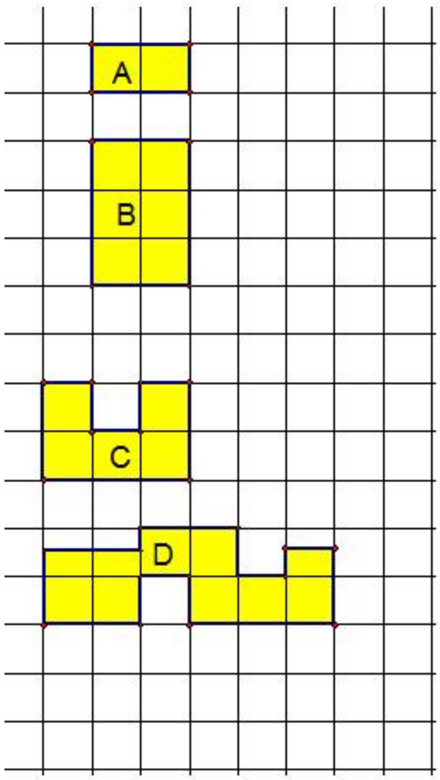


Figure A Perimeter: *6 units*

Figure A Area: *2 square units*

Figure B Perimeter: *10 units*

Figure B Area: *6 square units*

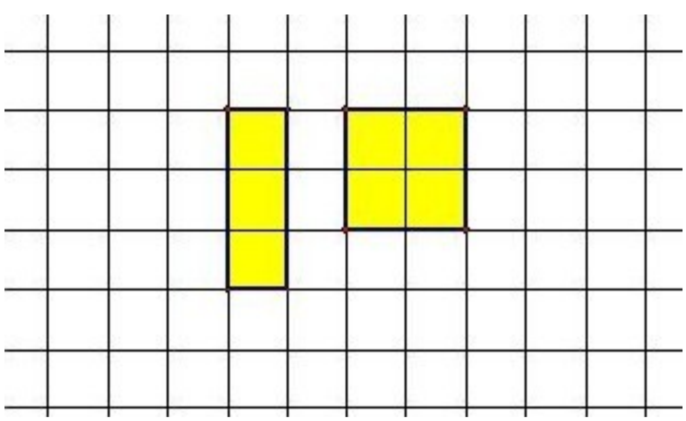
Figure C Perimeter: *12 units*

Figure C Area: *5 square units*

Figure D Perimeter: *Range 19-19 1/2 units*

Figure D Area: *Range 8 1/4-9 square units, approx. 8 3/4 square units*

5. Draw two objects that have the same perimeter but different areas.



Sample Drawing

Unit Test Scoring

Q1—Shapes and their parts
Score each correct answer 1/2 point. The total number of points possible for this question is 11 1/2.

Q2—Measurement: Whole and half inches
Score each correct answer 1 point. The total number of points possible for this question is 4.

Q3—Creating congruent shapes

Score	Description
4	Student has accurately measured and created equal segments in drawing, approximately equal angles, and has given an explanation to support the method used and possible discrepancies in angles or segments.
3	Student's drawing has approximately equal segments and explains the use of the ruler but fails to mention the angles.
2	Student has a fairly accurate drawing but is unable to provide an explanation OR student has given an accurate explanation but has major discrepancies in his/her drawing.
1	Student's drawing and explanations are completely flawed.
0	Student has not responded to either part of the question.

Q4—Determining perimeter and area
Score each correct answer 1 point. The total number of points possible for this question is 8.

Q5—Applying knowledge of perimeter and area

3	Student's figures have the same perimeter but different area.
2	Student's figures are close in perimeter with different areas OR have same perimeter and same area OR have same area but different perimeter.
1	Student's figures are exactly the same or show that the student did not understand any part of the question.
0	Student has not responded to the question or draws only one figure.

Teacher ____________ School ____________ City ____________ State ______ Date ____________

Please indicate the average length of time (in minutes) students needed to complete the test: ☐ 30 or less ☐ 40 ☐ 50 ☐ 60

☐ Pretest ☐ Posttest

Geometry and Measurement Grouping Guide

Name (Last Name, First Name)	**Q1 (11.5 pts)** Shapes and their parts				**Q2 (4 pts)** Measurement Whole and half inches				**Q3 (4 pts)** Creating congruent shapes				**Q4 (8 pts)** Determining perimeter and area				**Q5 (3 pts)** Applying knowledge of perimeter and area			
	score	L	M	H	score	L	M	H	score	L	M	H	score	L	M	H	score	L	M	H
1																				
2																				
3																				
4																				
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9																				
10																				
11																				
12																				
13																				
14																				
15																				

Teacher ____________ School ____________ City ____________ State ______ Date ____________

Please indicate the average length of time (in minutes) students needed to complete the test: ☐ 30 or less ☐ 40 ☐ 50 ☐ 60

☐ Pretest ☐ Posttest

Geometry and Measurement Grouping Guide

Name (Last Name, First Name)	**Q1 (11.5 pts)** Shapes and their parts				**Q2 (4 pts)** Measurement Whole and half inches				**Q3 (4 pts)** Creating congruent shapes				**Q4 (8 pts)** Determining perimeter and area				**Q5 (3 pts)** Applying knowledge of perimeter and area			
	score	L	M	H	score	L	M	H	score	L	M	H	score	L	M	H	score	L	M	H
16																				
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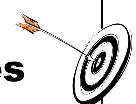
30																				

TIME TO SHAPE UP—
CLASSIFYING TWO-DIMENSIONAL SHAPES

LESSON 1

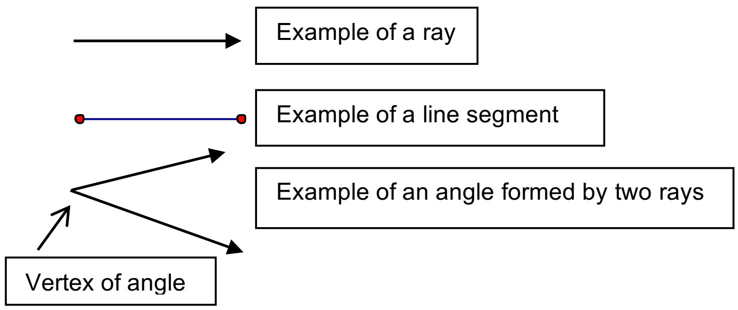
Big Mathematical Ideas

Objects have different shapes. Some are round, some are rectangular, some are square, and some are much more irregularly shaped and have no conventional name. Understanding similarities and differences in naming and describing shapes is essential for identifying objects in the world around us.

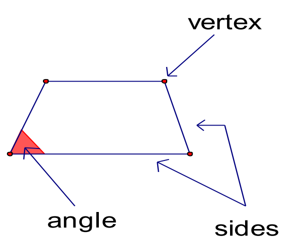
Lesson Objectives 	• Students will identify properties of given shapes. • Students will describe the similarities and differences among two-dimensional shapes.
Materials 	• Student Page—*Comparing Shapes* [SMJ page 1] • Student Page—*How Many Sides?* [SMJ page 3] • Popsicle sticks • Construction paper • Rulers • Scissors • Glue • Other art supplies if available

Mathematical Language

- **Line Segment:** Part of a line with two endpoints.
- **Side:** Any line segment that forms a two-dimensional polygon.
- **Sides:** The line segments that make up a polygon.
- **Angle:** A figure that is formed by two sides of a shape with a common endpoint.
- **Ray:** Part of a line with one endpoint that goes on forever in one direction. (The sun's rays begin at the sun and go on in one direction.)



- **Vertex (vertices)**: The point where the rays of an angle meet. The point(s) where the sides of a shape meet.



Lesson Preview

Students explore two-dimensional shapes by constructing "shape puppets." They compare the similarities and differences of shapes using the Mathematical Language. **(See Appendices A and B for information on math written communication and talk moves.)**

Initiate

1. **How many shapes can you name?**
Ask students to name as many shapes as they can. Invite students to the board to draw these shapes and describe their characteristics. Students' knowledge of quadrilaterals may be limited to "square" and "rectangle." Some might think all are squares. Ask students to name or to draw other four-sided figures.

Students may believe that triangles have to look a certain way. Ask students to draw triangles that “look different” from one another. Use this initiation as an informal assessment for deciding which groups students will be in during the investigation.

Investigate

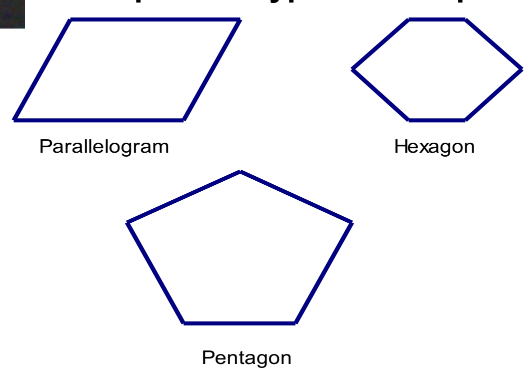
2. **Making shape puppets**

Students who were able to think beyond square and rectangle and exhibited knowledge of the shape’s parts in the initiation should be assigned shapes from Hypatia’s Shapes. Students whose knowledge or participation was limited during the initiation should be assigned shapes from Pythagoras’ Shapes. Assign each student a shape from the lists below, then group students by common shape (e.g., all of the parallelograms in one group). Provide each group materials for creating shape puppets. Ask each student to draw and cut out one shape according to his or her assignment. Students can create characters by drawing faces on their puppets. Challenge students by asking them to make their puppets different sizes and, if possible, different shapes (i.e., obtuse and acute triangles) than the other members of their group.

Pythagoras’ Shapes: Square Dancers, Rectangle Squad, Triangle Club

Hypatia’s Shapes: Parallelogram Posse, Pentagon People, Hexagon Herd

Examples of Hypatia’s Shapes:



While students construct their puppets, make a table on the board or overhead with columns labeled *Name of Shape, Number of Sides, Number of Angles,* and *Number of Vertices*. This will be used in the next activity.

3. Distinguishing characteristics

Begin by discussing the Mathematical Language. For each of the terms listed, discuss the meaning of the term and have students identify that part on their shape puppets and count how many of each they have. For example, give students the definition of **side** then ask each group to report its shape's name and number of sides. Record students' responses on the board or overhead in the table you created.

4. Game: I'm similar because…I'm different because…

Instruct students from two different shape groups to stand up and give one similarity between the two shapes and one difference. Encourage students to use phrases like "Both have fewer than 5 sides" in describing similarities. Some examples of pairs are given below.

a. Square and Parallelogram
b. Hexagon and Pentagon
c. Rectangle and Parallelogram
d. Triangle and Hexagon
e. Pentagon and Square
f. Rectangle and Square
g. Pentagon and Triangle

> Does anyone notice the right angles?

Conclude

5. A shape that you're not

Direct students to choose a shape other than their own and to report the number of sides, angles, and vertices for that shape. Then, extend this task by drawing an octagon and asking students to report the number of sides, angles, and vertices.

Challenge mathematically talented students by having them learn about a shape's diagonals. They can independently examine the diagonals of a quadrilateral, pentagon, and hexagon to look for a pattern.

Look Ahead

6. Similarities of the square and the rectangle

In the next lesson, students explore the right angle in greater detail. Point out that the square and rectangle each have four right angles. Tell students that they will be asked to give examples of where they see right angles in the next lesson. Encourage them to look for right angles on the bus ride home, in their houses, and when they go out.

Assess

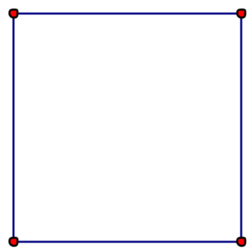
7. **Comparing shapes**
Students complete the *Comparing Shapes* Student Page **[SMJ page 1]**.

8. **Extension to number and operation**
Students complete the *How Many Sides?* Student Page **[SMJ page 3]**. This assignment focuses on using shapes as visual models to write and solve multiplication problems. Even students who have not learned how to multiply yet should be able to complete the assignment.

Shape Inspector: ___________________________________

Comparing Shapes

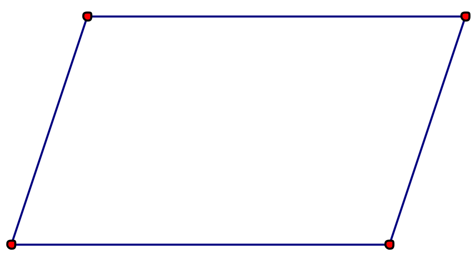
Directions: For each shape, write the number of sides, angles, and vertices.



Number of Sides: _____

Number of Angles: _____

Number of Vertices: _____



Number of Sides: _____

Number of Angles: _____

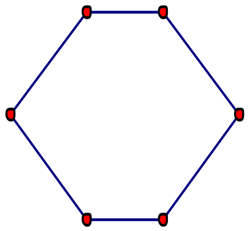
Number of Vertices: _____

Directions: Use the two shapes above to tell whether each statement below is TRUE or FALSE.

Statement	True or False?
1. Both shapes have four sides.	
2. Both shapes have four angles.	
3. Both shapes have six vertices.	
4. All sides are the same length on both shapes.	

SMJ page 1

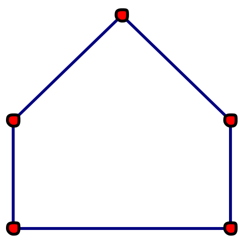
Directions: For each shape, write the number of sides, angles, and vertices.



Number of Sides: ______

Number of Angles: ______

Number of Vertices: ______



Number of Sides: ______

Number of Angles: ______

Number of Vertices: ______

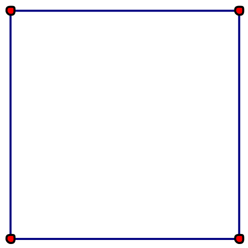
Directions: Use the two shapes above to tell whether each statement below is TRUE or FALSE.

Statement	**True or False?**
1. Both shapes have six sides.	
2. Both shapes have six angles.	
3. Both shapes have *more than* four vertices.	
4. One shape has *fewer than* six sides.	

SMJ page 2

Comparing Shapes
ANSWER KEY

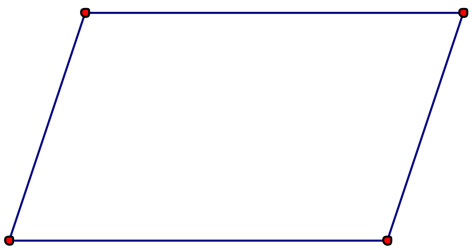
Directions: For each shape, write the number of sides, angles, and vertices.



Number of Sides: ___*4*___

Number of Angles: ___*4*___

Number of Vertices: ___*4*___



Number of Sides: ___*4*___

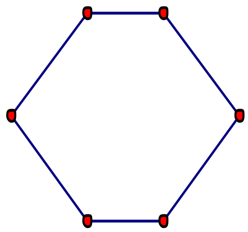
Number of Angles: ___*4*___

Number of Vertices: ___*4*___

Directions: Use the two shapes above to tell whether each statement below is TRUE or FALSE.

Statement	**True or False?**
1. Both shapes have four sides.	*TRUE*
2. Both shapes have four angles.	*TRUE*
3. Both shapes have six vertices.	*FALSE*
4. All sides are the same length on both shapes.	*FALSE*

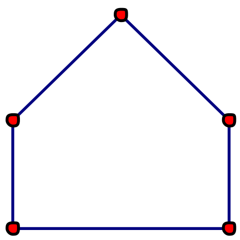
Directions: For each shape, write the number of sides, angles, and vertices.



Number of Sides: ____*6*____

Number of Angles: ____*6*____

Number of Vertices: __*6*____



Number of Sides: ____*5*____

Number of Angles: ____*5*____

Number of Vertices: __*5*____

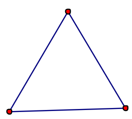
Directions: Use the two shapes above to tell whether each statement below is TRUE or FALSE.

Statement	**True or False?**
1. Both shapes have six sides.	*FALSE*
2. Both shapes have six angles.	*FALSE*
3. Both shapes have *more than* four vertices.	*TRUE*
4. One shape has *fewer than* six sides.	*TRUE*

Shape Multiplier: ________________________________

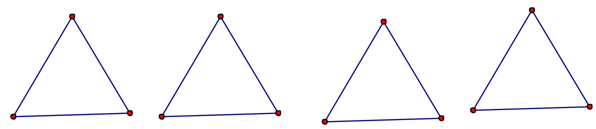
How Many Sides?

For each problem below, fill in the blanks. Then represent the problem with pictures and with a multiplication problem.

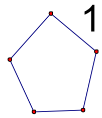


Example: This shape has 3 sides. If I drew 4 of these shapes, I would have 12 sides all together.

Represent with pictures:



Represent with multiplication: *3 sides x 4 shapes = 12 sides all together*

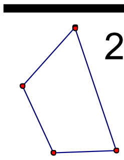


1. This shape has ______ sides. If I drew 2 of these shapes, I would have ______ sides all together.

Represent with pictures:

Represent with multiplication:

______ *sides x* ______ *shapes =* ______ *sides all together*



2. This shape has _______ sides. If I drew 8 of these shapes, I would have ______ sides all together.

Represent with pictures:

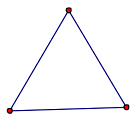
Represent with multiplication:

______ *sides x* ______ *shapes =* ______ *sides all together.*

SMJ page 3

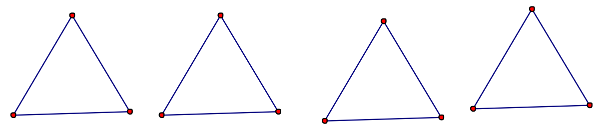
How Many Sides?
ANSWER KEY

For each problem below, fill in the blanks. Then represent the problem with pictures and with a multiplication problem.

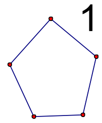


Example: This shape has 3 sides. If I drew 4 of these shapes, I would have 12 sides all together.

Represent with pictures:

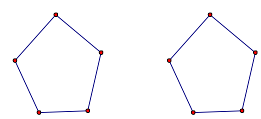


Represent with multiplication: *3 sides x 4 shapes = 12 sides all together*



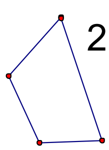
1. This shape has ___*5*___ sides. If I drew 2 of these shapes, I would have ___*10*___ sides all together.

Represent with pictures:



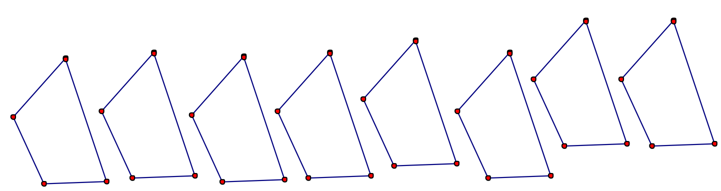
Represent with multiplication:

___*5*___ *sides x* ___*2*___ *shapes =* ___*10*___ *sides all together*



2. This shape has ___*4*___ sides. If I drew 8 of these shapes, I would have ___*32*___ sides all together.

Represent with pictures:

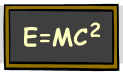


Represent with multiplication:

___*4*___ *sides x* ___*8*___ *shapes =* ___*32*___ *sides all together.*

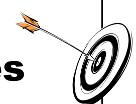
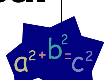
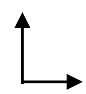
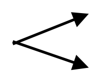
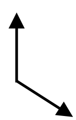
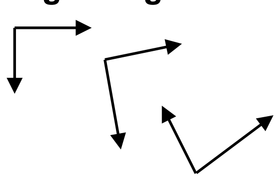
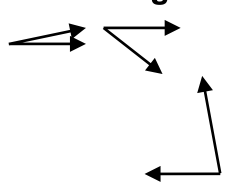
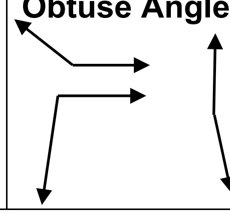
TIME TO SHAPE UP—
WHAT'S THE RIGHT ANGLE?

LESSON 2



Big Mathematical Ideas

Angles are all around us. Right angles make up many of the angles that we see when we look around. Right angles are especially important when it comes to building things like houses and bridges. Without right angles, structures could be unsafe.

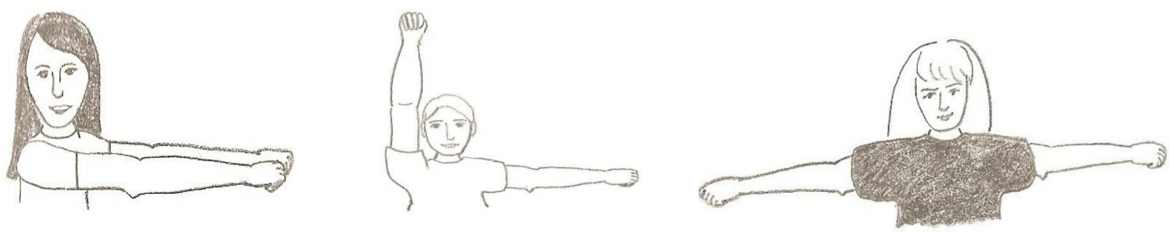
<table>
<tr><td>Lesson Objectives </td><td colspan="3">Students will be able to identify right angles in real-world contexts.
Students will be able to create triangles with and without right angles.</td></tr>
<tr><td>Materials </td><td colspan="3">Student Page—Am I Right? (Pythagoras and Euclid) [SMJ pages 5 & 11]
Student Page—Angles Eat, Too! [SMJ page 17]
Bendy straws
Patty paper 4 3/4 x 5 (available from Eastern Paper Bag Co., buy.easternbag.com)</td></tr>
<tr><td>Mathematical Language </td><td colspan="3">Right Angle: An angle that is 90°. 
Acute Angle: An angle that is greater than 0° but less than 90°. 
Obtuse Angle: An angle that is greater than 90° but less than 180°. </td></tr>
<tr><td></td><td>Examples of Right Angles
</td><td>Examples of Acute Angles
</td><td>Examples of Obtuse Angles
</td></tr>
</table>

Lesson Preview

Students begin to work with angle size. They examine right angles in relation to the world around them. Students learn to classify angles as smaller than right angles (acute) or larger than right angles (obtuse).

Initiate

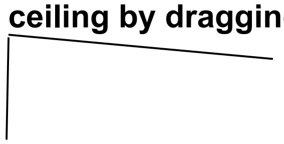
1. **The angle cheer**
Students can use their arms to demonstrate angle size and distinguish between 0° and 180°. Ask students to stand up and perform the 0-90-180 cheer using the moves below and calling out angle measures.



0° 90° 180°

Extend this activity by challenging students to use their arms to show what they think a 50° angle or a 120° might look like. Ask students to explain their thinking.

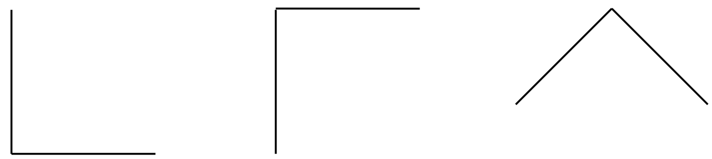
2. **Not quite right**
Pose the question:
What would happen if the corner of the classroom looked like this? (Focus students' attention to the corner where the wall meets the ceiling by dragging finger along the angle formed by the two.)



The goal of this initiation question is to help students understand that right angles are key components to the proper construction of structures. Follow-up questions might include:

- What if one leg of your chair was shorter than the others?
- The corner of the room is L shaped. Where else do you notice L-shaped corners?
(Only write down a few responses…more suggestions will be taken after you tell students it is called a right angle.)

Tell students that the angle in the corner of the room is called a "Right Angle." Right angles are shaped like the letter L. Here are some pictures of right angles:



Show students how to check if an angle is right using the corner of a paper (assuming they have a standard sheet of paper). One way to do this is to fit the corner of the paper into the corner of the board to show how neatly it fits.

Continue the list of right angles in the real world on the board. Tell students you challenge them to come up with 20 different places they see right angles in the real world.

Investigate

3. **Angles out of straws**
Give each student a straw with a bendy end.

- Tell them to bend the straw so that it is shaped like a right angle.
- Define **right angle** and draw some examples.
  - Provide a context by asking which letters of the alphabet have right angles. Examples include E, F, H, L, and T.
- Next tell them to keep bending the straw. As the sides of the straw get closer together, tell them the angle is SMALLER THAN a right angle.
- Define **acute angle** and draw some examples of varying sizes.
  - Ask which letters of the alphabet have acute angles. Examples include A, V, and M.
- Instruct students to put their straws back at right angles.
- Show students how to bend their straw the other way to create an angle bigger than a right angle.
- Define **obtuse angle** and draw some examples of varying sizes.
- Recap: Closing the straw creates angles SMALLER than right angles (acute), and opening the straw creates angles BIGGER than right angles (obtuse).

4. **Angles in our classroom**
Students work in pairs to create two lists of the different types of angles that can be seen in the classroom.
List I: Acute angles I see in the classroom.
List II: Obtuse angles I see in the classroom.

Share the lists students came up with and have them add suggestions from other groups that are not on their lists.

Conclude

5. ***Am I Right?***

Students work in pairs to complete the *Am I Right?* Student Pages **[SMJ pages 5 & 11]**. Students who were easily able to identify acute and obtuse angles in the classroom should be directed to the Euclid version. Students who struggled or did not participate in the discussion should be directed to the Pythagoras version. Information from question #1 on the pre-assessment can also help guide grouping decisions.

Non-visual learners may struggle drawing the triangles with no right angle, one right angle, and two right angles (#4, #5, & #6). Help them get started by suggesting the following:

- No right angle—Draw an acute or obtuse angle then connect.
- One right angle—Draw a right angle and then connect.
- Two right angles—Refer students to a sheet of paper and explain how the bottom has two right angles (there is no way to connect the sides and still have a three-sided figure).

Assess

6. **Collect *Am I Right?* Student Page**

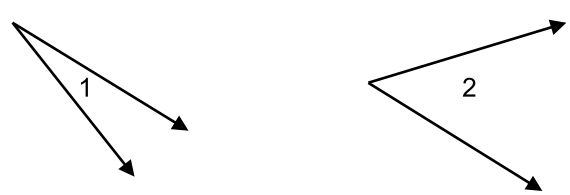
This can be used as the assessment for this section.

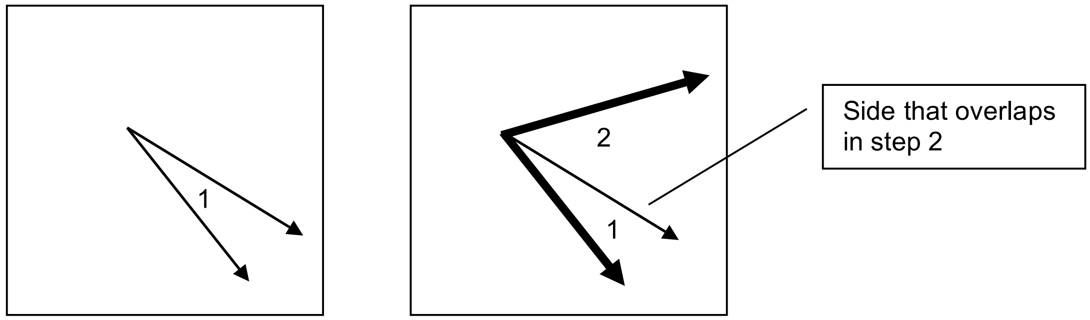
7. **Extension to number and operation**

Have students complete the *Angles Eat, Too!* Student Page **[SMJ page 17]**. This assignment focuses on using multi-digit addition in a fun, creative way. Students also are asked to put the addition problem into a silly sentence about angles.

This assignment asks students to draw the new angle that would be formed from combining two angles. This task can be done without a protractor by using patty paper or tracing paper. How to use patty paper to combine angles:

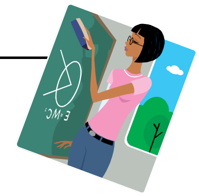
1. Place the patty paper over one of the angles on the Student Page and trace it.
2. Place the same patty paper over the second angle on the Student Page so that the vertex of the traced angle is on top of the vertex of angle 2 and one side overlaps.
3. Trace the second angle. (The resulting angle is shown in bold).





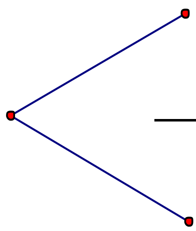
Incorporate some other grade three Mathematical Language into this activity. Ask students to turn, or rotate, the patty paper to line up the vertex and side of the traced angle with the vertex and side of the second angle.

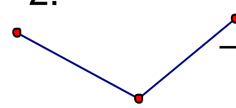
Angle Expert: ______________________________

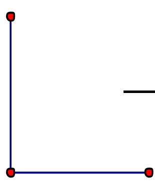


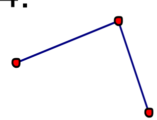
AM I RIGHT?

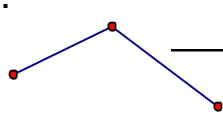
Directions: Next to each angle, write whether it is RIGHT or NOT RIGHT.

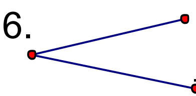
1.  ____________________

2.  ____________________

3.  ____________________

4.  ____________________

5.  ____________________

6.  ____________________

SMJ page 5

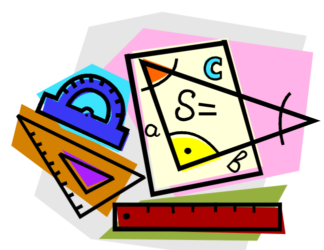
Draw each shape and circle whether or not it has right angles.

7. RECTANGLE: Right Angles? YES NO

8. SQUARE: Right Angles? YES NO

9. CIRCLE: Right Angles? YES NO

10. Draw a triangle with 1 right angle if possible, or write NOT POSSIBLE.



11. Draw a triangle with no right angles if possible, or write NOT POSSIBLE.

SMJ page 7

12. Draw a triangle with 2 right angles if possible, or write NOT POSSIBLE.

13. DESIGN TEAM TIME:

- ❖ Draw the front of a house. Do not use ANY RIGHT ANGLES in your drawing.
- ❖ Write 3 sentences about the person who lives in this house.

____

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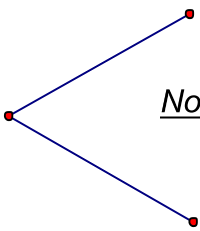
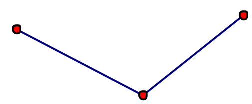
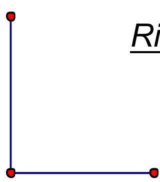
SMJ page 9

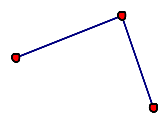
AM I RIGHT?
ANSWER KEY

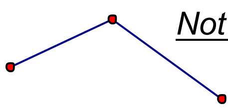
Directions: Next to each angle, write whether it is RIGHT or NOT RIGHT.

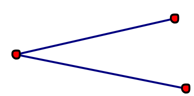
1.  *Not Right*

2.  *Not Right*

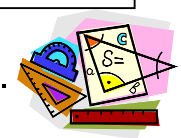
3.  *Right*

4.  *Right*

5.  *Not Right*

6.  *Not Right*

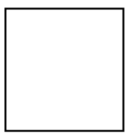
Pythagoras



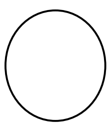
Draw each shape and circle whether or not it has right angles.

7. RECTANGLE: Right Angles? (YES) NO

8. SQUARE: Right Angles? (YES) NO

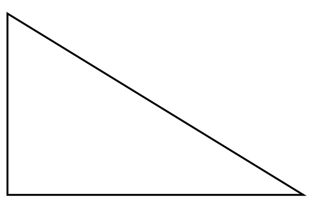


9. CIRCLE: Right Angles? YES (NO)



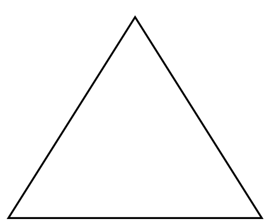
10. Draw a triangle with 1 right angle if possible, or write NOT POSSIBLE.

Drawings may vary. One example:



11. Draw a triangle with no right angles if possible, or write NOT POSSIBLE.

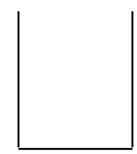
Drawings may vary. One example:



Pythagoras

12. Draw a triangle with 2 right angles if possible, or write NOT POSSIBLE.

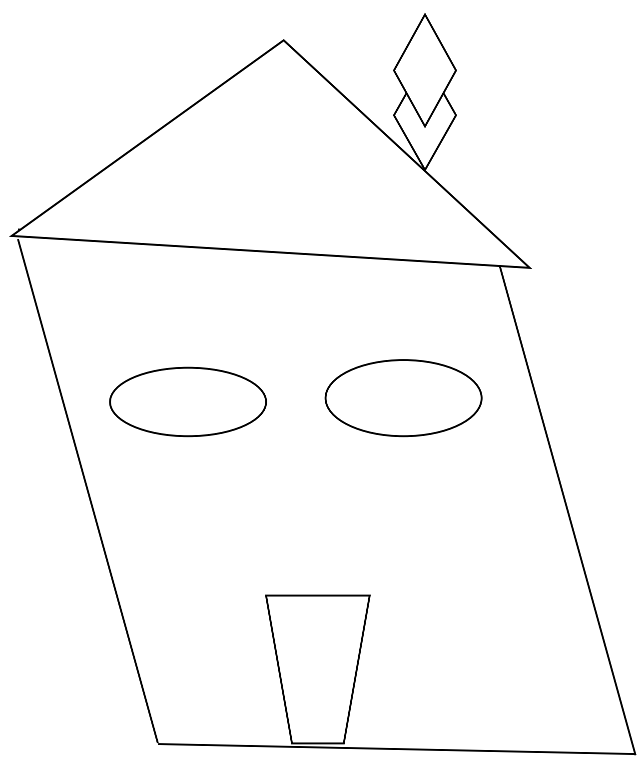
Not possible because once two right angles have been made the sides cannot be connected to make a triangle.



13. DESIGN TEAM TIME:

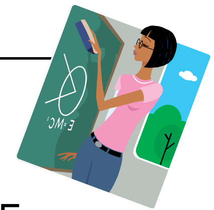
- ❖ Draw the front of a house. Do not use ANY RIGHT ANGLES in the drawing.
- ❖ Write 3 sentences about the person who lives in this house.

Sample drawing: Stories will vary.



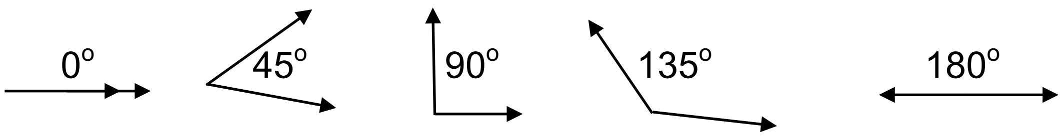
Euclid

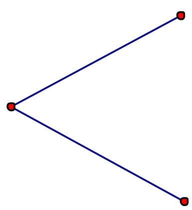
Angle Expert: ______________________________________

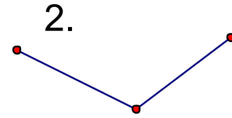


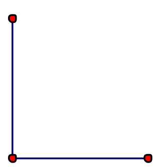
AM I RIGHT?

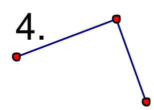
Directions: An angle measure tells how open an angle is. For example, if it is not open at all, it is 0^o. If it is open all the way, it is 180^o. The pictures below show some angles and their measures. Use these angles to estimate the measures of the angles below.

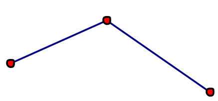


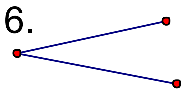
1.  ________________

2.  ________________

3.  ________________

4.  ________________

5.  ________________

6.  ________________

7. Acute angles are less than 90^o. Which angles are acute?

____

8. Obtuse angles are greater than 90^o. Which angles are obtuse?

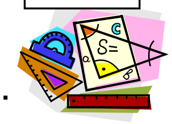
____

9. Right angles are exactly 90^o. Which angles are right?

____

SMJ page 11

Euclid



Draw each shape and circle whether or not it has right angles.

10. RECTANGLE: Right Angles? YES NO

11. SQUARE: Right Angles? YES NO

12. CIRCLE: Right Angles? YES NO

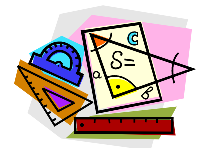
13. Draw a triangle with 1 right angle if possible, or write NOT POSSIBLE.

14 Draw a triangle with no right angles if possible, or write NOT POSSIBLE.

SMJ page 13

Euclid

15. Draw a triangle with 2 right angles if possible, or write NOT POSSIBLE.



16. DESIGN TEAM TIME:

- ❖ Draw the front of a house. Do not use ANY RIGHT ANGLES in the drawing.
- ❖ Write 3 sentences about the person who lives in this house.

____

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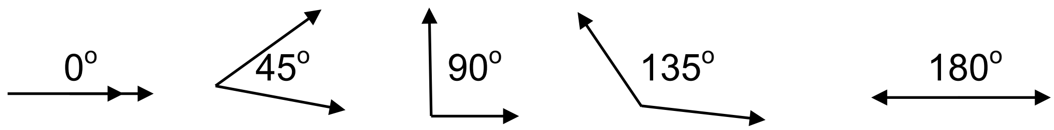
____

SMJ page 15

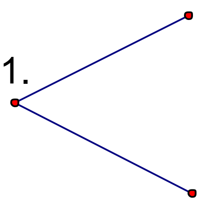
Euclid

AM I RIGHT?
ANSWER KEY

Directions: An angle measure tells how open an angle is. For example, if it is not open at all, it is 0°. If it is open all the way, it is 180°. The pictures below show some angles and their measures. Use these angles to estimate the measures of the angles below.

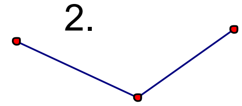


1.



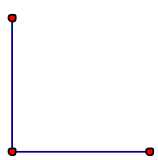
Acceptable Range: 35° - 60°

2.



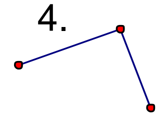
Acceptable Range: 130° - 150°

3.



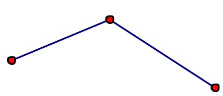
Acceptable Range: 85° - 95°

4.



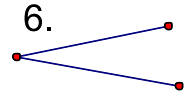
Acceptable Range: 85° - 95°

5.



Acceptable Range: 130° - 150°

6.



Acceptable Range: 20° - 45°

7. Acute angles are less than 90°. Which angles are acute?

   Angles 1 & 6 (3 & 4 possible based on estimates)

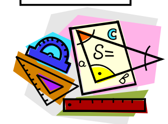
8. Obtuse angles are greater than 90°. Which angles are obtuse?

   Angles 2 & 5 (3 & 4 possible based on estimates)

9. Right angles are exactly 90°. Which angles are right?

   Angles 3 & 4 (may vary based on estimates)

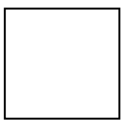
Euclid



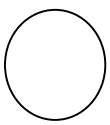
Draw each shape and circle whether or not it has right angles.

10. RECTANGLE: Right Angles? **YES** NO

11. SQUARE: Right Angles? **YES** NO

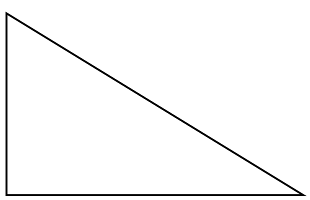


12. CIRCLE: Right Angles? YES **NO**



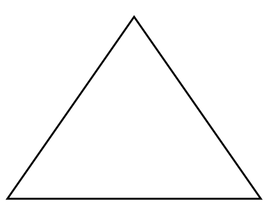
13. Draw a triangle with 1 right angle if possible or write NOT POSSIBLE.

    Drawings may vary. One example:



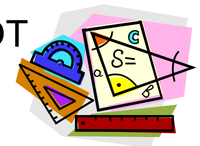
14. Draw a triangle with no right angles if possible or write NOT POSSIBLE.

    Drawings may vary. One example:

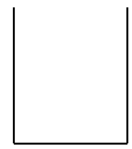


Euclid

15. Draw a triangle with 2 right angles if possible or write NOT POSSIBLE.



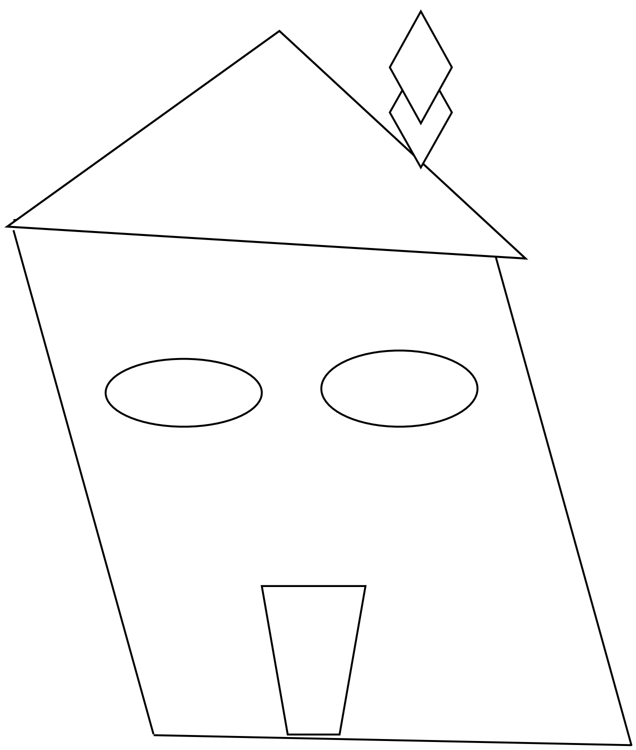
It is not possible because once two right angles have been made the sides cannot be connected to make a triangle.



16. DESIGN TEAM TIME:

- ❖ Draw the front of a house with NO RIGHT ANGLES.
- ❖ Write 3 sentences about the person who lives in this house.

Sample drawing. Stories will vary.



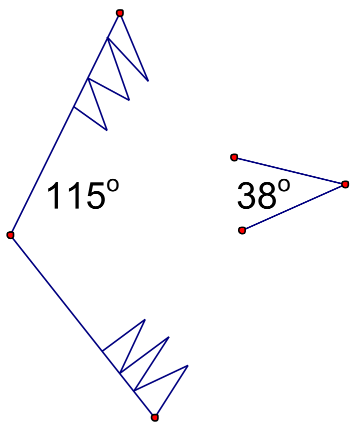
Angle Adder: ____

Angles Eat, Too!

Hungry Angle Postulate: When an angle eats another angle, it gains that many degrees. (It is similar to people eating food to grow taller.)

Use the Hungry Angle Postulate to figure out how big each angle will be after it eats the smaller angle. Write an addition problem and a sentence. Then, draw the new angle formed by combining both.

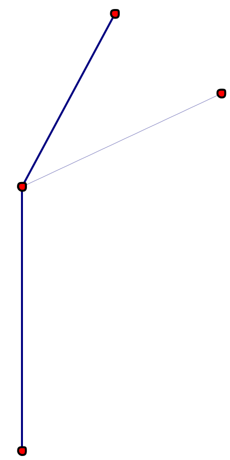
EXAMPLE:

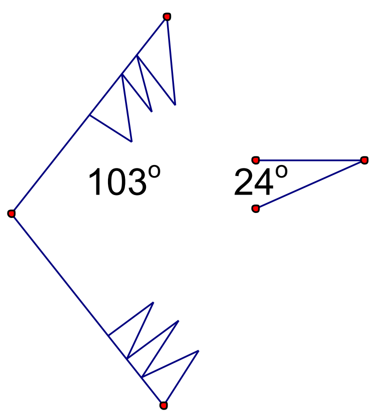


Addition Problem:

$115^\circ + 38^\circ = \underline{153^\circ}$

Sentence:
When a 115° angle eats a 38° angle, it becomes a 153° angle.



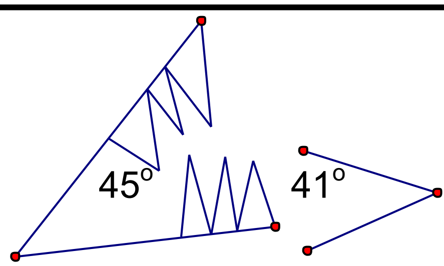


Addition Problem:

Sentence:

____

____



Addition Problem:

Sentence:

____

____

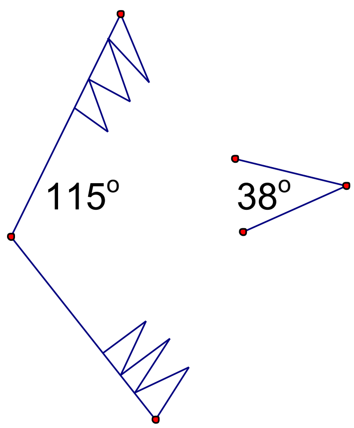
SMJ page 17

Angles Eat, Too!
ANSWER KEY

Hungry Angle Postulate: When an angle eats another angle, it gains that many degrees. (It is similar to people eating food to grow taller.)

Use the Hungry Angle Postulate to figure out how big each angle will be after it eats the smaller angle. Write an addition problem and a sentence. Then, draw the new angle formed by combining both.

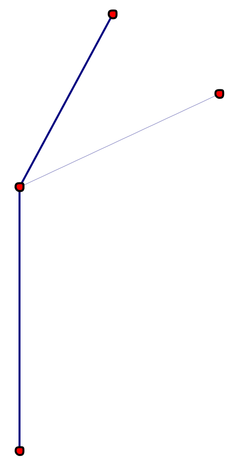
EXAMPLE:

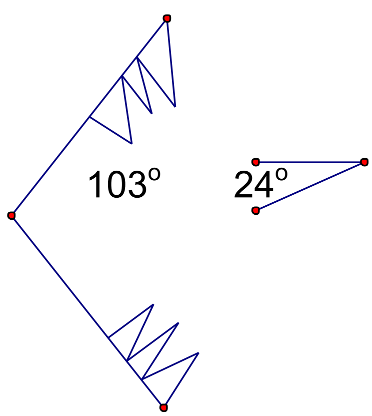


Addition Problem:

$115^\circ + 38^\circ = \underline{153^\circ}$

Sentence:
When a 115° angle eats a 38° angle, it becomes a 153° angle.

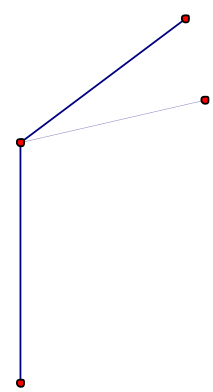


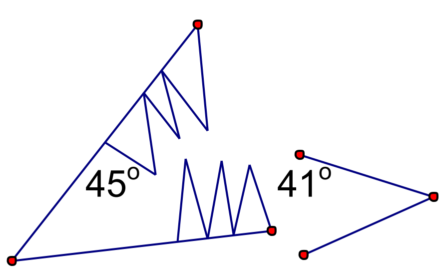


Addition Problem:

$103^\circ + 24^\circ = \underline{127^\circ}$

Sentence:
When a 103° angle eats a 24° angle, it becomes a 127° angle.

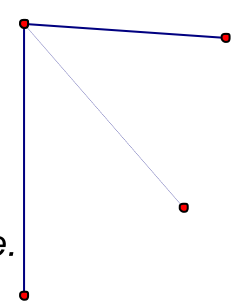




Addition Problem:

$45^\circ + 41^\circ = \underline{86^\circ}$

Sentence:
When a 45° angle eats a 41° angle, it becomes a 86° angle.



LESSON 3

TIME TO SHAPE UP—
THE GREEDY TRIANGLE

Big Mathematical Ideas

Students are familiar with classic three- and four-sided shapes, but what happens when shapes have more sides? Do they have names? Students look beyond triangles and learn the names of multi-sided polygons through children's literature.

Lesson Objectives	• Students will be able to determine a shape's name using a given number of sides. • Students will be able to relate the number of sides to the number of angles. • Students will be able to create a story that highlights different types of quadrilaterals.
Materials	• Student Page—*The Greedy Triangle* [SMJ page 19] • Student Page—*Design Your Own Shape Story* [SMJ page 23] ○ Make extra copies of the story pages for students who write longer stories. • Student Page—*The Annual Shape Party* [SMJ page 29] • Children's Book—*The Greedy Triangle* by Marilyn Burns
Mathematical Language	• **Polygon:** A closed figure formed by three or more line segments. *[Students may think of these as shapes; however, it is important to note that round shapes are not polygons.]* • **Triangle:** A polygon with three sides. • **Quadrilateral:** A polygon with four sides. • **Pentagon:** A polygon with five sides. • **Hexagon:** A polygon with six sides. • **Heptagon:** A polygon with seven sides. • **Octagon:** A polygon with eight sides. • **Nonagon:** A polygon with nine sides. • **Decagon:** A polygon with ten sides.

Lesson Preview

Students explore polygons through children's literature. They learn the pronunciations and meanings of shapes as classified by the number of sides. This lesson provides an introduction to the relationship between sides and angles of geometric figures.

Initiate

1. **Pre-assess knowledge of polygon names**
   Direct students to *The Greedy Triangle* Student Page **[SMJ page 19]**.

   Direct students to fill in (IN PENCIL) the names of the shapes that they know. Circulate during this activity to determine students' familiarity with the polygon names. This information will be helpful in determining task readiness in Lesson 4. During the story, students fill in the names of the remaining polygons and make corrections to this work.

Investigate

2. ***The Greedy Triangle***
   Read students the story *The Greedy Triangle.* As you read, direct students to fill in their tables with the name of each polygon. Each time the polygon is transformed, stop the story and write the new polygon name on the board to demonstrate correct spelling. Practice pronouncing the polygon names as a class.

3. ***The Greedy Triangle* questions**
   Direct students to complete *The Greedy Triangle* Student Page **[SMJ page 19]** questions and the *Design Your Own Shape Story* Student Page **[SMJ page 23]**.

Conclude

4. **Student stories**
   Ask students to share the stories they wrote as part of **Investigate #3**. Focus conversation on how all shapes in their stories are quadrilaterals because they have four sides, but they are different types of quadrilaterals.

Look Ahead

5. **General vs. specific polygons**
   Polygons can have general names, such as *quadrilateral*, as seen in *The Greedy Triangle,* or more specific names, such as *square* and *rectangle*. Now that students are acquainted with polygons as named by the number

of sides, they can look more at specific types of quadrilaterals in the following lesson.

Assess

6. **A quick check on polygon names**
Challenge students to remember as many of the polygon names as they can before the next class. When they come in the next day, determine how much they have learned using questions like, “What do you call a polygon with 5 sides?”

7. **Extension to number and operation**
Direct students to complete *The Annual Shape Party* Student Page **[SMJ page 29]**. This lesson focuses on the use of repeated addition for students who have not yet learned to multiply. Students who know basic multiplication tables may be able to complete the assignment more efficiently. Emphasis should be placed on the methods students use to get their answers as different solution methods are possible.

Shape Author: ____

The Greedy Triangle (Burns, 1995)

Directions: Write the names of the shapes you know and then fill in the rest as your teacher reads the story.

Name of Shape	Number of Sides	Number of Angles
	3	
	4	
	5	
	6	
	7	
	8	
	9	
	10	

SMJ page 19

Directions: Use your table and details from the story to answer each question.

1. What is a shape with 8 sides called? ________________________

2. How many sides does a pentagon have? ____________________

3. How many angles does a heptagon have? ___________________

4. What is true about the number of sides and the number of angles in a shape?

____

5. Why do you think the book is called *The Greedy Triangle*?

____

____

____

____

Burns, M. (1995). *The greedy triangle.* New York, NY: Scholastic Press.

SMJ page 21

The Greedy Triangle (Burns, 1995)
ANSWER KEY

Directions: Write the names of the shapes you know and then fill in the rest as your teacher reads the story.

Name of Shape	Number of Sides	Number of Angles
TRIANGLE	3	*3*
QUADRILATERAL	4	*4*
PENTAGON	5	*5*
HEXAGON	6	*6*
HEPTAGON	7	*7*
OCTAGON	8	*8*
NONAGON	9	*9*
DECAGON	10	*10*

Directions: Use your table and details from the story to answer each question.

1. What is a shape with 8 sides called? ______*Octagon*____________
2. How many sides does a pentagon have? _______*5 Sides*________
3. How many angles does a heptagon have? ______*7 Angles*_______
4. What is true about the number of sides and the number of angles in a shape?

   Sample answer: the number of sides and the number of angles are equal.

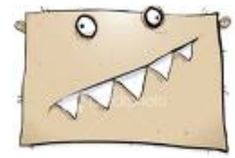
5. Why do you think the book is called *The Greedy Triangle* (Burns, 1995)*?*

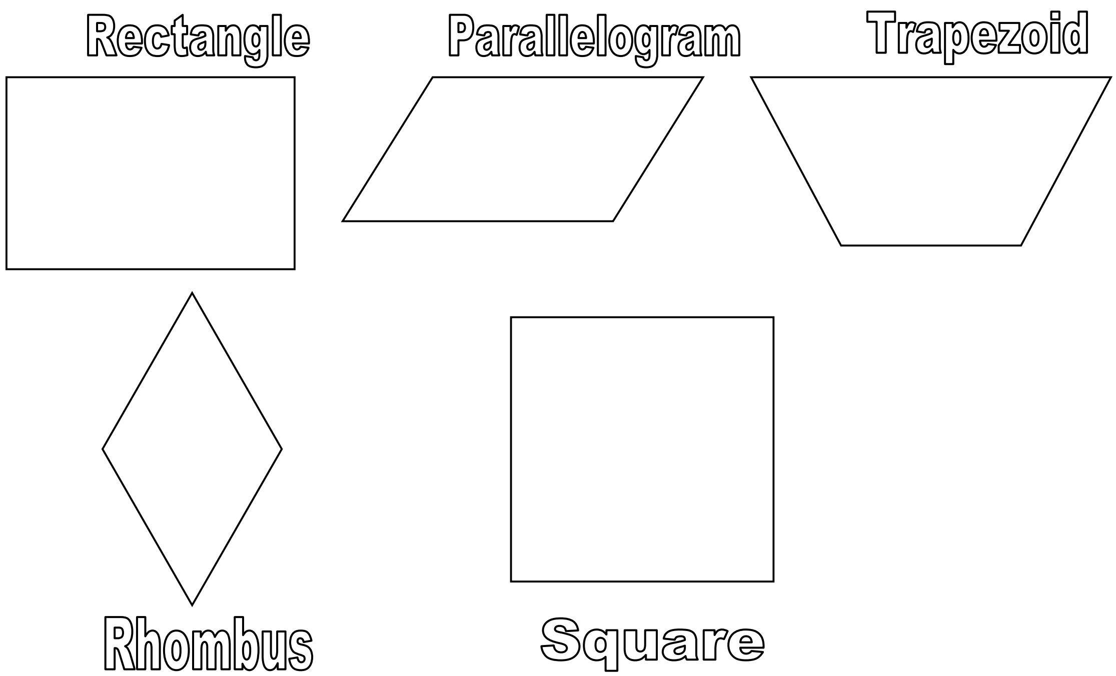
   Sample answer: the book is called the greedy triangle *because the shape always wants more sides and greedy means to want more.*

Burns, M. (1995). *The greedy triangle.* New York, NY: Scholastic Press.

Author: ____

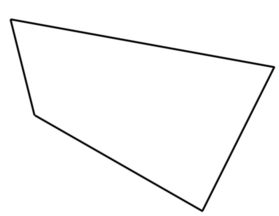
Design Your Own Shape Story





Directions:

1. Write a story about the quadrilateral in the picture below. (Don't forget to give your shape a name!)



2. In your story the quadrilateral must change into each of the 5 different quadrilaterals at the top of this page.
3. As the quadrilateral changes, tell what makes it similar to or different from its previous shape using terms like sides, angles, and vertices.
4. You may cut the pictures out to use in your story, or you may draw your own.
5. In the beginning of your story, tell why the quadrilateral is not happy and wants to change.

SMJ page 23

Use the lined pages to write your story and the blank pages for illustrations. Staple your pages together.

Shape Author: ______________________________

SMJ page 25

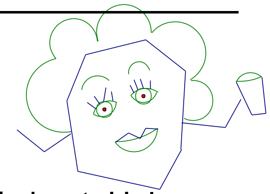
SMJ page 26

SMJ page 27

SMJ page 28

Angle Accountant: ____

The Annual Shape Party



At the annual shape party, shapes gather to have fun. This year's host, Helena Heptagon, has offered a prize to the first person who figures out how many angles are in the room. No guessing!! Helena wants to see how you got your answer.

The guests include:

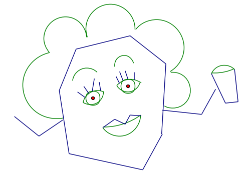
2 circles, 4 triangles, 3 quadrilaterals, 7 pentagons, 1 hexagon, 2 heptagons (including Helena), and 3 octagons

Use the space below to calculate the total number of angles in the room. You may draw pictures to help you.

There are ______ angles in the room all together.

SMJ page 29

The Annual Shape Party
ANSWER KEY

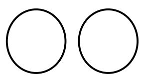


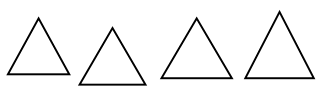
At the annual shape party, shapes gather to have fun. This year's host, Helena Heptagon, has offered a prize to the first person who figures out how many angles are in the room. No guessing!! Helena wants to see how you got your answer.

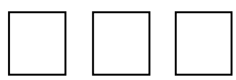
The guests include:
2 circles, 4 triangles, 3 quadrilaterals, 7 pentagons, 1 hexagon, 2 heptagons (including Helena), and 3 octagons

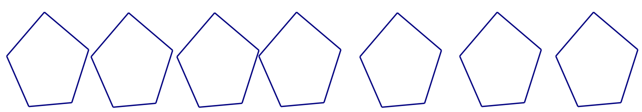
Use the space below to calculate the total number of angles in the room. You may draw pictures to help you.

Sample answer:

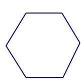
2 CIRCLES:  *0 ANGLES*

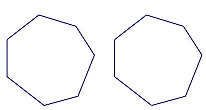
4 TRIANGLES:  *3 + 3 + 3 + 3 = 12 ANGLES*

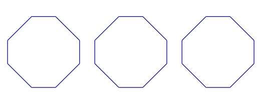
3 QUADRILATERALS:  *4 + 4 + 4 = 12 ANGLES*

7 PENTAGONS: 

5 + 5 + 5 + 5 + 5 + 5 + 5 = 35 ANGLES

1 HEXAGON:  *6 ANGLES*

2 HEPTAGONS:  *7 + 7 = 14 ANGLES*

3 OCTAGONS:  *8 + 8 + 8 = 24 ANGLES*

TOTAL ANGLES = 0 + 12 + 12 + 35 + 6 + 14 + 24 = 103 ANGLES

There are ___*103*___ angles in the room all together.

TIME TO SHAPE UP—
THE RECTANGLES ONLY CLUB!

LESSON 4

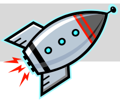
Big Mathematical Ideas

It is difficult to talk about shapes because they can be named very generally, such as *quadrilateral*, or very specifically, such as *rhombus*. One of the most difficult relationships for students to understand is that of square and rectangle. Many students believe that a rectangle must have sides of different lengths. By definition, however, rectangles have four sides and four right angles, a definition that also includes squares.

Lesson Objectives	● Students will be able to define attributes and distinguishing properties of squares, rectangles, and rhombuses.
Materials	● Student Page—*Play: The Rectangles Only Club!* [SMJ page 31] ● Student Page—*The Rectangles Only Club! (Hypatia and Euclid)* [SMJ pages 33-35] ● Student Page—*More Shapes, More Sides* [SMJ page 37] ● Check Up #1 [SMJ page 39]
Mathematical Language	✓ Revisit the definition of **quadrilateral** from previous lesson. ✓ Revisit the concept of **right angle**, especially in the context of a shape. ● **Congruent sides:** Sides of equal length. ● **Rhombus:** A quadrilateral with four sides of equal length. ● **Rectangle:** A quadrilateral with four equal angles. ● **Square:** A quadrilateral with four sides of equal length and four right angles.

Lesson Preview

Students grapple with the idea that a square is always a rectangle because it has four sides and four right angles. They discover that the converse is not true. A rectangle is only a square if it has four congruent sides.



Initiate

1. Is it more than a rectangle?

Draw a rectangle (that is not a square) on the board. Use the following questions to develop a temporary definition. Instruct students to refer to the table from the previous lesson. Do not give the “true” definition of a rectangle yet.

- What is the name of this shape? (*Quadrilateral, Rectangle*)
- Does it have any other names? (*Quadrilateral, Parallelogram*)
- How do you know it is that shape? (*Four sides, four right angles, opposite sides equal length*)
- Can you define it?
  - *Use student suggestions. Definition can be modified as students’ ideas develop throughout the lesson.*

Students’ responses during this initiation can help designate task assignments. Students who recall other names for the figure and its properties should be directed to the Euclid version of *The Rectangles Only Club* Student Page **[SMJ page 33]**. Students whose knowledge is limited to “rectangle” should work on the Hypatia version of *The Rectangles Only Club* Student Page **[SMJ page 35]**.

Investigate

2. A rectangle play

Read *The Rectangles Only Club* play with students. If students have never read a play before, go over the directions that precede the play. Students reading the roles can hold “shape puppets” from Lesson 1 to indicate whether they are playing the role of a square or a rectangle.

3. Comparing characteristics

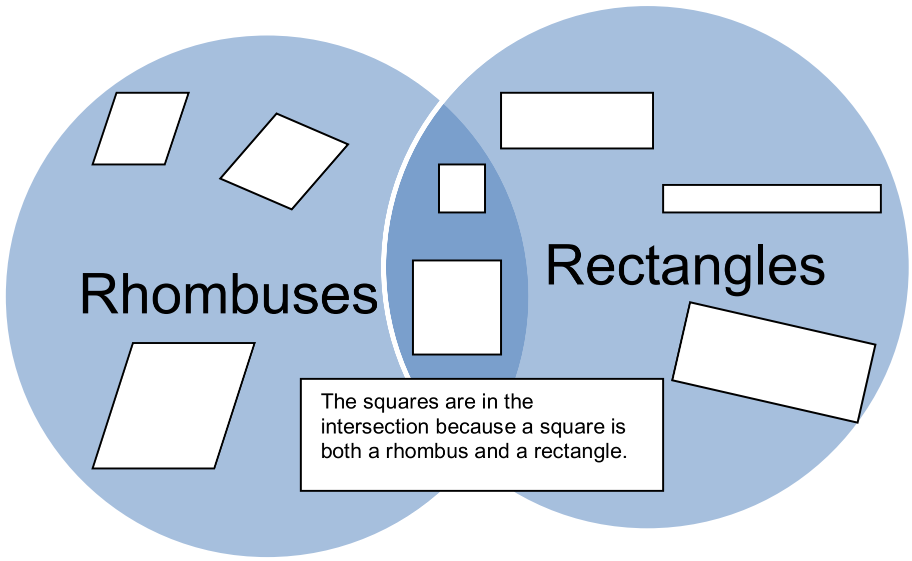
Once students have read the play, introduce the definitions of **rectangle**, **rhombus**, and **square**. Have students practice drawing rectangles and rhombuses. Be sure the definitions are posted clearly so students can refer to them as they answer the questions. Organize students into Hypatia and Euclid groups. Use student performance on the initiation and previous two lessons to designate group assignments.

- Have students answer questions on *The Rectangles Only Club* Student Pages **[SMJ pages 33-35]** in their groups.
- Discuss students’ responses by circulating around the room and talking to individual groups. Look for students to challenge each others’ thinking during this discussion and try to develop the understanding that a square is both a rhombus and a rectangle.

Conclude

4. The truth about squares

Do a final wrap-up to emphasize that squares are always rectangles and rhombuses, but that the reverse is not true. Rectangles and rhombuses DO NOT have to be squares. A simple Venn diagram might help visual learners.



- Challenge students to determine why the squares are in the intersection of the Venn diagram and why the other shapes are not.

Assess

5. Square, rectangle, rhombus

Distribute paper. Ask students to perform the following tasks:

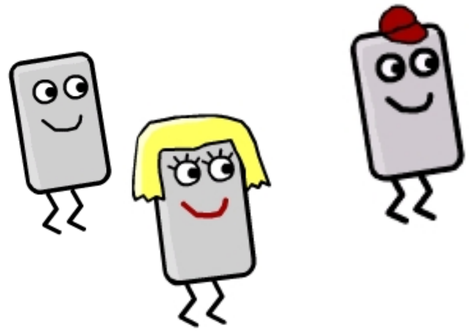
- Draw a rectangle that is not a square.
- Draw a rectangle that is also a rhombus. (*This should be a square.)*
  Questions to scaffold for student understanding:
  - *What does it mean to be a rectangle?*
  - *What does it mean to be a rhombus?*
  - *What does it mean to be both? What characteristics must be present?*
- Draw a rhombus that is not a square.
- Draw a square that is not a rhombus. *(This is impossible but reinforces the concepts of today's lesson. Allow students to try this and discuss reasons why this is impossible.)*

Assess student knowledge on Lessons 1, 2, 3, and 4 by assigning Check Up #1 **[SMJ page 39]**.

6. Extension to number and operation

Students complete the *More Shapes, More Sides* Student Page **[SMJ page 37]**. This exercise gets students thinking about the efficiency of multiplication. It also requires some "backwards thinking," the basis for inverse operations.

Play: The Rectangles Only Club!



Characters:
Rosie Rectangle
Robert Rectangle
Rashawn Rectangle
Sally Square

How to Read a Play:

1. The name in front of the colon tells who should read the line.
   EXAMPLE—Rosie would read this line without reading her name:
   Rosie: I'm bored.
2. The *italicized* words ARE NOT read out loud! They tell the reader WHAT to do.

Play: The Rectangles Only Club!

Rosie: I'm bored. School is over and I have nothing to do. Being a rectangle is so boring. Maybe I'll call Robert.
(Holds up pretend phone and calls Robert.)

Robert: Hello.

(Holds up pretend phone.)

Rosie: Hi, Robert. I'm just calling because I'm bored. What should rectangles like us do on such an afternoon?

Robert: I'm bored, too. Let me call Rashawn, and we can all meet at my house to decide what to do.

Rosie: Good idea. I'll see you there.

(Robert calls Rashawn.)

SMJ page 31

Rashawn: Hello.

(Holds up pretend phone.)

Robert: Hi, Rashawn. Rosie is coming over and we want you to come, too. That way, we'll have three rectangles together, and we can find something to do.

Rashawn: Good idea. I'll be right over.

(The 3 rectangles meet at Robert's house.)

Rosie: So, what should we do?

Rashawn: How about building a clubhouse?

Robert: Yeah! What should we call it?

Rosie: How about the Rectangles Only Club?

Robert: That's a great name!

(The 3 rectangles begin building their clubhouse. Sally Square walks by.)

Sally: Hey, what are you doing?

Rashawn: We're building a Rectangles Only Club. Do you want to help?

Sally: Wow, that sounds fun, but I'm just a square.

Rosie: You're not just a square! You're a rectangle, too!

Sally: What do you mean? My sides are all the same length and yours are different.

Robert: Yeah, but to be a rectangle, all you need are 4 sides and 4 right angles!

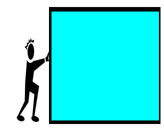
Sally: Wow! Does that mean you are squares?

SMJ page 32

Euclid

Quadrilateral Judge: ______________________________

The Rectangles Only Club!



1. How should Robert answer Sally at the end of the play? Why?

____

____

____

____

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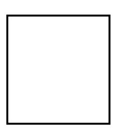
2. The Rectangles and the Rhombuses have decided to have their own baseball teams. The rules for joining are:
   - To be on the Rectangle team, you must fit the definition of Rectangle.
     Rectangle—A quadrilateral with 4 right angles.
   - To be on the Rhombus team, you must fit the definition of a Rhombus.
     Rhombus—A quadrilateral with 4 sides the same length.

   Samuel Square would like to join one of the teams. Which team can he join? Explain your answer.



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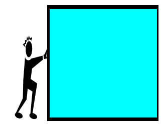
SMJ page 33

Euclid



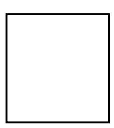
The Rectangles Only Club!
ANSWER KEY

1. How should Robert answer Sally at the end of the play? Why?

   Sample Answer: The definition of rectangle states that you have to be a quadrilateral with four right angles. That means you are a rectangle. Squares are special. They have to have 4 sides the same length while ours are different. That means we can't be squares.

2. The Rectangles and the Rhombuses have decided to have their own baseball teams. The rules for joining are:
   - To be on the Rectangle team, you must fit the definition of Rectangle.
     Rectangle—A quadrilateral with 4 right angles.
   - To be on the Rhombus team, you must fit the definition of a Rhombus.
     Rhombus—A quadrilateral with 4 sides the same length.

   Samuel Square would like to join one of the teams. Which team can he join? Explain your answer.



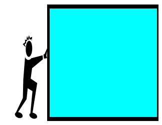
Sample Answer: Samuel Square can join either team. Therefore, acceptable answers include rectangle, rhombus, or either. Look for the following justifications:

- *Samuel Square can join the rectangle team because a square is a quadrilateral with 4 right angles.*
- *Samuel Square can join the rhombus team because a square is a quadrilateral with 4 sides the same length.*
- *Samuel Square can join either team because a square is a quadrilateral and has 4 right angles like a rectangle and 4 sides the same length like a rhombus.*

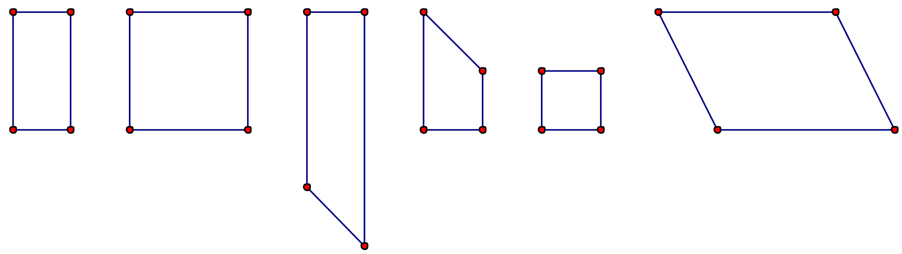
Hypatia

Quadrilateral Judge: ______________________________

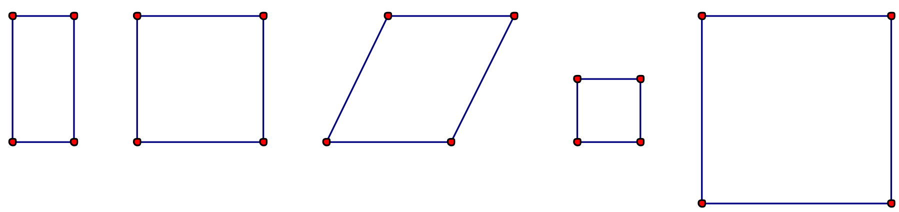
The Rectangles Only Club!



1. A rectangle has to have 4 sides and 4 right angles. Circle all of the rectangles you see.



2. A square has to have 4 sides that are the SAME LENGTH and 4 right angles. Circle all of the squares you see.



3. Is it possible to draw a square that is not a rectangle?

   Yes No

4. Is it possible to draw a square that is not a rhombus?

   Yes No

5. Is it possible to draw a rectangle that is not a square?

   Yes No

6. How should Robert answer Sally at the end of the play? Why?

____

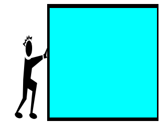
____

____

____

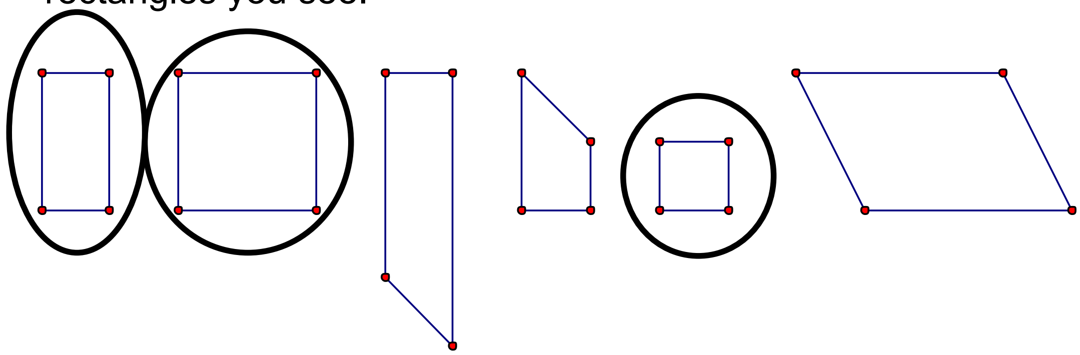
SMJ page 35

Hypatia

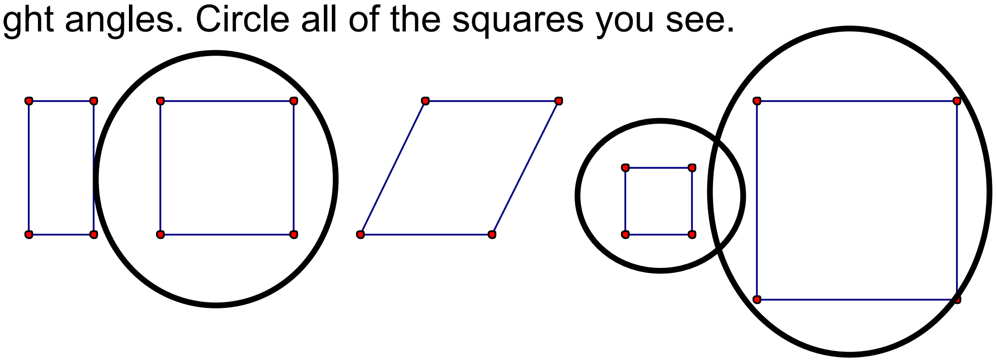


The Rectangles Only Club!
ANSWER KEY

1. A rectangle has to have 4 sides and 4 right angles. Circle all of the rectangles you see.



2. A square has to have 4 sides that are the SAME LENGTH and 4 right angles. Circle all of the squares you see.



3. Is it possible to draw a square that is not a rectangle?

   Yes (No)

4. Is it possible to draw a square that is not a rhombus?

   Yes (No)

5. Is it possible to draw a rectangle that is not a square?

   (Yes) No

6. How should Robert answer Sally at the end of the play? Why?

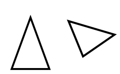
   Sample Answer: The definition of rectangle states that you have to be a quadrilateral with four right angles. That means you are a rectangle. Squares are special. They have to have 4 sides the same length while ours are different. That means we can't be squares.

Side Searcher: ___________________________________

More Shapes, More Sides

Directions: Fill in the columns below so that the number of triangles matches the total number of sides.		*Directions:* Fill in the columns below so that the number of quadrilaterals matches the total number of sides.	
Number of Triangles	Total Number of Sides	Number of Quadrilaterals	Total Number of Sides
2	6	7	28
5	15	3	12
7		5	
9		1	
1		12	
10		8	
14		15	
	9		8
	12		16
	18		24

Draw pictures to help you fill in the boxes!



2 triangles have 6 sides

Explain why you cannot have a group of quadrilaterals with a total of 13 sides.

____

____

____

____

SMJ page 37

More Shapes, More Sides
ANSWER KEY

Directions: Fill in the columns below so that the number of triangles matches the total number of sides.		*Directions:* Fill in the columns below so that the number of quadrilaterals matches the total number of sides.	
Number of Triangles	Total Number of Sides	Number of Quadrilaterals	Total Number of Sides
2	6	7	28
5	15	3	12
7	*21*	5	*20*
9	*27*	1	*4*
1	*3*	12	*48*
10	*30*	8	*32*
14	*42*	15	*60*
3	9	*2*	8
4	12	*4*	16
6	18	*6*	24

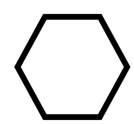
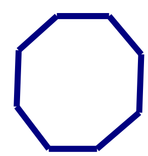
Explain why you cannot have a group of quadrilaterals with a total of 13 sides.

Sample Answer: You cannot have a group of quadrilaterals with a total of 13 sides because 1 quadrilateral has 4 sides, 2 quadrilaterals have 8 sides, 3 quadrilaterals have 12 sides, and 4 quadrilaterals have 16 sides. Since there cannot be more than 3 and fewer than 4 quadrilaterals, there is no way to make 13 sides.

Name: ____________________________ Date: _________________

Check Up #1

Fill in the missing parts of the table.

Name of Shape	Number of Sides	Number of Angles	Drawing of Shape
Triangle			
Quadrilateral			
	5		
			
			

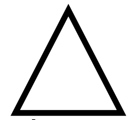
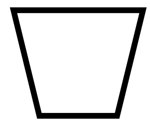
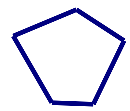
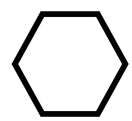
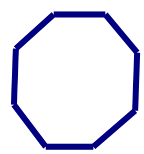
Draw an example of each type of angle.

Acute	Obtuse	Right

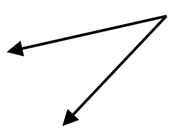
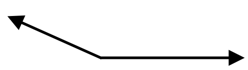
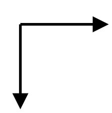
SMJ page 39

Check Up #1
ANSWER KEY

Fill in the missing parts of the table.

Name of Shape	Number of Sides	Number of Angles	Drawing of Shape
Triangle	*3*	*3*	 *Drawings will vary*
Quadrilateral	*4*	*4*	 *Drawings will vary*
Pentagon	5	5	 *Drawings will vary*
Hexagon	*6*	*6*	
Octagon	*8*	*8*	

Draw an example of each type of angle.

Acute	Obtuse	Right
 *Drawings will vary*	 *Drawings will vary*	 *Drawings will vary*

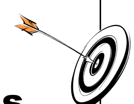
LESSON 5

TIME TO SHAPE UP—
FLIPPING, TURNING, AND SLIDING (OPTIONAL LESSON)

E=MC²

Big Mathematical Ideas

It is amazing that new technologies such as Mapquest and GPS allow us to type in addresses and get directions from one location to another in a matter of seconds. Just ten years ago, we were writing directions like "turn left at the 3rd light." The ability to give or follow a set of directions is a highly desirable skill in today's world. Mathematical transformations in coordinate space can help develop spatial reasoning like that needed to give directions.

Lesson Objectives 	● Students flip, turn, and slide geometric figures.
Materials 	● Student Page—*Turn, Turn, Turn* [SMJ page 41] ● Student Page—*Flipping, Turning, and Sliding* [SMJ page 43] ● Student Page—*Flipping, Turning, and Sliding Directions 1* [SMJ page 45] ● Student Page—*Flip, Turn, and Slide to the Finish* [SMJ page 47] ● Student Page—*Flipping, Turning, and Sliding Directions 2* [SMJ page 49] ● Student Page—*Create Your Own: Flip, Turn, and Slide* [SMJ page 51] ● Student Page—*Flipping, Turning, and Sliding Directions 3* [SMJ page 53] ● Tangrams (one set per student) ● Drawing paper ● Ruler

Mathematical Language	Many grade three curricula use the terms *flip*, *turn* and *slide* to identify mathematical transformations. Students may be interested to learn the terms used by mathematicians. These are in parentheses. ● **Transform:** To change the position of something. ● **Flip (reflect):** To turn over. ● **Turn (rotate):** To rotate around a point. ● **Slide (translate):** To move an item in any direction without rotating it. ● **Clockwise:** A rotation in the same direction that the hands of a clock move. ● **Counterclockwise:** A rotation in the opposite direction than the hands of a clock move.

Lesson Preview

Students flip, turn, and slide tangrams to create fun puzzles.

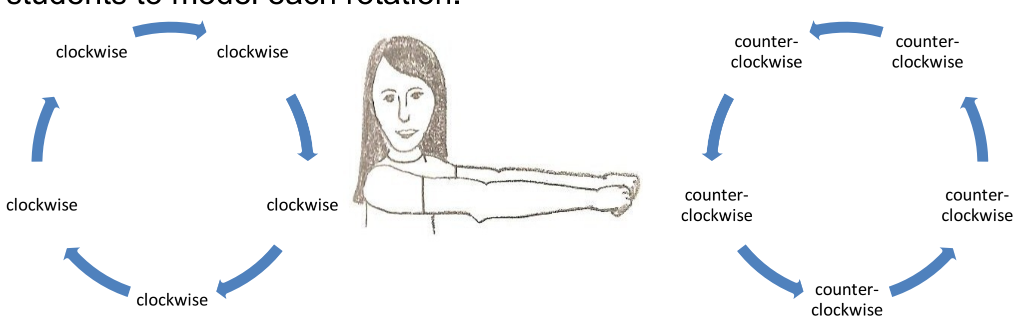
Initiate

1. **Flip, turn, and slide**

Give each student a set of tangrams. Tell students that they should have two large triangles, one medium triangle, two small triangles, one square and one parallelogram. You can have them verify their sets (and practice their mathematical vocabulary) at the beginning and end of the lesson by holding up each figure.

Ask students to separate the square from the rest of the tangrams on the desk. Show students how to slide (or translate) up, down, left, and right using the square. Ask students to flip (or reflect) the square. Point out that you can *hear* a difference between flipping and sliding.

Ask students to stand up and make a zero degree angle with their hands. Show them how to move their arms “clockwise” and “counterclockwise,” explaining that clockwise means to move in the same direction as the hands on a clock. It may help to stand facing the same direction as the students to model each rotation.



Ask students to take out the large triangle and turn (or rotate) it clockwise. Ask them to turn (or rotate) it counterclockwise.

Investigate

2\. **The 90° rotation**

Ask students to turn to the *Turn, Turn, Turn* Student Page **[SMJ page 41]**. Tell them to take one of their small triangles and line it up on the triangle marked with the number one. Instruct students to put their finger on the bottom left-hand vertex (the one formed by the right angle) and turn (or rotate) the shape clockwise onto the triangle numbered two. Explain that this is a 90^{o} rotation. You can go back and show students that if they rotate the first triangle 90^{o} *counterclockwise* they end up in the position of the number four triangle.

Challenge students by asking how many degrees Triangle 1 gets rotated to return to its original position. Students can repeatedly add 90^{o} rotations to figure this out. One answer to this challenge is 360^{o} although any multiple of 360 would work. The same is true whether students rotate the shape clockwise or counterclockwise.

3\. **Transforming to the finish**

Ask students to turn to the *Flipping, Turning, and Sliding* Student Page **[SMJ page 43]**. Tell them they will use flips, turns, and slides to move the triangle to the number 4 spot. Have students place a small triangle on the START position. Then use the set of questions below to help guide students in the process of using appropriate mathematical language. (NOTE: The decision to use *flip* versus *reflect*, *turn* versus *rotate*, or *slide* versus *translate* should be determined by the school's curriculum.)

- The first move we'd like to make is to move the triangle to Triangle 1. Should we use a flip, turn, or slide to get there?
  - *A slide should be used since the triangle is facing the same direction after the move.*
- If each box is one unit, how many units are we moving the triangle? Which direction? [Scaffold this task for students by having them mark the left-hand vertex with a pen or pencil and the same left-hand vertex on the image triangle. Then they can count by physically moving the triangle from one mark to the other, watching the vertex as they go.]
  - *The triangle is being moved 7 units to the right.*
- Ask students to record the first direction on the *Flipping, Turning, and Sliding Directions* Student Page **[SMJ page 45]**. They may record, "Slide the triangle 7 units to the right" or something similar as their first direction.

- Figure out how to get from Triangle 1 to Triangle 2. Is it a flip, turn, or slide? [If students are having trouble differentiating between the three moves, emphasize that the shape does not change direction in a slide.]
    - *It is a slide again, this time moving down.*
- How many units and in which direction?
    - *This time the slide is three units down.*
- Ask students to record the second direction on the Student Page.
- Would the move from Triangle 2 to Triangle 3 be a flip, turn, or slide? [Students should place their tangram triangle on Triangle 2 and play with ways to get from Triangle 2 to Triangle 3.]
    - *This is a 90^o clockwise turn.*
- Do we use a flip, turn, or slide to get from Triangle 3 to Triangle 4?
    - *This is a flip to the left (or reflection over the longest side of the triangle).*
- Ask students to record the final two directions for the move.
- If these are new concepts for students, have them pair up, assigning one student to read directions while the other moves the triangle to perform the translation. This will give them a chance to practice their mathematical language with kinesthetic movement.

Conclude

4. Finding a new way (optional)

A second Student Page, *Flip, Turn, and Slide to the Finish* **[SMJ page 47]** and accompanying directions pages *Flipping, Turning, and Sliding Directions 2* **[SMJ page 49]** have been provided for students to develop a different set of directions to get the triangle to the "END." These can be used as homework, extra practice, or for differentiation purposes.

Ask students to "test" the directions of their classmates. Each Student should use a set of mathematical transformation directions written by a classmate to see if they are able to get to the correct ending location.

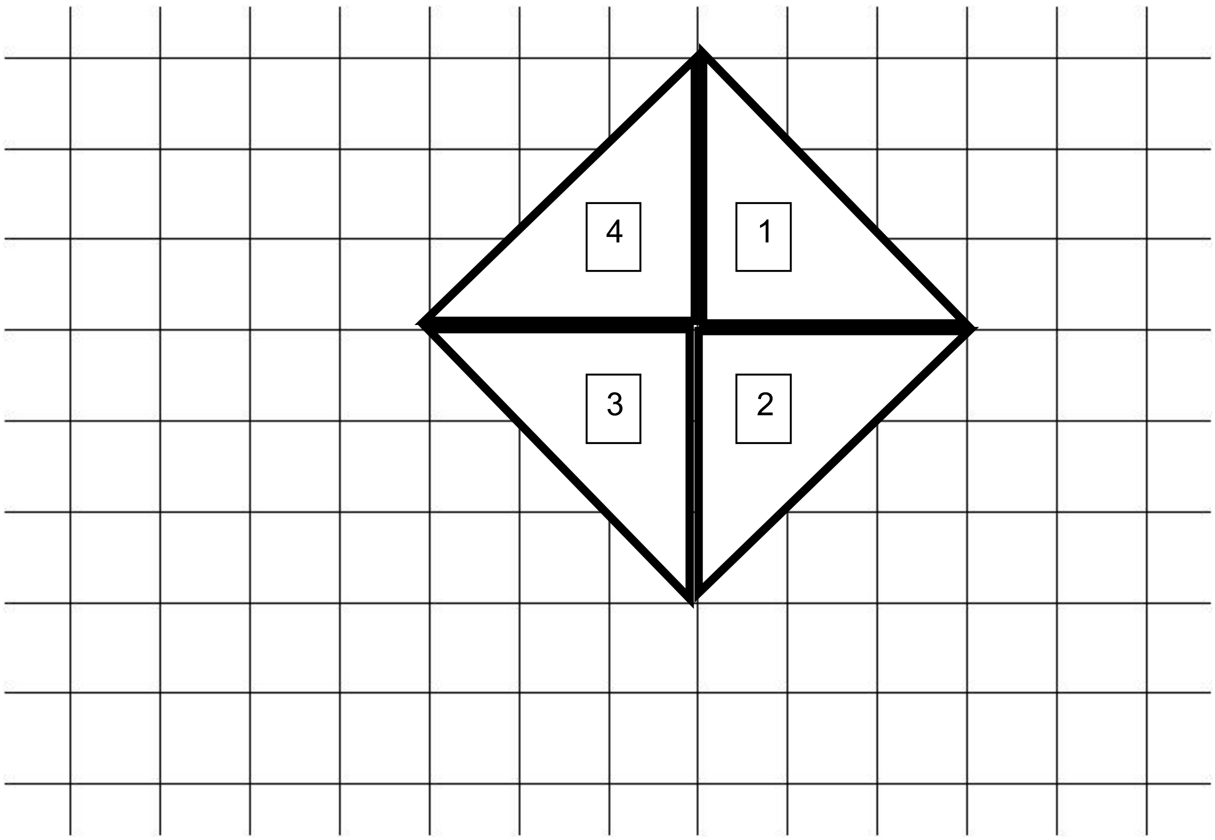
Assess

5. Create your own

Students complete the *Create Your Own: Flip, Turn, Slide* Student Page **[SMJ page 51]** and accompanying directions pages *Flipping, Turning, and Sliding Directions 3* **[SMJ page 53]**. Students can use these to create a transformation for a classmate to try *or* they can give directions and provide the solution as a classroom assessment. Teachers can also use these to create their own assessment based on the needs of students in the classroom.

Mathematical Transformer: ___________________________________

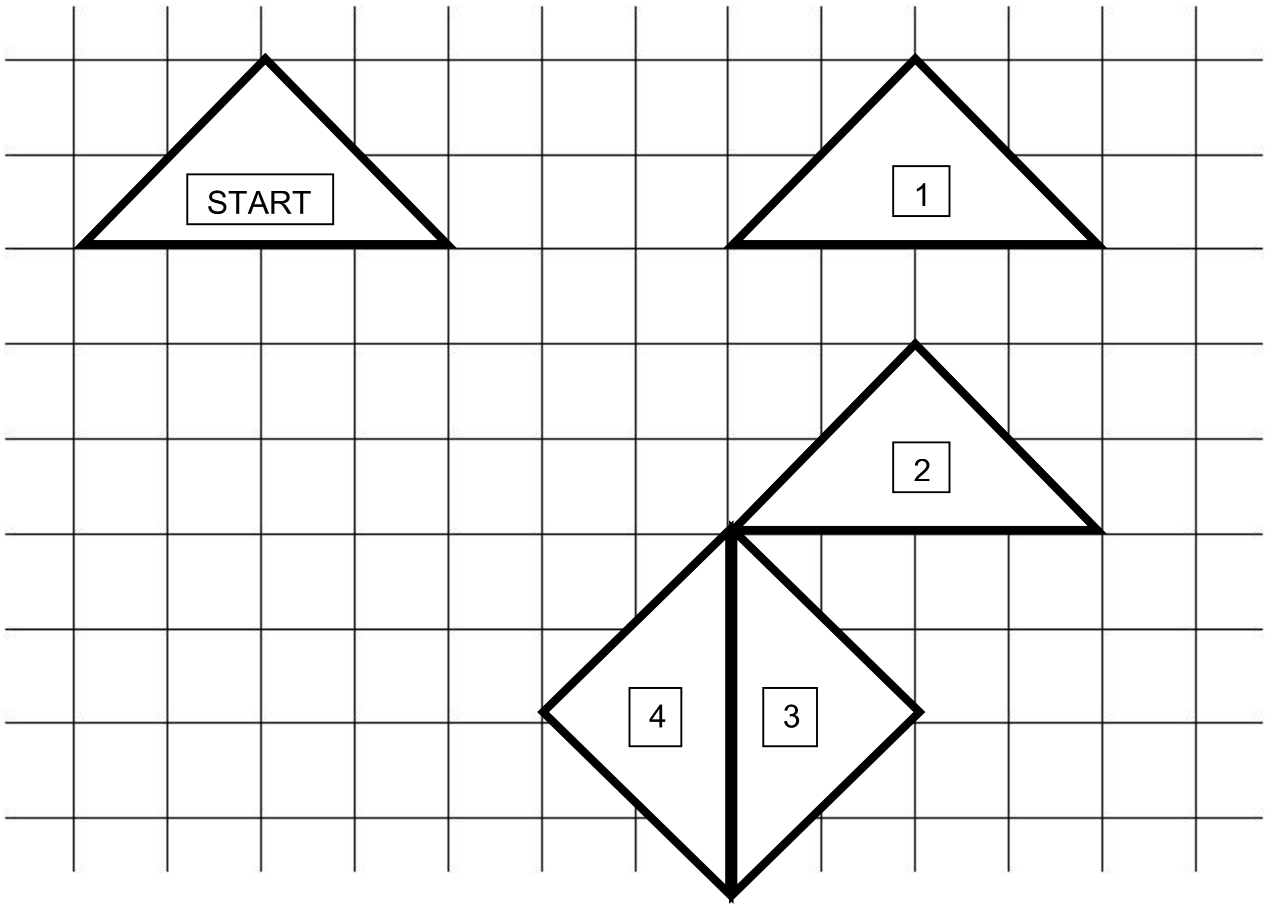
Turn, Turn, Turn



SMJ page 41

Mathematical Transformer: ___________________________________

Flipping, Turning, and Sliding



SMJ page 43

Mathematical Transformer: ______________________________

Flipping, Turning, and Sliding Directions 1

My directions from START to Triangle 4:

1. ____

____

2. ____

____

3. ____

____

4. ____

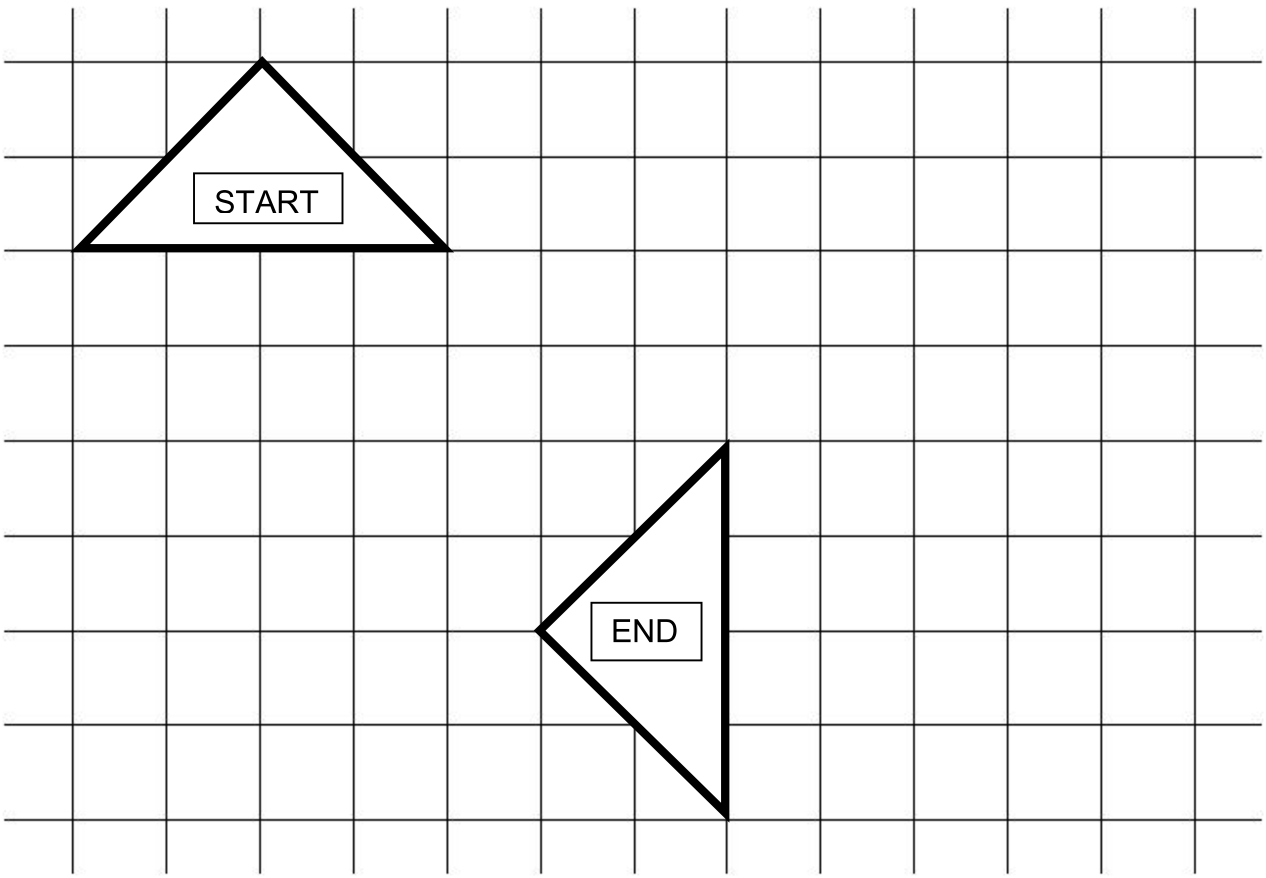
____

SMJ page 45

Mathematical Transformer: ________________________________

Flip, Turn, and Slide to the Finish

Can you find another way to get your triangle from START to END? Write your directions using terms like FLIP, TURN, SLIDE, UP, DOWN, LEFT, and RIGHT.



SMJ page 47

Mathematical Transformer: ________________________________

Flipping, Turning, and Sliding Directions 2

My *different* directions from START to END:

1. ____

____

2. ____

____

3. ____

____

4. ____

____

5. ____

____

6. ____

____

SMJ page 49

Mathematical Transformer: ________________________________

Create Your Own: Flip, Turn, Slide

SMJ page 51

Mathematical Transformer: ______________________________

Flipping, Turning, and Sliding Directions 3

My *different* directions from START to END:

1. ____

____

2. ____

____

3. ____

____

4. ____

____

5. ____

____

6. ____

____

SMJ page 53

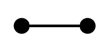
TIME TO SHAPE UP—

YOUR HEIGHT CAN CHANGE YOUR LIFE

LESSON 6

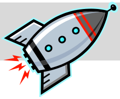
Big Mathematical Ideas

A person's height is one of the physical measurements taken at a routine visit to the doctor's office. Often, this measurement is used to typify a group of people. For example, we might say that basketball and volleyball players are generally tall or that gymnasts and horse jockeys are generally short. The ability to perform linear measurements extends far beyond the height of people. It is a critical skill in many fields including construction, travel, and set design.

Lesson Objectives	• Students will be able to draw objects with given heights (using half and whole inches).
Materials	• Student Page—*One Inch Tall* [SMJ page 55] • Student Page—*If I Were ____ Tall (Pythagoras and Hypatia)* [SMJ pages 59 & 61] • Student Page—*Segment Addition* [SMJ page 63] • 12-inch Rulers (quarter-inch intervals)
Mathematical Language	• **Inch:** A unit of standard measurement. • **Vertical Line Segment:** A line segment drawn in the up-down direction.  • **Horizontal Line Segment:** A line segment drawn in the left-right direction. 

Lesson Preview

Students imagine what it would be like to be much smaller than their actual heights. In this lesson, students draw themselves in varying heights and reflect poetically on how their lives change in accordance with those heights.



Initiate

1.

One Inch Tall

Read the poem “One Inch Tall” by Shel Silverstein. Provide students with rulers (quarter-inch rulers have been provided). Direct students to the *One Inch Tall* Student Page **[SMJ page 55]**. Demonstrate the task in question 1 that asks students to draw a one-inch tall person. Focus on starting measurement at the line and not at the beginning of the ruler. Check to see that students understand the phrase **vertical line segment**.

Ask students to answer questions 2, 3, and 4 independently. Students who are able to answer these questions (particularly #4) easily should be challenged with the Hypatia version of the *If I Were* ____ *Tall* Student Pages **[SMJ pages 59 & 61]**. Students demonstrating limited prior knowledge of measurement involving half inches should work on the Pythagoras version. Before moving on, demonstrate how to perform measurements such as 1/2” (like in #4) and 1 1/2”.

Investigate

2.

If I Were ____ Tall

Direct students to the *If I Were* ____ *Tall* Student Pages **[SMJ pages 59 & 61]** according to the mathematician name designated in the lesson initiation. Students can work in pairs when completing the drawing section but should write their poems independently. Circulate the room to assist pairs of students with understanding 1/4 and 3/4 inch measurements. Focus on conceptual understanding that each inch is divided into four pieces and 1/4 represents one out of four while 3/4 represents three out of four.

Students may question why the middle line is 1/2 and not 2/4. Use pictorial examples to show the equivalence of the two fractions.

Conclude

3.

Sharing poems

Invite a student to the board to draw the person that goes with his/her poem. Then have the student read his/her poem. While the poem is read, ask the class to think about whether or not they agree with the lines in the poem for a person of that size. Be sure to have students defend their responses with measurement reasoning.

Assess

4.

Inches and half inches

Assess students’ understanding of inches and half inches by asking them to draw segments with teacher-selected lengths before leaving the class.

Since students are working with 12-inch rulers, the difficulty of this task can be increased by having students draw lengths greater than 12 inches.

5. **Extension to number and operation**
Students complete the *Segment Addition* Student Page **[SMJ page 63]**. Explain that they can draw **horizontal line segments** on this page and demonstrate the difference between horizontal and vertical. This extends their knowledge of linear measurement to consider what happens when two segments of given length are combined.

Height Changer: ____

One Inch Tall

One Inch Tall

If you were only one inch tall, you'd ride a worm to school.
The teardrop of a crying ant would be your swimming pool.
A crumb of cake would be a feast
And last you seven days at least,
A flea would be a frightening beast
If you were one inch tall.

If you were only one inch tall, you'd walk beneath the door,
And it would take about a month to get down to the store.
A bit of fluff would be your bed,
You'd swing upon a spider's thread,
And wear a thimble on your head
If you were one inch tall.

You'd surf across the kitchen sink upon a stick of gum.
You couldn't hug your mama, you'd just have to hug her thumb.
You'd run from people's feet in fright,
To move a pen would take all night,
(This poem took fourteen years to write—
'Cause I'm just one inch tall).

Shel Silverstein

Silverstein, S. (1974). *Where the sidewalk ends.* New York, NY: HarperCollins.

SMJ page 55

1. Draw a vertical segment that is one inch long. Next to that segment, draw a one-inch tall person.

2. Would a one-inch tall person fit under the door of your classroom? Explain why or why not.

____

____

____

3. Choose one other line from the poem. Explain whether or not the line makes sense for a one-inch tall person.

____

____

____

4. Draw a person who is only 1/2 inch tall.

SMJ page 57

Pythagoras

Height Adjuster: ____

If I Were ____ Tall

Under each measurement, draw a person with the given height.

2 inches	3 1/2 inches	5 inches

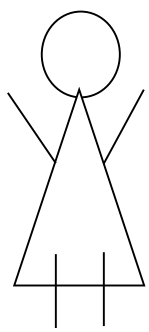
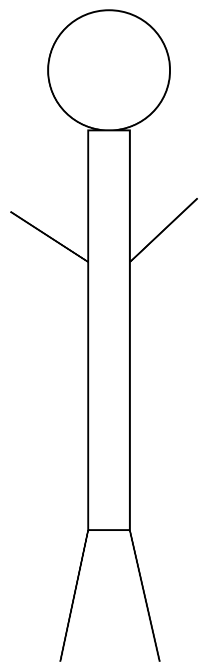
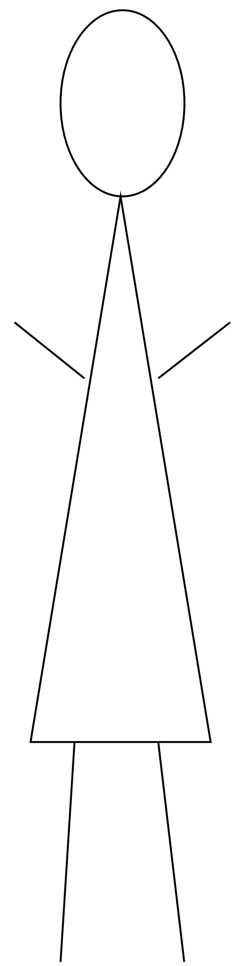
Choose one of the people you drew above and write a poem about what life is like for that person.

SMJ page 59

Pythagoras

If I Were ____ Tall
ANSWER KEY

Under each measurement, draw a person with the given height.

2 inches	3 1/2 inches	5 inches
		

Choose one of the people you drew above and write a poem about what life is like for that person.

Poems will vary.

Hypatia

Height Adjuster: ______________________________________

If I Were ____ Tall

Under each measurement, draw a person with the given height.

2 1/2 inches	3 1/4 inches	4 3/4 inches

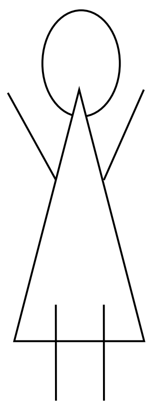
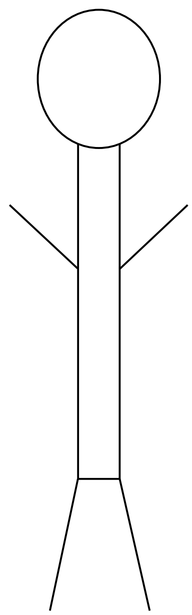
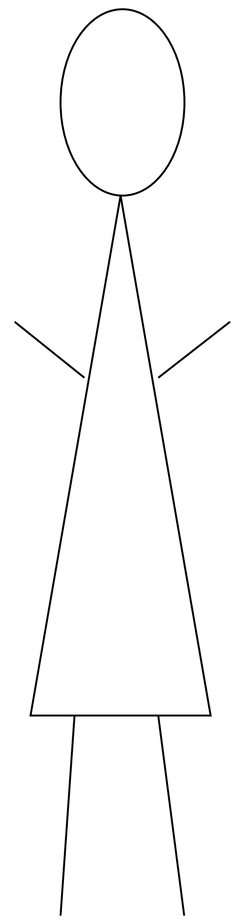
Choose one of the people you drew above and write a poem about what life is like for that person.

SMJ page 61

Hypatia

If I Were ____ Tall
ANSWER KEY

Under each measurement, draw a person with the given height.

2 1/2 inches	3 1/4 inches	4 3/4 inches
		

Choose one of the people you drew above and write a poem about what life is like for that person.

Poems will vary.

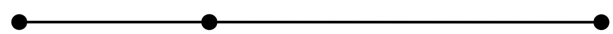
Segment Adder: ____

Segment Addition

Recall: A **line segment** is a part of a line with two endpoints.

The **Segment Addition Postulate** says that if you add two pieces of a segment, you can find the length of the whole segment.

EXAMPLE: Combine a 1 inch segment and a 2 inch segment.



What is the length of the whole segment? 1 inch + 2 inches = 3 inches

---

1. Combine a 2 inch sement and a 3 inch segment.

   What is the length of the whole segment? ___________________

2. Combine a 2 inch segment and another 2 inch segment.

   What is the length of the whole segment? ___________________

3. Combine a 2 1/2 inch segment and a 3 1/2 inch segment.

   What is the length of the whole segment? ___________________
   (Hint: Measure the length if you cannot find it by adding!)

SMJ page 63

Segment Addition
ANSWER KEY

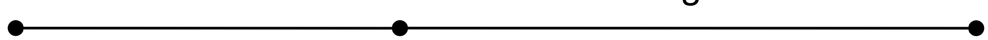
Recall: A **line segment** is a part of a line with two endpoints.

The **Segment Addition Postulate** says that if you add two pieces of a segment you can find the length of the whole segment.

EXAMPLE: Combine a 1 inch segment and a 2 inch segment.

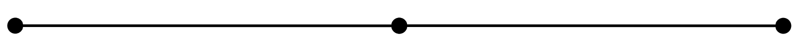
What is the length of the whole segment? 1 inch + 2 inches = 3 inches

1. Combine a 2 inch sement and a 3 inch segment.



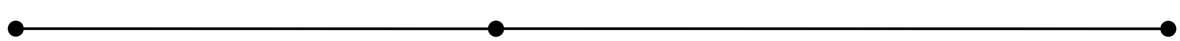
   What is the length of the whole segment? *2 inches + 3 inches = 5 inches*

2. Combine a 2 inch segment and another 2 inch segment.



   What is the length of the whole segment? *2 inches + 2 inches = 4 inches*

3. Combine a 2 1/2 inch segment and a 3 1/2 inch segment.



   What is the length of the whole segment? *2 1/2 inches + 3 1/2 inches = 6 inches*

TIME TO SHAPE UP—
SPEAKING FRACTIONAL LANGUAGE

LESSON 7

Big Mathematical Ideas

Fractional reasoning can be difficult because meanings are highly context-dependent. For example, half an inch is completely different than half of a cookie, yet both represent some way of dividing something into two parts. Within the concept of measurement, one must distinguish between half an inch and half a foot. Understanding fractions begins with understanding how and why something is divided.

Lesson Objectives	• Students will understand the meaning of the numerator and denominator of a fraction. • Students will be able to write and speak the language of fractions.
Materials	• Student Page—*Fractional Paths* [SMJ page 65] • Blue painter's tape • Measuring tape (or similar) • Sticky notes • Rulers
Mathematical Language	• **Fraction:** Part of a group, number, or whole. • **Numerator:** Number on the top of a fraction showing how many parts of the whole. • **Denominator:** Number on the bottom of a fraction showing the number of parts into which the whole is divided. • **Equivalent fractions:** Fractions that are equal, like 1/2 and 2/4.

Lesson Preview

Students walk along paths that have been divided into different fractional parts to better understand the meaning of a fraction's numerator and denominator.

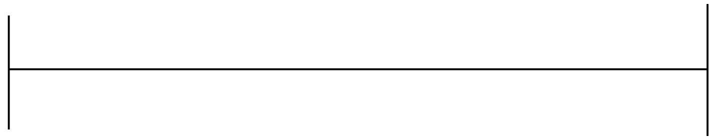
Initiate

1. Classroom setup

On the classroom floor set up seven “paths.” Each path will be 12 feet long, so find an area that has plenty of space. Use the following set of instructions to create your paths.

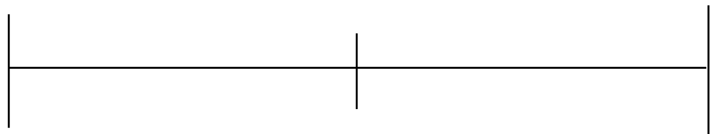
1. Lay out a piece of blue painter’s tape 12 feet in length. Put another piece of tape on each end perpendicular to the first as shown in the diagram, measuring from the center of each piece.

2. Repeat the first step SIX more times, aligning the ends with the first as shown in the series of diagrams on the next page.
3. Divide each path into fractional parts using the following guidelines.
   - Path 1—No additional tape
   - Path 2—One piece of tape at 6 feet
   - Path 3—Two pieces of tape, each at 4 feet and 8 feet
   - Path 4—Three pieces of tape, each at 3 feet, 6 feet, and 9 feet (Check for accuracy—Does the halfway point of this path align with the halfway point of Path 2?)
   - Path 5—Five pieces of tape, each at 2 feet, 4 feet, 6 feet, 8 feet, and 10 feet
   - Path 6—Seven pieces of tape, each at 1.5 feet, 3 feet, 4.5 feet, 6 feet, 7.5 feet, 9 feet, and 10.5 feet
   - Path 7—Eleven pieces of tape, one at every foot
4. Label one end of each path “Home” and the other end “School.”



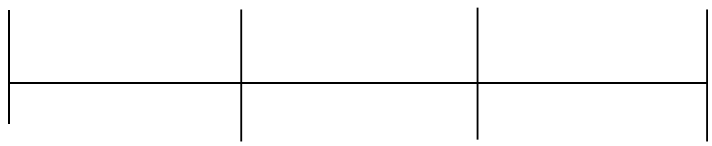
#1

Home School



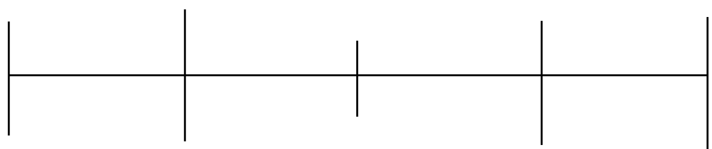
#2

Home School



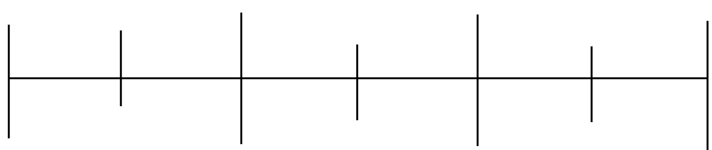
#3

Home School



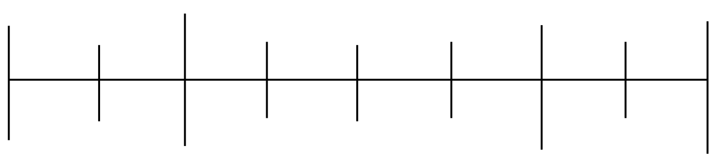
#4

Home School



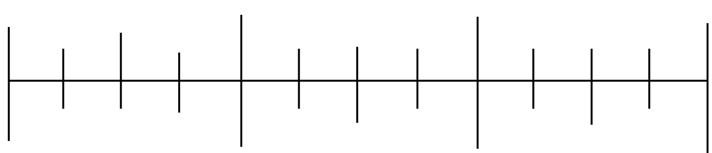
#5

Home School



#6

Home School



#7

Home School

Investigate

2. Speaking the language of fractions

Gather students around the fractional paths. Ask a volunteer to walk along the first path from home to school. Use the following activities and questions to frame students' introduction to fractional language.

- Do you think that the distance from home to school is the same on each path? Why or why not?
  - *Sample response: The distance from home to school is the same because each path appears to start and end at the same spot.*
- How do the paths differ?
  - *Sample responses: They have different numbers of marks. They are divided into different sections.*
- Ask a student to walk along the second path by starting on the mark for HOME. Ask students to count each mark, counting "zero, one, two."
- Why do you think we count the first mark as zero instead of one?
  - *Sample response: Since we are looking at distance, the first mark represents the spot where we haven't moved yet. Zero represents the starting point here, similar to the first mark on the ruler representing the place where you start measuring.*
- Draw students' attention to the halfway mark and ask what they think this mark should be called.
  - *Sample responses: The middle, halfway, 1/2, midpoint, center.*
- Explain that each mark can be labeled with its distance. Using sticky notes, label the first mark 0/2, the second mark 1/2, and the third mark 2/2.
- Tell students the bottom number, 2, is called the denominator. Can somebody explain why the denominator is 2?
  - *Sample response: The denominator is 2 because we start with zero. When the person is at home, it is zero because he/she hasn't moved yet. Then 1 is the first mark and 2 is the second mark.*
- Would this number change if we were to switch paths?
  - *If we switch paths, we would have a different denominator because there are more sections. It all depends on how the path is broken up.*
- Ask a student to walk the same path, stopping on each mark for students to repeat the following phrases as they apply:
  - [Student's Name] has walked zero out of two spaces.
  - [Student's Name] has walked one out of two spaces.
  - [Student's Name] has walked two out of two spaces.
- Repeat the previous activity using more common language:
  - [Student's Name] has not left his/her home yet.
  - [Student's Name] is 1/2 of the way from home to school.
  - [Student's Name] is at school.

Break students into five groups. Then assign each group to a single path that was not used already. Paths with more marker lines may be used to challenge students who quickly grasped the concepts in the previous activity.

Direct students to the *Fractional Paths* Student Page **[SMJ page 65]**. Let students know that they are completing the directions for their assigned paths only. They will complete *Fractional Paths* to gain expertise and then share this expertise with classmates.

Circulate among groups to discuss questions as students work. Encourage students to walk along the paths and use fractional language. For example, students might say, "I am on mark one out of three" or "I am one-third of the distance from home to school."

3. Challenge another group

Have the students remove their sticky notes. Question 10 of the Student Page asks each group to come up with five questions related to its path to ask a group from another path. Encourage students to use vocabulary words such as **zero, numerator, denominator,** and **fraction**. If you know students who need an additional challenge, create questions that will get these students thinking. An example of a challenge question might be:

- Can you use the paths to find another fraction that is equal to 1/2? Why do you believe it is equal?
  - *Sample response: 4/8 is equal to 1/2. I can tell because it is exactly in-between home and school OR because it lines up with the 1/2 on Path #2. They are in the same spot; only the number of sections changes.*

Conclude

4. Connecting to equivalent fractions

Use the paths to teach students about equivalent fractions and the concepts of greater than and less than. The following tasks can serve as a guide. For each task have students explain both verbally and physically why they believe the fractions are equal. Encourage students to explain their responses in terms of distance from "home." The last question addresses the misconception that one can just look at the denominator when comparing fractions. Feel free to add additional questions or have students observe their own equalities and inequalities. Here are a few suggestions:

- List three other fractions that are equal to 1/2.
- List two other fractions that are equal to 1/4.
- List two other fractions that are equal to 3/4.

- List one other fraction that is equal to 2/6.
- Which fraction is greater, 1/4 or 3/4?
- Which fraction is greater, 2/6 or 5/8?
- Which fraction is less, 2/3 or 3/12?

Assess

5.

Connecting back to measurement

Ask students to draw three parallel line segments on a sheet of paper, each 6 inches in length. Ask students to divide (1) one into halves, (2) one into thirds, and (3) one into sixths.

Scaffold the task by asking students to number the marks on each line segment starting with zero.

Have students color (1) the mark for 1/2 GREEN, (2) the mark for 2/3 RED, (3) the mark for 2/6 BLUE.

EXTEND: Find the mark on the third path that is equal to 1/2. Color it PURPLE and write the fraction that is equal to 1/2 below the mark. Ask which mark on the “thirds” segment is equal to 2/6.

Fractioneers: ____

Fractional Paths

Follow the steps below.

Remember to count the HOME mark as "zero."

1. The WALKER starts on the mark labeled "HOME."

2. The WALKER counts each mark from HOME to SCHOOL.

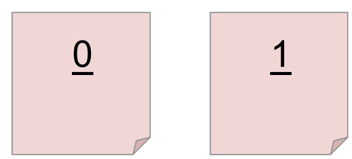
3. The RECORDER records the number of marks counted.

   Number of marks counted: ____________

4. The WALKER walks along the path again. The whole group counts the marks starting with zero.

5. Put a sticky note on each of the marks along your path.

6. At the top of each sticky note, write the number for that mark. Underline the number. This is called the **numerator** of your fraction.



7. Under the **numerator**, write the **denominator**. This is the total number of marks counted from question 3. The **denominator** is the same for all of your fractions.

   BE THE EXPERT: Why is the denominator the same for all of the fractions?

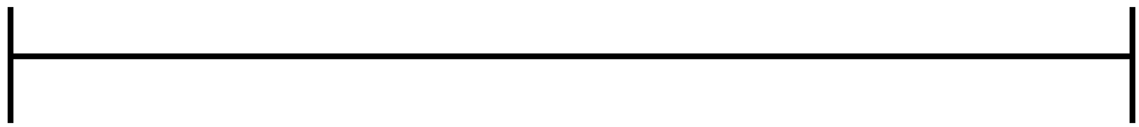
____

____

____

SMJ page 65

8. Finish the picture below so that it looks like your path. Write the correct fraction above each mark in the picture.



HOME SCHOOL

9. Choose FOUR of the fractions from your path and write them in the first column. Then tell how to say each fraction and what it means. An example is given to help you.

FRACTION	HOW I SAY IT	WHAT IT MEANS
EXAMPLE: 3/8	Three-eighths	The third mark out of eight

10. Write five questions to ask anther group, about your group's fractional path.

____

____

____

Segment Addition
ANSWER KEY

7. BE THE EXPERT: Why is the denominator the same for all of the fractions?

   Sample Answer: The denominator is the same for all of the fractions because the denominator is the total number of marks along the path, and the path doesn't change, so the number of marks, or the denominator, doesn't change. The numerator changes as you move along the path, but the denominator doesn't.

8. Finish the picture below so that it looks like your path. Write the correct fraction above each mark in the picture.

   Notes to teacher: Make sure that each group's picture accurately represents its fractional path and that students have properly labeled the fractions above each fractional mark on the path.

9. Choose FOUR of the fractions from your path and write them in the first column. Then tell how to say each fraction and what it means. An example is given to help you.

   Examples of possible student responses:

FRACTION	HOW I SAY IT	WHAT IT MEANS
2/3	*two-thirds*	*the second mark out of three*
3/4	*three-fourths (or three-quarters)*	*the third mark out of four*
5/6	*five-sixths*	*the fifth mark out of six*
7/12	*seven twelfths*	*the seventh mark out of twelve*

10. Write five questions to ask anther group, about your group's fractional path.

    Possible questions students might come up with:

    <u>Sample questions for any group</u> (refer to p. 157 in Teacher's Manual for a visual):
    Which mark is a bigger walk along the path: 2/3 or 4/6? (answer: they are the same)

    If I haven't started walking down our path yet, what is the numerator of my fraction? (answer: zero)

    <u>Sample questions for group #5</u> (refer to p. 157 in Teacher's Manual for a visual):
    What is the denominator for all the fractional parts of our group's path? (answer: 6)

    With a denominator of 6, what will be the numerator if I walk from 0/6 to one-third of the way down our group's path? (answer: 4, so the fraction would be 4/6, or 1/3)

    What two fractions would I have for walking to the third mark on my group's fractional path? (answer: 3/6 or 1/2)

    Which one is a longer distance: walking 1/3 of my group's path or walking 1/3 of your group's path? (answers will vary)

TIME TO SHAPE UP—

A WORLD WITHOUT CONGRUENCE

LESSON 8

Big Mathematical Ideas

We seldom think of how important congruent figures are to our everyday lives. This lesson presents some examples in which it is vitally important to have objects that are both the same size and shape.

Lesson Objectives	• Students will be able to explain why objects must be congruent for certain everyday things to function properly.
Materials	• Student Page—*What If…* [SMJ page 67] • Student Page—*It Depends on the Number of Humans* [SMJ page 75] • Blank paper
Mathematical Language	• While this lesson focuses on congruence, the term **congruent** is not introduced until the next lesson.

Lesson Preview

Students explore congruence through fairness and functionality. Sometimes when objects are not congruent it is not fair because one person gets more than another. In other cases, objects simply do not work if they lack congruence.

Initiate

1. **Why same size?**
   Pose the following problems:

   - Hold up two identical unsharpened pencils. These two pencils are from the same box. Why do you suppose the company who made these pencils made them all the same size? Record responses.
   - Can you think of other times when it is important for things to look exactly the same? Record responses. Ask students look around the classroom if they are having difficulty thinking of examples.
     - Students might notice *windows, drawers, desks*.

Investigate

2. Problem #1

Direct students to the *What If…* Student Page **[SMJ page 67]**. Review problem #1 by having students read the question aloud, respond to the question independently, share responses with a partner, and then discuss ideas as a class.

In this question, the focus should be on the size differences of candy bars offered to girls versus boys. If students use the word <u>same</u>, what do they mean? Size/Shape or both?

Sample responses Problem 1:

a. *The candy bar she gives to the girls is much bigger.*
b. *The teacher should give the same candy bar to everybody.*

3. Pair up and answer

Instruct students to complete problems 2 and 3 in pairs or small groups. Discuss the responses as a class.

Sample responses Problem 2:

a. *The car will be higher on one side and lower on the other. It probably won't be drivable.*
b. *The tires should be exactly the same size and shape.*

Sample responses Problem 3:

a. *The door will not fit because it is a different size (too small). The door will not fit because it is a different shape (corners are round).*
b. *The door must be the same size and shape as the opening.*

4. My non-congruent human

Once students have a good understanding of how size and shape affect certain objects, read problem 4 aloud. Have students complete their drawings and stories in class. If time permits, allow students to walk around and look at the drawings their classmates have created.

Conclude

5. My new human has trouble with that!

Now that students have created a new, non-congruent human, conclude by allowing students to read their stories aloud and display their drawings.

Look Ahead

6. **Creating same size, same shape**
Now that students have an understanding of the importance of same size and same shape, they can use this knowledge in the next lesson to create congruent shapes and judge whether or not two shapes are congruent.

Assess

7. **Congruence and fairness**
Give students a scrap piece of paper and instruct them to list five examples in which things wouldn't work or wouldn't fit if they were not the same size and shape.

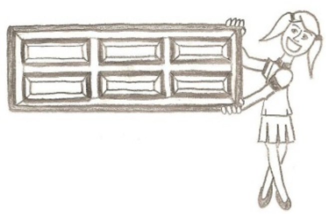
8. **Extension to number and operation**
Students complete the *It Depends on the Number of Humans* Student Page **[SMJ page 75]**. This activity links multiplication to concrete images. For example, if one human has 10 fingers, then two humans would have a total of 20 fingers. Thus, 2 humans x 10 fingers each = 20 fingers all together.

Shape Evaluator: ________________________________

What If…

Problem 1: Your teacher tells the class that she is going to give each student a candy bar.

What if your teacher gave all the girls in class 1 of these?



Then, all of the boys in class got 1 of these.

a. Explain why the teacher is being unfair.

____

____

b. What should the teacher do to make it fair?

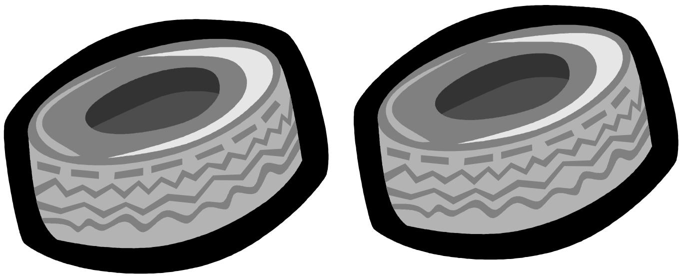
____

____

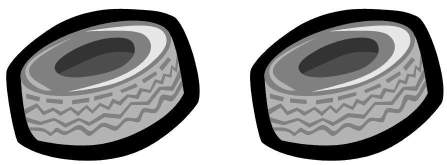
SMJ page 67

Problem 2: Andrea's mom just bought a new car and she got a really good deal.

The two tires on one side of the car looked like this:



The two tires on the other side of the car looked like this:



a. What is wrong with the tires on the new car?

____

____

b. What should Andrea's mom do to fix the car?

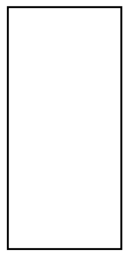
____

____

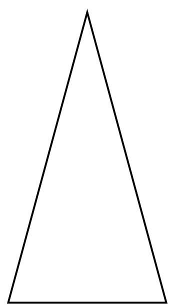
SMJ page 69

Problem 3: Tyrell is helping his father install the new door they bought.

The empty space where the door goes looks like this:



The new door looks like this:



a. Explain what is wrong with the door they bought.

____

____

b. What should they do before they go to buy another door?

____

____

SMJ page 71

Problem 4: DESIGN TEAM TIME: Making a new human.
Make a list of body parts that come in 2s.

a. LIST: We all have 2…

____

____

____

____

____

____

____

____

____

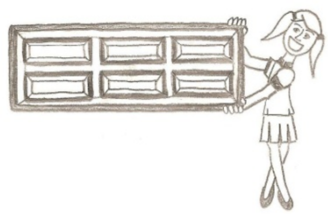
b. THINK about which of the body parts you listed are supposed to be the same size and shape.
c. DRAW a person making the body parts you listed different sizes, different shapes, or both.
d. WRITE a story about why life is harder for the person you drew.

SMJ page 73

What If…
ANSWER KEY

Problem 1: Your teacher tells the class that she is going to give each student a candy bar.

What if your teacher gave all the girls in class 1 of these?



Then, all of the boys in class got 1 of these.

a. Explain why the teacher is being unfair.

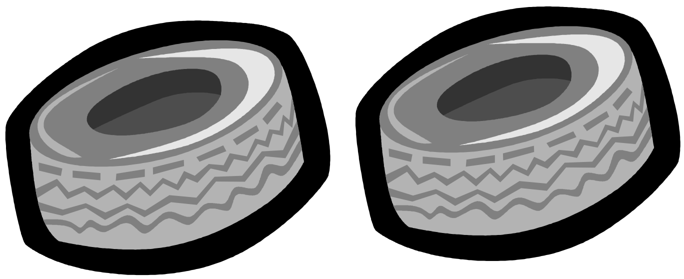
   Sample Answer: The teacher is being unfair because the candy bar that the girls receive is much bigger than the candy bar that the boys receive.

b. What should the teacher do to make it fair?

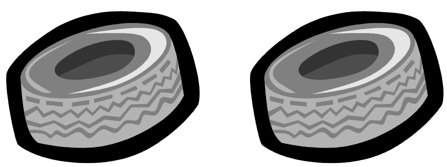
   Sample Answer: The teacher should give every student a candy bar that is the same size.

Problem 2: Andrea's mom just bought a new car and she got a really good deal.

The two tires on one side of the car looked like this:



The two tires on the other side of the car looked like this:



a. What is wrong with the tires on the new car?

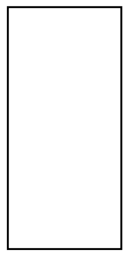
Sample Answer: The tires on one side of the car are bigger than the tires on the other side of the car.

b. What should Andrea's mom do to fix the car?

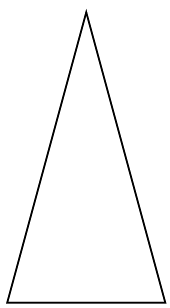
Sample Answer: Andrea's mom should get 4 tires that are exactly the same size.

Problem 3: Tyrell is helping his father install the new door they bought.

The empty space where the door goes looks like this:



The new door looks like this:



a. Explain what is wrong with the door they bought.

Sample Answer: The new door is not the same shape as the opening, so it will not fit.

b. What should they do before they go to buy another door?

Sample Answer: They should make sure they know the size and shape of the opening so that the new door matches exactly.

Problem 4: DESIGN TEAM TIME: Making a new human.
Make a list of body parts that come in 2's.

a. LIST: We all have 2…

Sample Answers

Eyes

Ears

Hands

Arms

Legs

Feet

Knees

Elbows

Stories will vary. Sample drawing is shown.

b. THINK about which of the body parts you listed are supposed to be the same size.
c. DRAW a person making the body parts you listed different sizes, different shapes, or both.
d. WRITE a story about why life is harder for the person you drew.

Body Builder: ____

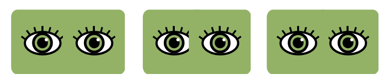
It Depends on the Number of Humans

For each problem below, write the number of body parts for the given number of humans. Then, write the multiplication problem you can use to figure out the total number of body parts for the given number of humans.

EXAMPLE: If I see 3 humans, then I see 3 noses, 6 eyes, and 30 fingers.

Multiplication problems:

Noses 3 humans x 1 nose each = 3 noses all together



Eyes 3 humans x 2 eyes each = 6 eyes all together

Fingers 3 humans x 10 fingers each = 30 fingers all together

---

1. If I see 5 humans, then I see ____ eyebrows, ____ tongues, and ____ toes.

Multiplication Problems:

Eyebrows ____ humans x ____ eyebrows each = ____ eyebrows all together

Tongues ____ humans x ____ tongue each = ____ tongues all together

Toes ____ humans x ____ toes each = ____ toes all together

SMJ page 75

2. If I see 7 humans, then I see ____ knees, ____ elbows, and ____ nails (fingers and toes).

Multiplication Problems:

Knees ____ humans x ____ knees each = ____ knees all together

Elbows ____ humans x ____ elbows each = ____ elbows all together

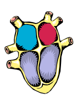
Nails ____ humans x ____ nails each = ____ nails all together

---

3. If I see 4 humans, then I know that they have ____ heart chambers, about ____ teeth, and about ____ bones.

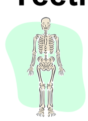
(HINT: Each human has 4 heart chambers, about 28 teeth, and about 208 bones.)

Multiplication Problems



Heart Chambers ____ humans x ____ heart chambers each = ____ heart chambers all together

Teeth ____ humans x ____ teeth each = ____ teeth all together



Bones ____ humans x ____ bones each = ____ bones all together

Why do you think the term “about” must be used when talking about teeth and bones?

____

____

____

SMJ page 77

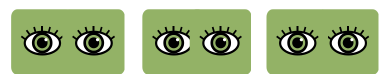
It Depends on the Number of Humans
ANSWER KEY

For each problem below, write the number of body parts for the given number of humans. Then, write the multiplication problem you can use to figure out the total number of body parts for the given number of humans.

EXAMPLE: If I see 3 humans, then I see 3 noses, 6 eyes, and 30 fingers.

Multiplication problems:

Noses 3 humans x 1 nose each = 3 noses all together



Eyes 3 humans x 2 eyes each = 6 eyes all together

Fingers 3 humans x 10 fingers each = 30 fingers all together

---

1. If I see 5 humans, then I see *10* eyebrows, *5* tongues, and *50* toes.

Multiplication Problems:

Eyebrows *5* humans x *2* eyebrows each = *10* eyebrows all together

Tongues *5* humans x *1* tongue each = *5* tongues all together

Toes *5* humans x *10* toes each = *50* toes all together

2. If I see 7 humans, then I see *14* knees, *14* elbows, and *140* nails (fingers and toes).

Multiplication Problems:

Knees *7* humans x *2* knees each = *14* knees all together

Elbows *7* humans x *2* elbows each = *14* elbows all together

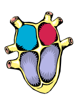
Nails *7* humans x *20* nails each = *140* nails all together

---

3. If I see 4 humans, then I know that they have *16* heart chambers, about *112* teeth, and about *832* bones.

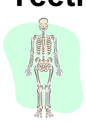
(HINT: Each human has 4 heart chambers, about 28 teeth, and about 208 bones.)

Multiplication Problems



Heart Chambers *4* humans x *4* heart chambers each = *16* heart chambers all together

Teeth *4* humans x *28* teeth each = *112* teeth all together



Bones *4* humans x *208* bones each = *832* bones all together

Why do you think the term “about” must be used when talking about teeth and bones?

Sample Response: Some people have fewer than 28 teeth because teeth fall out while others have more than 28 teeth (wisdom). Though we are born with about 300 bones, this number decreases with age. (Some students might be interested in investigating this claim further!)

TIME TO SHAPE UP—
SAME SIZE, SAME SHAPE

LESSON 9

Big Mathematical Ideas

Are all triangles the same shape simply because they are called triangles? Is it possible to make triangles that are shaped differently? It really depends on how one thinks of the phrase “same shape.” When we talk about congruence, it is essential that students think about both size *and* shape. Shape should be thought of not only the name of the figure but also the angles that define it.

Lesson Objectives	• Students will be able to create congruent right triangles using grid paper. • Students will be able to identify congruent and non-congruent shapes.
Materials	• Student Page—*Are You My Twin?* [SMJ page 79] • Student Page—*Triangle Sum Theorem* [SMJ page 83] • Scissors • Blank white paper • Graph paper • Rulers
Mathematical Language	• **Congruent Figures:** Figures that have the same size and shape.

Lesson Preview

Students explore congruent figures by drawing triangles. They begin to see the role that angles play in congruence.

Initiate

1. **What does *same size and shape* really mean?**
Distribute blank white paper and rulers to students. Ask them to perform the following tasks. If they cannot perform the task, ask them to describe why it is not possible.

- Draw two squares that are different sizes.
  (This is possible. Squares have to have four sides of the same length and four right angles, but one square might have sides 3 inches long and another might have sides 2 inches long.)
- Draw two squares that are different shapes.
  (This is not possible since the angles cannot be changed on squares. If a student changes the size of one side, the other sides must be changed as well so it maintains the same shape.)
- Draw two triangles that are different sizes.
  (This is possible. This task does not mention shape and should be fairly easy. Triangles may appear the same shape or not.)
- Draw two triangles that are different shapes.
  (Many students may think this is not possible but it depends on their interpretations of the phrase "same shape." This question is to try to get students to think about the angles in triangles and how an obtuse triangle and right triangle are "shaped" differently. Whether or not they believe it is possible is of less importance than the discussion on how changing the size of the angles affects a triangle's shape.)

Investigate

2. Right triangles: Is mine the same as yours?

Distribute sheets of graph paper. Working in pairs, students will draw a right angle by making one segment that is 6 units long and another that is 8 units long. Have them connect their segments to make a right triangle.

Check to be sure students have begun their triangles with a right angle and have correctly counted side lengths.

Discuss as a class:

- Do you think your triangle is the same size and shape as everyone else's in the class?
- Can you think of how we might check to be sure they are exactly the same size and shape?
  - Some students might suggest measuring to check. Give them a ruler. How does this prove that they are the same size and shape?
  - Some students might suggest cutting the triangles out and laying them on top of one another. Give students scissors to try this method.

Have students compare their triangles.

Students who easily complete the tasks in this section should be assigned to the OBTUSE group in the next activity as obtuse angles are more difficult to draw than acute angles.

3. **Acute and obtuse: Is mine the same as yours?**

Assign students to a group: ACUTE (Pythagoras) or OBTUSE (Hypatia). Pair students so that each pair has a Pythagoras member and a Hypatia member. Students will need rulers, pencils, and BLANK WHITE paper (graph paper lends itself to right angles and might confuse students here.) Instructions should be given as follows:

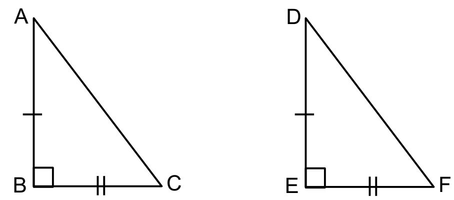
- We are starting with members of the ACUTE group.
- Everyone who is assigned to the OBTUSE group, put your pencils down. Your job right now is to watch and help your partner.
- ACUTE group, use your ruler to make a line segment 4 inches long.
- Once you have finished, have your partner check that the length is 4 inches.
- ACUTE group, make another segment connected to that segment that is also 4 inches AND makes an acute angle.
- Have your partner check the length and make sure the angle is acute.
- Connect the two sides to make a triangle.
- OBTUSE group, pick up your pencils. ACUTE group put pencils down.
- OBTUSE group, use your ruler to make a line segment that is 4 inches long.
- Once you have finished, have your partner check that the length is 4 inches.
- OBTUSE group, make another segment connected to that segment that is also 4 inches AND makes an obtuse angle.
- Have your partner check the length and make sure the angle is obtuse.
- Connect the two sides to make a triangle.
- BOTH groups, cut out your triangles.
- Students compare their triangles using the questions on the *Are You My Twin?* Student Page **[SMJ page 79]**. Give students the definition of **congruent figures** for question 1.

Conclude

4. **Which triangles were congruent?**

All the right triangles in the "investigate" section should be the same due to a theorem students will learn much later in their educational careers. Side Angle Side Theorem. This theorem states that if sides $\overline{AB}$ = $\overline{DF}$ and $\overline{BC}$ = $\overline{FE}$ and angles <AC = <DE then triangles △ ABC = △ DEF are the same.

Ex:



Discuss the acute and obtuse triangles and ask students whether these triangles turned out to be congruent or not. Students should have different triangles because there are so many obtuse and acute angles that can be made. (See *Are You My Twin?* Student Page **[SMJ page 79]** for clarification.)

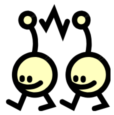
Look Ahead

5. **Transition to circles**
In the next lesson, students will leave the straightedge world and move to the world of circles. Students attempt to invent a way of making the perfect circle and are asked if all circles made in this way are the same size, once again tying in the notion of congruence.

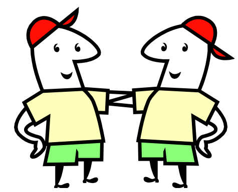
Assess

6. **Congruent and non-congruent triangles**
On a sheet of graph paper, have students create 3 right triangles that are congruent. Then have students draw 2 different right triangles that are non-congruent to the first 3.

7. **Extension to number and operation**
Students complete the *Triangle Sum Theorem* Student Page **[SMJ page 83]**. This activity uses multi-digit addition and inductive reasoning to predict a geometric theorem.



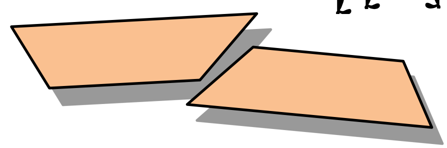
Size-Shape Specialist: ____________________________________



Are You My Twin?

1. Let's define **congruent figures**.

   CONGRUENT FIGURES:



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2. Do the triangles you and your partner made look the same?

____

____

3. Look at the third side that you drew. Which triangle has a LONGER third side, ACUTE or OBTUSE?

____

____

4. Are the two triangles CONGRUENT FIGURES? Why or why not?

____

____

5. COMPARE your acute triangle to the acute triangle of another group. Are they CONGRUENT FIGURES? Why or why not?

____

____

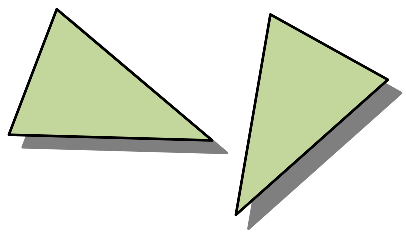
SMJ page 79

6. COMPARE your obtuse triangle to the obtuse triangle of another group. Are they CONGRUENT FIGURES? Why or why not?

____

____

7. Give directions for making two triangles that are CONGRUENT FIGURES.



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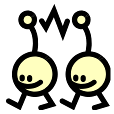
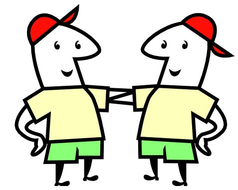
SMJ page 81

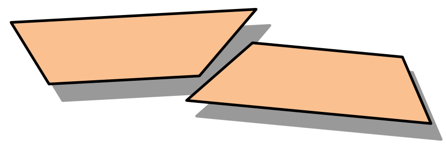




Are You My Twin?
ANSWER KEY

1. Let's define **congruent figures**.

   CONGRUENT FIGURES:

   Sample Answer: Congruent figures are figures that are exactly the same size and shape.

2. Do the triangles you and your partner made look the same?

   Sample Answer: No, the triangles do not look the same because one has an obtuse angle and one does not.

3. Look at the third side that you drew. Which triangle has a LONGER third side, ACUTE or OBTUSE?

   Sample Answer: The obtuse triangle has a longer third side.

4. Are the two triangles CONGRUENT FIGURES? Why or why not?

   Sample Answer: No, the two triangles are not congruent figures because they are not the same size and the angles are different.

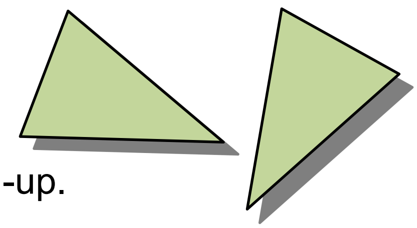
5. COMPARE your acute triangle to the acute triangle of another group. Are they CONGRUENT FIGURES? Why or why not?

   Sample Answer: No, they are not congruent figures because when we put the triangles on top of each other the sides did not match up. (Some students may have triangles that are so close in size that they will answer yes for this question.)

6. COMPARE your obtuse triangle to the obtuse triangle of another group. Are they CONGRUENT FIGURES? Why or why not?

   Sample Answer: No, they are not congruent figures because when we put the triangles on top of each other the sides did not match up. (Some students may have triangles that are so close in size and shape that they will answer yes for this question.)

7. Give directions for making two triangles that are CONGRUENT FIGURES.



- Encourage students to recall the lesson warm-up.

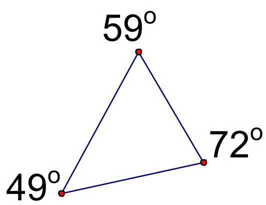
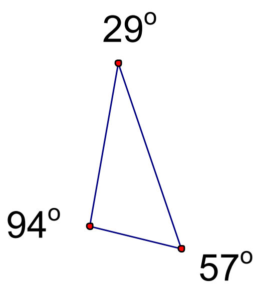
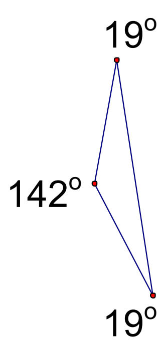
   Sample Answer: Make two right angles using 3 inch segments, then connect the endpoints to make two triangles.

Triangler: ____________________________________

Triangle Sum Theorem

In Geometry, a **theorem** is something that can be proven. One theorem is called the **Triangle Sum Theorem**.

ADD THE 3 ANGLES in each triangle then predict what you think that theorem might say.

1. 	Your Work:
2. 	Your Work:
3. 	Your Work:

4. What do you think the Triangle Sum Theorem says about the sum of the three angles in any triangle?

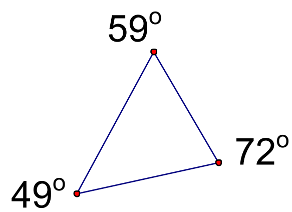
SMJ page 83

Triangle Sum Theorem
ANSWER KEY

In Geometry, a **theorem** is something that can be proven. One theorem is called the **Triangle Sum Theorem**.

ADD THE 3 ANGLES in each triangle then predict what you think that theorem might say.

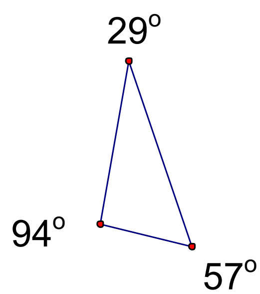
1.



Your Work:

$59^o + 72^o + 49^o = 180^o$

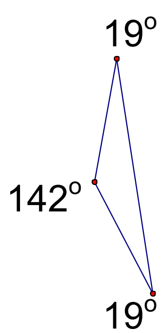
2.



Your Work:

$29^o + 57^o + 94^o = 180^o$

3.



Your Work:

$19^o + 142^o + 19^o = 180^o$

4. What do you think the Triangle Sum Theorem says about the sum of the three angles in any triangle?

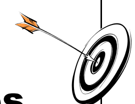
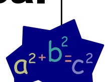
   Sample Answer: The sum of the angles in any triangle is 180^o.

TIME TO SHAPE UP—
GOING IN CIRCLES
(Optional Lesson)

LESSON 10

Big Mathematical Ideas

Why can't the teacher ever draw a perfect circle on the board? The compass is a tool for drawing circles. How does it work? Circles can be found in many places in the real world, like the tires on a car or the wheels on a bike. These objects rely on rotation around the center of the circle.

Lesson Objectives 	● Students will develop a method for drawing "the perfect circle."
Materials 	● Student Page—*Spot the Impostors* [SMJ page 85] ● Student Page—*Making a Circle My Way* [SMJ page 87] ● Student Page—*Three Methods for Making a Circle* [SMJ page 89] ● Student Page—*From Radius to Diameter and Back Again* [SMJ page 91] ● Check Up #2 [SMJ page 93] ● 1 piece of yarn (about 5 inches long) ● 2 different-size paper clips ● Four toothpicks ● Rubber bands ● At least 2 pencils per group ● Blank paper
Mathematical Language 	● **Circle:** Set of points a fixed distance from a center (Do not give this definition to students prior to activity). ● **Radius:** Line segment that starts from the center of a circle to any point on the circle. ● **Diameter:** Line segment passing through the center of a circle and has end points on the circle.

Lesson Preview

Students are challenged to draw a perfect circle using only the materials listed above. With a partner, students develop the concept that a perfect circle is dependent on keeping the pencil the same distance from the center.

Initiate

1. **The imperfect circle**
Students try to draw a perfect circle with only a pencil and piece of paper. Have them try over and over until they have “the perfect circle.” Challenge them with some of the following questions:

- How do you know that your circle is perfect?
- Consider the wheels on a bicycle. How can the person making them tell if they are perfect circles? What might happen if they are not?

The idea is for students to begin to define a perfect circle in their own minds. If the idea of “center” is not mentioned in the discussion, let students grapple with this on their own as they develop a way to create the perfect circle using a limited supply of materials.

A discussion like the following might occur:

Teacher:	Was anybody able to draw a perfect circle?
Sara:	Mine is almost perfect.
Teacher:	What makes it almost perfect?
Sara:	Well, it’s pretty round.
Teacher:	What needs to be changed to make it perfect?
Sara:	This side right here needs to come out more.
Teacher:	What do you mean by “out?”
Sara:	It doesn’t match up with the other side.
Teacher:	Who can explain what Sara means by “match up with the other side?” Bernard?
Bernard:	If you fold your paper through the middle of the circle it would be on top of itself.
Teacher:	Maybe we should try that. Who wants a pair of scissors to try what Bernard suggested?

Have students complete the *Spot the Impostors* Student Page **[SMJ page 85]**. This sheet challenges students to find 3 real circles out of the 8 shapes. Discuss the concept of “perfect circle” further using the shapes on this page as discussion points.

Investigate

2. **The compass alternative**
Break students into groups of three. Similar ability or mixed ability grouping works for this lesson. Give each group the *Making a Circle My*

Way Student Page **[SMJ page 87]**. Ask each group to find a way to make the perfect circle using some or all of the materials. These materials are paperclips (one large, one small), string or yarn, four toothpicks, a rubber band, and two pencils. No matter how they answer, pretend you are unconvinced! Require proof. If students develop a method quickly, challenge them further with the following prompts:

- If you use the same method again, will you get a circle the same size as the first?
- Can you create 3 different-size circles?
- Can you think of a different method to make a circle?

For students who are struggling to find a method, an additional *Three Methods for Making a Circle* Student Page **[SMJ page 89]** has been included. Once students have had some time to work on the task on their own, this page can provide further scaffolding for groups who need it.

Conclude

3. **Inventors share**

Discuss the different methods students used to make circles. Ask why they think their methods resulted in a circle. Have students try other methods that they did not use to see if they agree with their classmates.

A discussion similar to the following might occur:

Teacher:	Who wants to share his or her method for making a perfect circle? How about Selena's group?
Selena:	We used a paper clip and two pencils.
Teacher:	Interesting. How did you do that?
Selena:	Tony put his pencil in the paper clip to hold it in place. Then I put my pencil in the other end and moved it in a circle around his.
Teacher:	Did you make the perfect circle on your first try?
Selena:	No. At first, I kept hitting his arm but once I stood up, I was able to get all the way around.
Teacher:	Tony, do you agree that your circle is perfect?
Tony:	Yes.
Teacher:	How do you know?
Tony:	I think because I never moved my pencil.
Teacher:	How did that make your circle perfect?
Tony:	The paper clip made sure Selena stayed perfectly around my pencil the whole way.
Teacher:	That's really neat. For students who did not use the paper clip in making a circle, try Tony and Selena's method now. Once you've finished, we can discuss some other methods people used.

Assess

4. Explain the compass concept

Students essentially developed make-shift compasses in this lesson. Ask students to choose the method that they understood best based on class discussions and explain why that method produced a “perfect circle.”

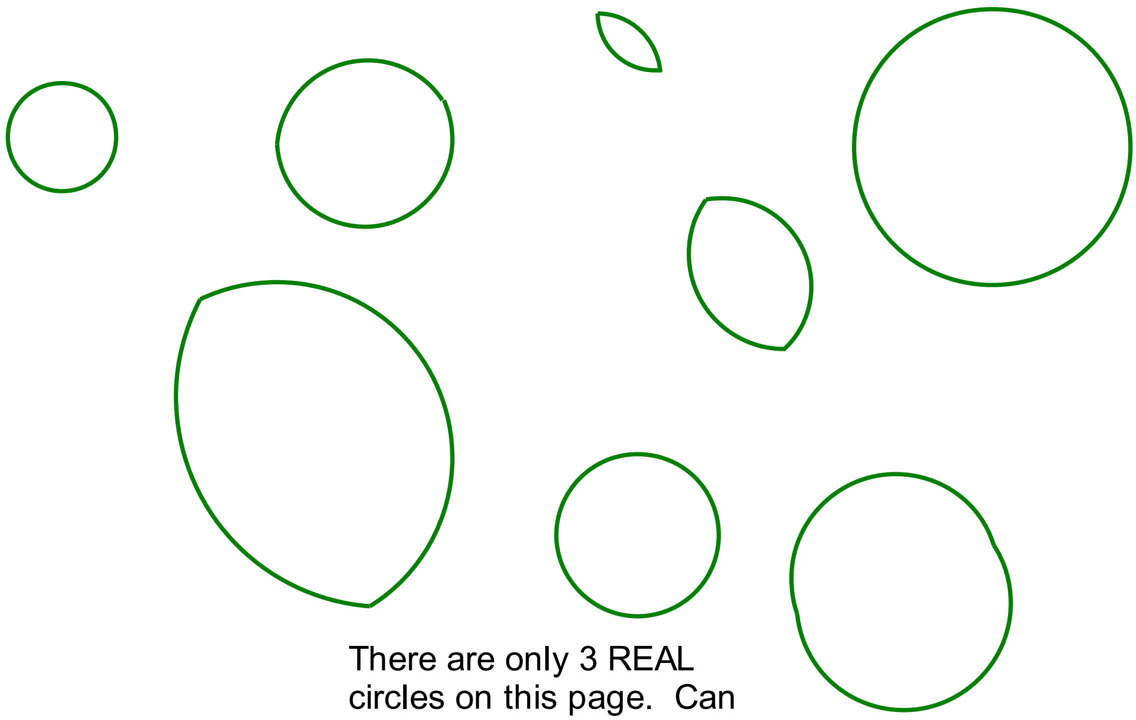
Assess student knowledge on Lessons 5, 6, 7, 8, and 9 by assigning Check Up #2 **[SMJ page 93]**.

5. Extension to number and operation

Students complete the *From Radius to Diameter and Back Again* Student Page **[SMJ page 91]**. This activity focuses on doubling and halving numbers. Discuss definitions of radius and diameter with the students. Use available objects or drawings to practice determining the radius and diameter.

Circle Spotter: ____

Spot the Impostors



There are only 3 REAL circles on this page. Can you spot the impostors? Color the 3 circles.

Explain how you were able to tell which shapes were NOT circles.

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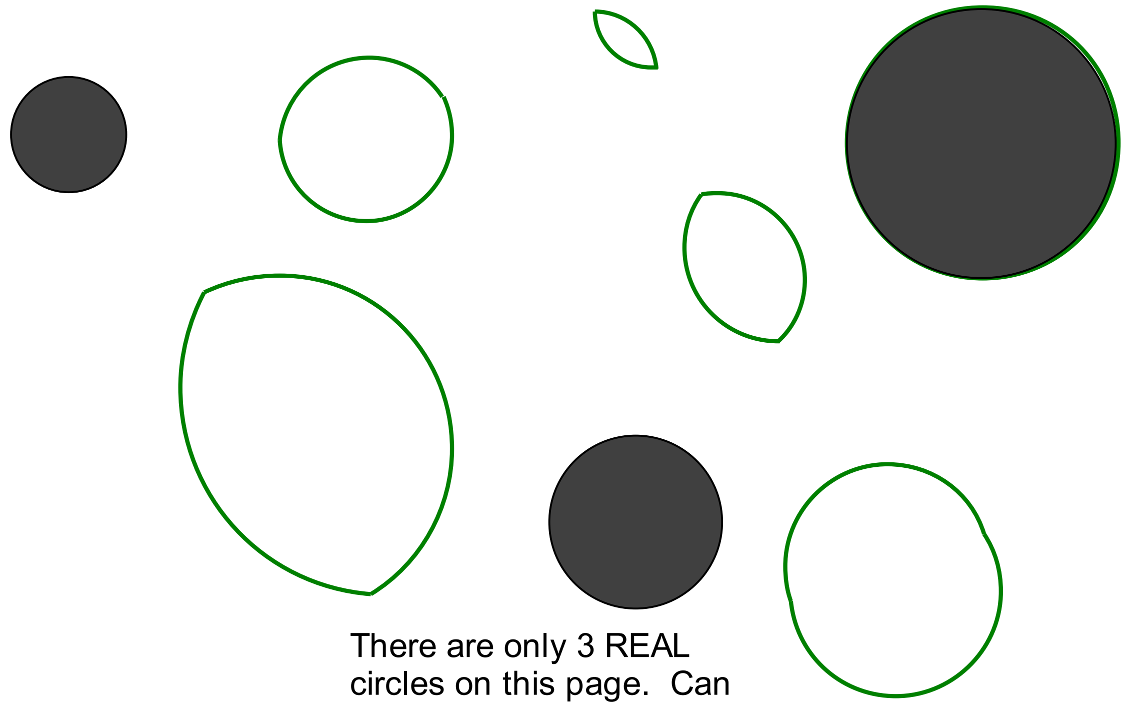
SMJ page 85

Spot the Impostors
ANSWER KEY



There are only 3 REAL circles on this page. Can you spot the impostors? Color the 3 circles.

Explain how you were able to tell which shapes were NOT circles.

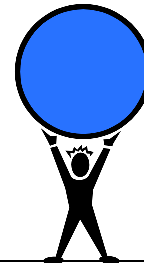
Sample Answer: Many of the shapes had pointy edges that circles do not have. Others I had to imagine folding the circle and realized that the sides would not match up when folded down the middle.

Inventors: ____

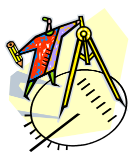
Making a Circle My Way

Develop a way to make a perfect circle using the materials provided. Write instructions for how to make a circle the way you invented.

HOW TO MAKE A PERFECT CIRCLE:



1. ____



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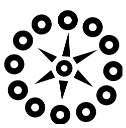
2. ____



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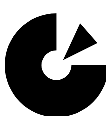
3. ____



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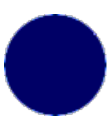
4. ____



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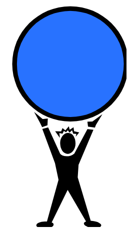
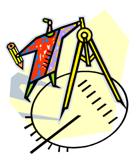
SMJ page 87

Making a Circle My Way
ANSWER KEY

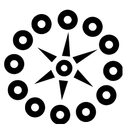
Develop a way to make a perfect circle using the materials provided. Write instructions for how to make a circle the way you invented.

HOW TO MAKE A PERFECT CIRCLE:
(Answers will vary significantly. A sample is provided.)

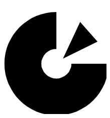




1. *Begin by tying the string around your pencil.*



2. *One person holds the untied end of the string in the center of the paper.*



3. *A second person holds the pencil so that the string is tight and moves it in a circle.*

Inventors: ____

Three Methods for Making a Circle

There are three different methods for making a circle described below. Test each method and decide which works the best.

Method #1

1. Draw a point in the middle of a blank piece of paper.
2. Place a toothpick so that one end touches the point.
3. Make another point at the other end of the toothpick.
4. Rotate the toothpick a little bit and make another point.
5. Continue rotating and making more points until you can draw your circle.

Method #2

1. Put one pencil through a rubber band so that the point is in the middle of a piece of paper.
2. Put another pencil at the other end of the rubber band with its point also on the paper.
3. Ask a partner to turn the paper while you hold the pencils steady.

Method #3

1. Use the same method you used for the rubber band, but this time use a PAPER CLIP.

Which method do you think made the best circle? Explain your thinking.

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SMJ page 89

Three Methods for Making a Circle
ANSWER KEY

Answers will vary.

Diameter Detector: ______________________________

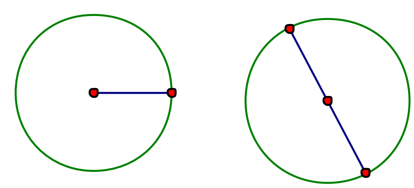
From Radius to Diameter and Back Again

A circle has special parts called the Radius and the Diameter. Use patterns to help you complete the table below.

Circle A shows a line segment that is the radius. Circle B shows a line segment that is the diameter.

A circle with radius…	**Has diameter…**
1 inch	2 inches
2 inches	4 inches
3 inches	6 inches
4 inches	_____ inches
5 inches	_____ inches
_____ inches	12 inches
7 inches	_____ inches
_____ inches	_____ inches
_____ inches	_____ inches

Circle A Circle B



Questions

1. Find the diameter of a circle with radius 24 inches.

2. Find the radius of a circle with diameter 100 inches.

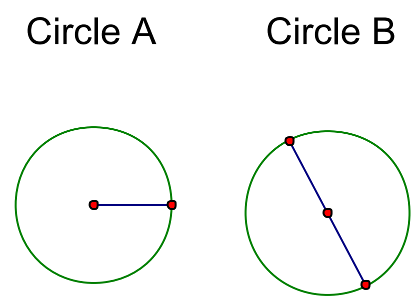
SMJ page 91

From Radius to Diameter and Back Again
ANSWER KEY

A circle has special parts called the Radius and the Diameter. Use patterns to help you complete the table below.

Circle A shows a line segment that is the radius. Circle B shows a line segment that is the diameter.

A circle with radius…	Has diameter…
1 inch	2 inches
2 inches	4 inches
3 inches	6 inches
4 inches	*8* inches
5 inches	*10* inches
6 inches	12 inches
7 inches	*14* inches
8 inches	*16* inches
9 inches	*18* inches



Questions

1. Find the diameter of a circle with radius 24 inches.

   The diameter of a circle with radius 24 inches would be 48 inches.

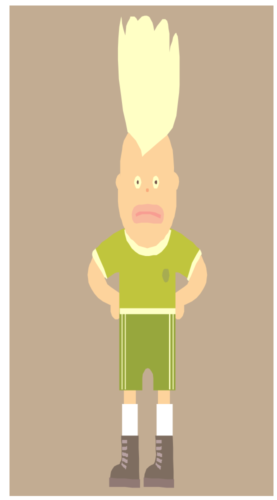
2. Find the radius of a circle with diameter 100 inches.

   The radius of a circle with diameter 100 inches would be 50 inches.

Name: ______________________________ Date: ____________

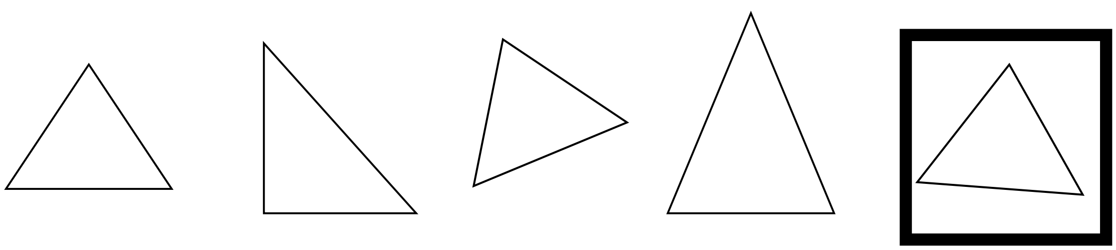
Check Up #2

1. Measure the height of each person in inches. Record your answer below.

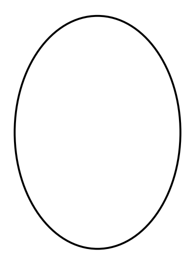


____________________ ____________________

2. Circle the triangles that are congruent to the triangle in the box.



3. Explain how you know the picture IS NOT a perfect circle.



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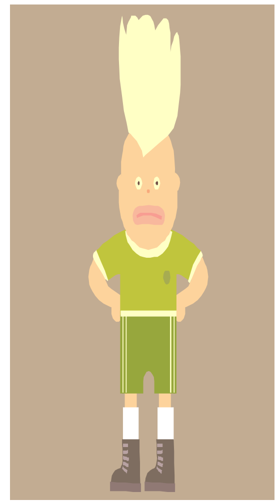
SMJ page 93

Check Up #2
ANSWER KEY

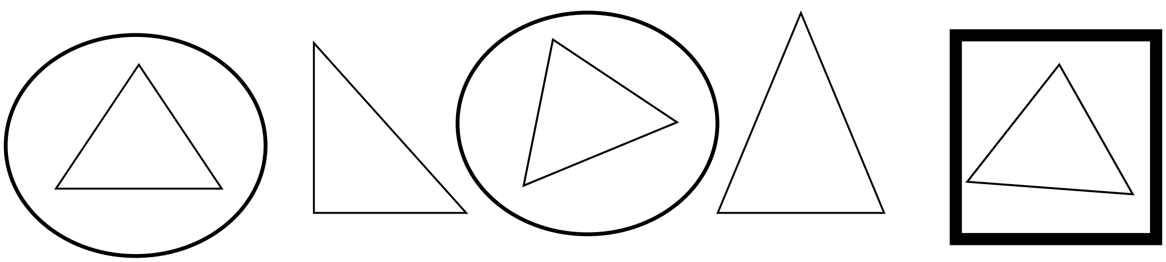
1. Measure the height of each person in inches. Record your answer below.



2 inches

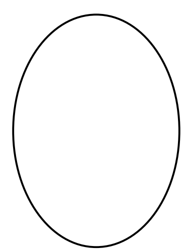
2 1/2 inches

2. Circle the triangles that are congruent to the triangle in the box.



3. Explain how you know the picture IS NOT a perfect circle.

   Sample Answers:
   (a) If you find the center with your pencil, one side of the shape is farther away from the center than the other.
   (b) This shape is longer vertically than horizontally.



TIME TO SHAPE UP—
INFINITE LINES OF SYMMETRY

LESSON 11

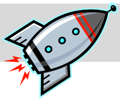
Big Mathematical Ideas

Have you ever stopped to consider how important symmetry is to our world? Would you want to fly in an asymmetrical airplane? Deepen students' understanding of congruence by exploring it in the context of object symmetry.

Lesson Objectives	• Students will fold and draw lines of symmetry for different shapes. • Students will understand that the number of lines of symmetry varies for different shapes.
Materials	• Patty paper shapes (trapezoid, parallelogram, square, heptagon, regular hexagon, regular octagon, circle, and oval) • Patty paper—whole sheet • Foam shapes (symmetrical and asymmetrical) or a set of two-dimensional objects that ate symmetrical/asymmetrical • Rulers • Scissors • Construction paper
Mathematical Language	• **Symmetry:** An object is symmetrical when one half is a mirror image of the other half. • **Regular Polygon:** A polygon with all sides and angles congruent. • **Diagonals of a Polygon:** Line segments in a polygon connecting non-consecutive vertices. (For the purposes of this lesson, students will only discuss diagonals of a square.) • **Asymmetrical:** Not symmetric.

Lesson Preview

Students categorize different shapes by the number of lines of symmetry they have.



Initiate

1. **My line of symmetry**

Ask students to define symmetry. What does it mean for something to have a line of symmetry? Using talk moves such as “add on” or “rephrase,” develop a solid working definition, helping students to understand the following key ideas and connections:

- A line of symmetry creates two equal halves of a figure.
- A line of symmetry divides a figure into two congruent parts.
- When you fold a shape on its line of symmetry, sides and vertices match up.
- Some figures have no lines of symmetry.
- Some figures have several lines of symmetry.

Ask students whether or not humans have any lines of symmetry. Ask them to stand up and draw their lines of symmetry using their index fingers. Raise one arm up and leave one arm down and ask students whether or not you have a line of symmetry. They should indicate that you are not symmetric because you are not the same on both sides when you stand that way. Tell all students to stand “asymmetrically,” or not symmetric. Then ask them to stand symmetrically. Some students may point out that certain clothing patterns make them not symmetric.

Investigate

2. **Symmetry of different polygons**

Use students’ performance in the Initiate tasks to help decide how they should be assigned to the next task. Distribute patty paper shapes for this activity. The following choices are available for each grouping level:

Pythagoras—trapezoid, parallelogram
Hypatia & Euclid—regular heptagon, regular hexagon, regular octagon

Each student should have his/her own shape to fold. If students sit in pairs or groups, give each student within the group a different shape. Ask students whether or not they believe their shapes have any lines of symmetry. If they do, ask them to sketch one line of symmetry in pencil. Then ask how they can “prove” that the line is a line of symmetry. Students can check their symmetry lines by folding and seeing if the shapes, sides, and vertices match up.

One common error that students make is to fold their shapes over and over again without opening to check the lines. This may happen in groups working with figures with several symmetry lines. If students do this, they create several lines on their shapes some of which are not lines of symmetry. If this happens, have students open their shapes and check the lines. Offer students a new shape to start over.

Use the following questions and extensions to guide students in different groups (optional section for groups/students who finish early):

Pythagoras—
- Did your shape have any lines of symmetry? How do you know?
- Does your shape have any other lines of symmetry?
- Can you draw a different quadrilateral that might have more lines of symmetry?

Hypatia & Euclid—
- Do you think your shape has any more lines of symmetry? Sketch and fold them.
- Can you draw a heptagon/hexagon/octagon with *fewer* lines of symmetry? With *more* lines of symmetry?

If it becomes difficult to manage the different groups, hold a class discussion focusing on the questions above. Have groups share findings about their different shapes. Students should understand the following concepts prior to the next activity:

- A symmetry line divides a figure into two congruent figures
- Symmetry can be checked by folding
- Some figures have more lines of symmetry than others

3. **Symmetry of the "round" shapes**

Explain to students that the next shape is an oval, also called an ellipse. Ask students to make an oval with their hands to make sure everyone knows what an oval looks like. Then, ask them to predict the number of lines of symmetry for the oval. Give students patty paper ovals and tell them to "test their theories."

Discuss the number of lines of symmetry on the oval. Students should have found two lines, though a common error is to count each line twice in which case they would have counted four. If this error occurs, hold up an oval with the lines drawn in and show them physically why they may have counted each twice. Explain that even the best mathematicians sometimes make this mistake.

Next, ask students to predict the number of lines of symmetry of a circle. Predictions tend to be very large as some students are already able to visualize the many symmetry lines. Give each student a patty paper circle to test his/her prediction. Ask students to fold and count the lines of symmetry. [If a student notices that the number of folds will be infinite during this task, challenge that student to think of a way to convince the class of his/her idea.] As students are folding, prompt their thinking by asking, “Are you sure you’ve found them all?” Once students get to a point where they believe they are finished or have decided that they can continue folding and/or drawing lines, ask for answers. Try starting with a student who only found one or two lines of symmetry and then continue to elicit different responses. Use the following questions to help guide the discussion:

- How many lines of symmetry did you find?
- Why does a circle have so many lines of symmetry?
- Can you draw a line of symmetry anywhere on the circle? [Some students will answer yes to this question. Ask them to see if they can find a place on the circle to fold it that is *not* a line of symmetry. Any line that does not go through the circle’s center will not be a line of symmetry.]

4. The rectangle-square challenge

Tell students that the next shape they will get is a square. Ask them to predict the number of lines of symmetry of the square. Tell students that they have to open up their square after each fold before they fold a new line of symmetry. If they continue to fold the shape without opening it, they will generate lines that are not symmetry lines for the original square.

Use an overhead or chalkboard to have students draw and count the lines of symmetry for the square. Tell students that one of the cool things about a square is that its diagonals are lines of symmetry.

Explain to students that you have a challenge for them. Give each student a whole sheet of 4 3/4 x 5 patty paper and ask them to use “folding” to determine whether or not the patty paper is a square. If students come up with an answer quickly, pretend you are unconvinced and ask them to use a ruler to provide additional evidence. Use the following prompts to guide a discussion of the activity:

- What did you find out? Is it a square? If not, what shape is it?
- Give me one way that you determined it is not a square.
- Did anybody use a different method to show it is not a square?

A discussion might look something like the following:

Teacher: Gary, you said the shape is not a square. Can you explain why not?
Gary: When we folded the square like this *(folds his patty paper)*, it was a line of symmetry. But it didn’t work for this shape.
Teacher: Did anybody have a similar finding?
Brandy: I did the same thing. I remembered that we learned the diagonal of the square is a symmetry line but it didn’t work for this shape.

Teacher: Did anybody prove it in a different way?
Lola: I measured it with a ruler and found out one side was 4 3/4 inches and the other side was 5 inches.
Teacher: It sounds like we agree that it is not a square. What shape is it?
Benny: It's a rectangle.
Teacher: How do you know?
Benny: Because it looks like a square with the four right angles but when you measure it, you find out that its sides are not all equal. Just these two and these two *(points to opposite sides).*

Conclude

5. Offer students an additional sheet of patty paper to take home. Have them ask somebody at home what shape they think it is. Tell students to teach the person (or let him/her try for him/herself) how you can use folding to show whether or not a figure is a square.

Assess

6. **Symmetry sort (optional performance assessment)**
Assign students to pairs or groups of three. It would be helpful to group students by ability according to how they performed on tasks related to congruence (Unit Test question 3 or the previous lessons). Give each pair or group a set of foam shapes and a piece of construction paper. Ask students to create labels using the scissors and construction paper. The signs should have the following labels:

- 0 Lines of Symmetry
- 1 Line of Symmetry
- 2 Lines of Symmetry
- 3 Lines of Symmetry
- 4 Lines of Symmetry
- 5 Lines of Symmetry
- 6 Lines of Symmetry

Ask students to find one of each shape. The shapes they should have are butterfly, snowflake, flower, star, boat, heart, and tree. Tell them to put each shape with the appropriate label. For example, if a shape has no lines of symmetry it should be placed by the label "0 Lines of Symmetry."

The students should have the following "sort."

- 0 lines of symmetry—the boat
- 1 line of symmetry—the heart, the tree, the butterfly
- 5 lines of symmetry—the star
- 6 lines of symmetry—the flower, the snowflake

If students have any shape sorted incorrectly, probe the group by asking them to show you the lines of symmetry. Alternately, you might ask them to trace the shape and draw the lines of symmetry to see if they are missing any or have too many.

Increase the level of challenge using the following tasks:

- I see that you have no shapes in the 2, 3, and 4 lines of symmetry categories. Can you draw a shape that would fit into each of those categories?
- What could you change about the boat so it would have one line of symmetry? What could you change about the butterfly so it would have two lines of symmetry?

LIVING ON THE EDGE—
THE ANTS GO MARCHING

LESSON 12

Big Mathematical Ideas

The distance around an object is called its perimeter. For circular objects, the term is circumference. In the real world, we see fences, tracks, sidewalks, and borders that go around perimeters.

Lesson Objectives	• Students will be able to apply the concept of perimeter to complete a task.
Materials	• Student Page—*Exercising Ants (Hypatia and Euclid)* [SMJ pages 95 & 99] • Student Page—*Which Pool Has the Biggest Perimeter?* [SMJ page 103] • Rulers • Calculator
Mathematical Language	• **Perimeter:** The distance around a figure. • **Unit:** A standard of measurement.

Lesson Preview

Students look at perimeter as the distance around the outside edge of a rectangular figure. They calculate the distance around the outside of a swimming pool then determine how long it takes "exercising ants" to make their way around.

Initiate

1. **Distance around my desk**

What is the distance around the outside edge of your desk? Distribute rulers and have students determine the distance around the outside edge of their desk in inches. If the desks are not uniform in size, pair students and instruct them to measure a specific type of desk. Have them record this measurement.

Explain to students that they have measured the perimeter and provide them with the definition.

Investigate

2. Working as a class

Pair students into Hypatia and Euclid pairs based on measurement proficiency (use pre-assessment and/or Lesson 5 as indicators). Ask each pair to complete the *Exercising Ants* Student Pages **[SMJ pages 95 or 99]**. Help students develop justifications for their answers.

Once students have finished, discuss the answers to Tasks 1-3. Focus on the methods students selected to determine their answers. Solution strategies might include:

- Add up all four sides.
- Multiply the long side by 2, the short side by 2, and then add.
- Pictures—divide the existing picture into 3-foot intervals.
- Calculate one side of the pool at a time and then add.
- Estimate because an exact solution strategy cannot be determined by the student.
- Other methods effectively employed by students.

Conclude

3. Bringing in the real world

Ask students to identify some real-world examples of places where people or animals are timed to go around something. Some examples include:

- Runners
- Race car drivers
- Horses

Look Ahead

4. Perimeter

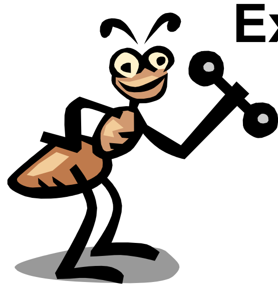
Students continue work with perimeter as they focus on “units” represented by graph paper boxes. They also participate in a perimeter lab involving estimation rather than precision.

Assess

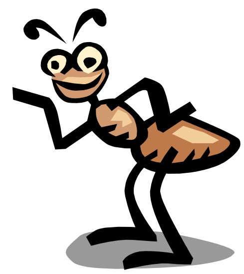
5. Extension to number and operation

Students complete the *Which Pool Has the Biggest Perimeter?* Student Page **[SMJ page 103]**. This activity allows students to practice multi-digit addition in the context of perimeter.

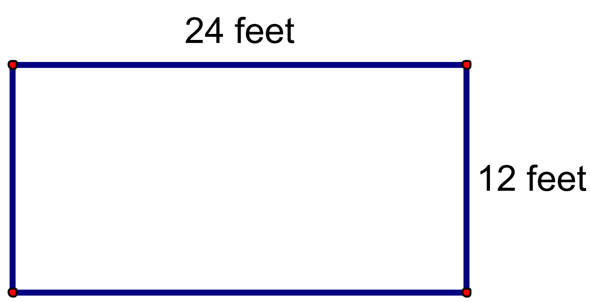
Ant Coach: ____



Exercising Ants



Task 1
Annie the ant wants to start exercising. She decides to do one lap around the rectangular pool each day.



How many feet will Annie walk if she goes all the way around the edge of the pool? Explain your answer.

Task 2
Annie's brother, Arthur Ant, decides that he, too, will start exercising. He decides to walk around the pool twice. How many feet does he walk?
(Hint: Use your answer from Task 1 to help you.)

SMJ page 95

Task 3
Angela Ant brags that she walked around the pool many times and went 360 feet, but she forgot how many times she went around. Help Angela figure out how many laps she did. Show your work below.

1 time around the pool is ______ feet.

2 times around the pool is ______feet.

3 times around the pool is ______feet.

4 times around the pool is ______feet.

5 times around the pool is ______feet.

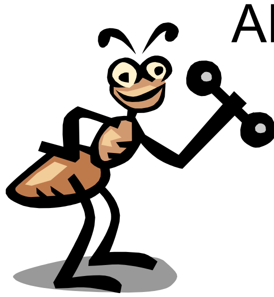
6 times around the pool is ______feet.

Using the information above, I can tell that Angela went around the pool ____ times.

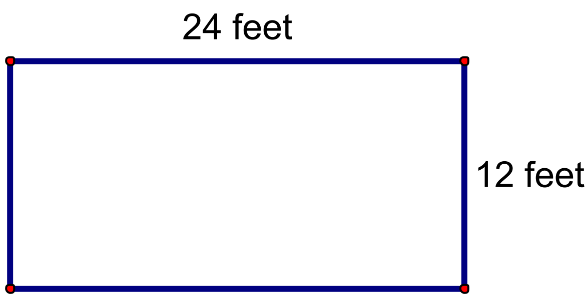
SMJ page 97

Hypatia

Exercising Ants
ANSWER KEY

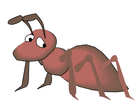



Task 1
Annie the ant wants to start exercising. She decides to do one lap around the rectangular pool each day.



How many feet will Annie walk if she goes all the way around the edge of the pool? Explain your answer.

Sample Answer: 24 + 12 + 24 + 12 = 72
Annie will walk 72 feet if she goes around the edge of the pool.



Task 2
Annie's brother, Arthur Ant, decides that he, too, will start exercising. He decides to walk around the pool twice. How many feet does he walk?
(Hint: Use your answer from Task 1 to help you.)

Sample Answer: Annie walks 72 feet. Since Arthur is walking around the pool two times, he will walk 72 + 72 or 144 feet.

Task 3

Angela Ant brags that she walked around the pool many times and went 360 feet, but she forgot how many times she went around. Help Angela figure out how many laps she did. Show your work below.

1 time around the pool is *72* feet.

2 times around the pool is *144* feet.

3 times around the pool is *216* feet.

4 times around the pool is *288* feet.

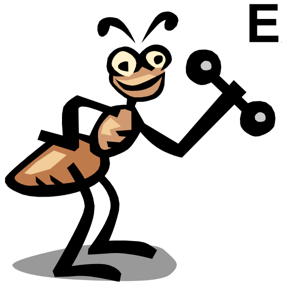
5 times around the pool is *360* feet.

6 times around the pool is *432* feet.

Using the information above, I can tell that Angela went around the pool *5* times.

Euclid

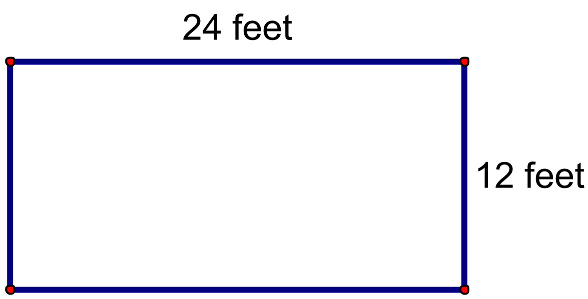
Ant Coach: ____



Exercising Ants

Task 1

Annie the ant wants to start exercising. She decides to do one lap around the rectangular pool each day.



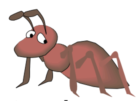
How many feet will Annie walk if she goes all the way around the edge of the pool? Explain your answer.

Task 2

Annie's brother, Arthur Ant, decides that he, too, will start exercising. He decides to walk around the pool twice. How many feet does he walk?

SMJ page 99

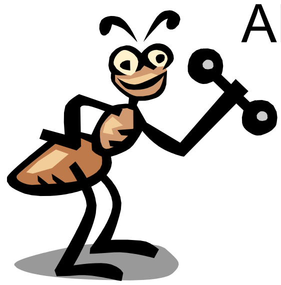
Task 3
Angela Ant brags that she walked around the pool many times and went 360 feet, but she forgot how many times she went around. Help Angela figure out how many laps she did. Show your work below.

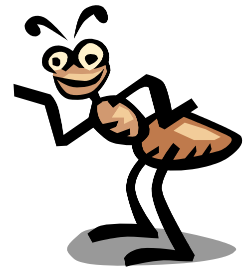


Challenge Task
If Annie can walk 3 feet in 1 minute, how many minutes will it take her to get around the pool one time? Explain your answer.

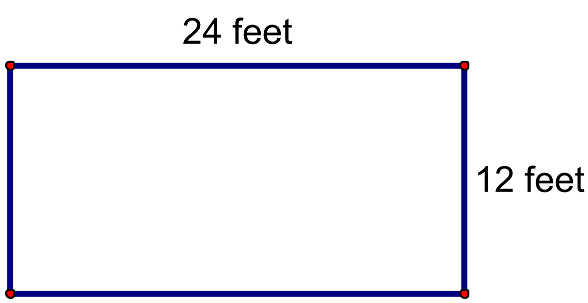
SMJ page 101

Exercising Ants
ANSWER KEY



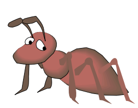


Task 1
Annie the ant wants to start exercising. She decides to do one lap around the rectangular pool each day.



How many feet will Annie walk if she goes all the way around the edge of the pool? Explain your answer.

Sample Answer: 24 + 12 + 24 + 12 = 72
Annie will walk 72 feet if she goes around the edge of the pool.



Task 2
Annie's brother, Arthur Ant, decides that he, too, will start exercising. He decides to walk around the pool twice. How many feet does he walk?

Sample Answer: Annie walks 72 feet. Since Arthur is walking around the pool two times, he will walk 72 + 72 or 144 feet.

Task 3

Angela Ant brags that she walked around the pool many times and went 360 feet, but she forgot how many times she went around. Help Angela figure out how many laps she did. Show your work below.

Sample Answer: In Task 3, I found out that 2 times around the pool is 144 feet. That means that 4 times around the pool is 144 + 144 = 288 feet. Then, 288 + 72 = 360, so 4 times around the pool plus 1 time (72 feet) equals the 360. Angela walked around the pool 5 times.

Challenge Task

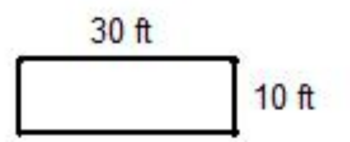
If Annie can walk 3 feet in 1 minute, how many minutes will it take her to get around the pool? Explain your answer.

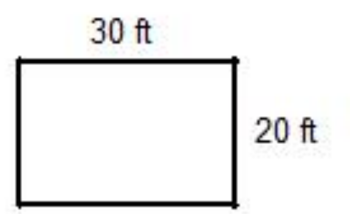
Sample Answer: 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 = 24. It will take 8 minutes to walk the side that is 24 feet since each 3 represents 1 minute. 3 + 3 + 3 + 3 = 12 so it will take 4 minutes to walk the side that is 12 feet long. If I double both the 8 minutes and the 4 minutes because there are two sides of each length, it will take 8 + 8 + 4 + 4 = 24 minutes to walk around the pool.

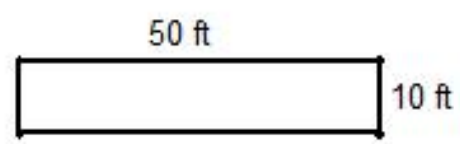
Distance Diver: ____

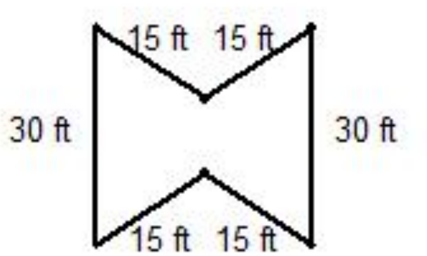
Which Pool Has the Biggest Perimeter?

Directions: Find the distance around the outside edge of each pool below.









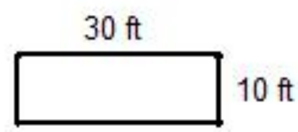
Which of the pools above has the biggest perimeter?

Do you think the pool with the biggest perimeter holds the most water? Explain.

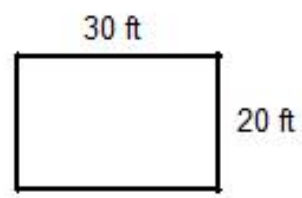
SMJ page 103

Which Pool Has the Biggest Perimeter?
ANSWER KEY

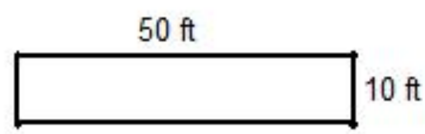
Directions: Find the distance around the outside edge of each pool below.



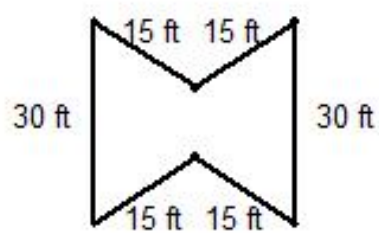
30 + 30 + 10 + 10 = 80 feet



30 + 30 + 20 + 20 = 100 feet



50 + 50 + 10 + 10 = 120 feet



30 + 30 + 15 + 15 + 15 + 15 = 120 feet

Which of the pools above has the biggest perimeter?

Sample Answer: C & D both have a perimeter of 120 feet which is larger than A & B.

Do you think the pool with the biggest perimeter holds the most water? Explain.

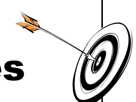
Sample Answer: Since the depth can vary from pool to pool, the amount of water may or may not be greater in the pool with the largest perimeter.

LIVING ON THE EDGE—
RULER OF THE RULER

LESSON 13

Big Mathematical Ideas

Rulers can be confusing. What are all those lines doing there? Those lines allow us to measure more precisely. Precise measurement is essential to many real-world applications including perimeter (previous lesson) and area (forthcoming lesson). Imagine a world in which only whole inches existed! This lesson gets students to contemplate the necessity of the "in-between" lines on a ruler.

Lesson Objectives 	• Students will be able to list reasons why standard measurement units are necessary. • Students will be able to explain why there are marks between numbers on a ruler. • Students will be able to measure to the nearest 1/4, 1/2, 3/4, or whole inch.
Materials 	• Student Page—*Who Built My House?* [SMJ page 105] • Student Page—*Ruler Without a Ruler* [SMJ page 107] • Rulers

Mathematical Language	**Pictorial representations of vocabulary words are sufficient for students in this lesson.** ● **Line Segment:** Part of a line that includes two points, called endpoints, and all the points between them.  ● **Endpoints:** The points on a line segment that show where it begins and ends. ● **Point:** An exact location in space, usually represented by a dot.  point A ● **Numerator:** the part of a fraction that is above the line and is divided by the denominator. ● **Denominator:** the part of a fraction that is below the line and divides the numerator.

Lesson Preview

This lesson gives an overview of the relationship between the number of marks in between whole numbers on a ruler and precision of measurement. Students learn to identify the markings for 1/4, 1/2, 3/4, and 1 inch.

Initiate

1. **Why guess when you can measure?**
Questions for student consideration:

- How many of you have been to the doctor lately?
- When you get there, does the doctor look at you and say, "You look like you're about 43 inches tall and around 80 pounds"?
- What if the doctors of *Student's Name* and *Student's Name* decided they would measure them using the number of hands?
- Call up two students and measure one using teacher's hands and the other student's hands. What is wrong with this method?

(Students should explain that doctors measure their height and weight using instruments appropriate for the measurement. Hands are different sizes. Example: 5 teacher hands is a different measurement than 5 student hands.)

A discussion such as the following might occur:

Teacher: Who wants to explain why doctors do not use their hands to measure patients' heights?

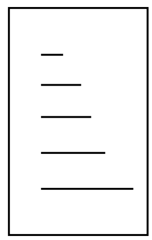
Trivon:	Doctors' hands might not all be the same size.
Teacher:	Why is that important?
Trivon:	One doctor might say ten hands and one might say eleven hands.
Teacher:	Can you think of a time when that would be a problem?
Trivon:	Maybe a sport in which they want you to be tall, like basketball.
Teacher:	That's a good example. What if two workers are building a house and measuring in hands?
Maria:	The house might be bent.
Teacher:	Bent?
Maria:	Like one side taller than the other.
Teacher:	Why?
Maria:	If one worker has different size hands than the other.

Investigate

2. Beyond halves and wholes

Give each student a ruler and a sheet of drawing paper. Ask students to draw segments with the following lengths on their papers (demonstrate on board as shown in picture):

1/2 inch
1 inch
1 1/2 inches
2 inches
2 1/2 inches

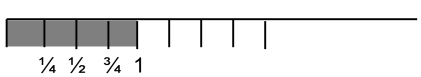


Ask students to perform the following task:

- Draw a line segment that is *longer than* the 1/2 inch segment but *shorter than* the 1 inch segment.
- Ask: How long is the segment you drew?
  - *Some students who received the Euclid version of the task in Lesson 5 may remember that 3/4 is between 1/2 and 1.*

If students have never worked with fractions before, it is important to provide a conceptual understanding of the numbers 1/4, 1/2, and 3/4. The following activity may help students grapple with these ideas.

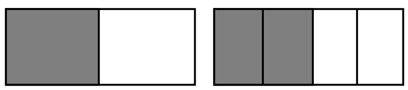
Draw a diagram representing the ruler from 0 to 2 inches on the board.



Shade the area from 0 to 1 inch. Tell students that in the fraction 1/4, 1 is called the **numerator** and 4 is called the **denominator** (label them off to the side). The fraction tells us that out of the 4 sections from 0 to 1, we are talking about only one. We can also say 1/4 as "one out of four." On the back of the drawing paper, ask students to draw a segment that is 1/4 inch long and label it "1/4 inch."

Repeat the dialogue with 3/4 inch, indicating that it represents 3 sections out of 4. Ask students to draw and label a 3/4 inch segment. Once they have finished, challenge them with the following question:

- Ask: Does anybody know another way of writing 1/2 so that it has a denominator of 4? *(Remember wait time. Encourage students to look for a pattern. Scaffold by writing the fraction with the top number missing if necessary.) (Answer: 2/4)*
- Reinforce the notion that 1/2 = 2/4 using pictures like those shown below.



Extend: Can anybody name a fraction equal to 1/2 and 2/4 that has a denominator of 8?

Finally, ask students to interpret the meaning of some other segments they drew.

- 1/2 inch segment—If the inch were broken into only 2 equal parts, 1/2 inch would be 1 out of 2.
- 1 1/2 inch segment—This segment is one whole inch plus another half.
- 2 1/2 inch segment—This segment is 2 whole inches plus half of another inch.

3. Who built my house?

Read the poem "Who Built My House?" and direct students to the accompanying *Who Built My House?* Student Page **[SMJ page 105]**. Ask students to draw a picture using the measurements in the poem. Tell students to draw roofs on their houses that they measure to fill in the blank spaces in the poem. Students can answer question #3 independently.

Conclude

4. When do we need exact measurements?

Ask students to identify some real-world examples of when exact measurements are necessary. Some examples may include:

- Construction
- Doctor's office (as in Initiate section)
- Manufacturing
- Football

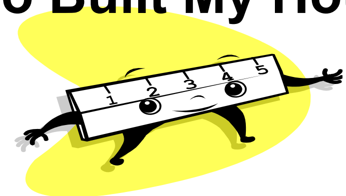
Assess

5. Extension to number and operation

Students complete the *Ruler Without a Ruler* Student Page **[SMJ page 107]** by using a segment of given length to estimate the measures of other line segments. Have students brainstorm strategies and try one before they take it home. It is important that students know these are estimates.

Measurement Authority: ________________________________

Who Built My House?



Read the poem “Who Built My House?” Answer the questions that follow.

Who Built My House?

I wonder who built my house
One side is three inches high
The other side three and a half
I cannot figure out why

One side of my door is one and a half
And the other one and three-fourths
From the ground to the top of each side
Measured in inches of course

I need a ladder to measure my roof
I’m worried that it will be wrong
One side measures _______ inches
The other is ________ inches long

SMJ page 105

1. Draw a picture of the house described in the poem using a ruler to make your measurements match those in the poem.

2. The length of each side of the roof is not given in the poem. Measure the sides of your roof in inches and write your answers in the blanks in the poem.

3. Explain why it is important for house builders to measure accurately.

____

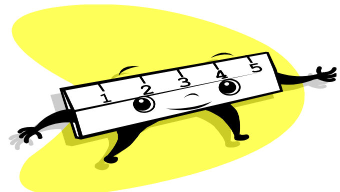
____

____

____

SMJ page 106

Who Built My House?
ANSWER KEY



Read the poem "Who Built My House?" Answer the questions that follow.

Who Built My House?

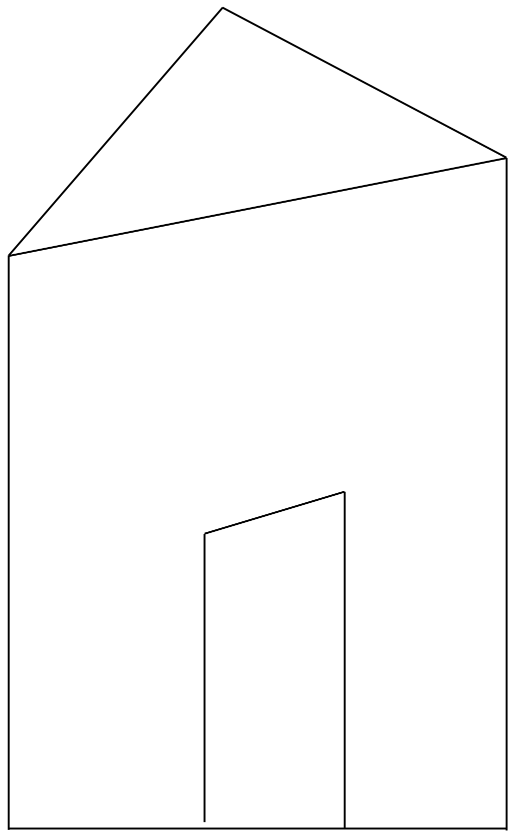
I wonder who built my house
One side is three inches high
The other side three and a half
I cannot figure out why

One side of my door is one and a half
And the other one and three-fourths
From the ground to the top of each side
Measured in inches of course

I need a ladder to measure my roof
I'm worried that it will be wrong
One side measures ___*1 1/4*_ inches
The other is _____*3/4*____inches long
(Sample answers for sample drawing. Students' responses will vary.)

1. Draw a picture of the house described in the poem using a ruler to make your measurements match those in the poem.

   Sample Drawing



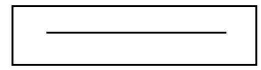
2. The length of each side of the roof is not given in the poem. Measure the sides of your roof in inches and write your answers in the blanks in the poem.

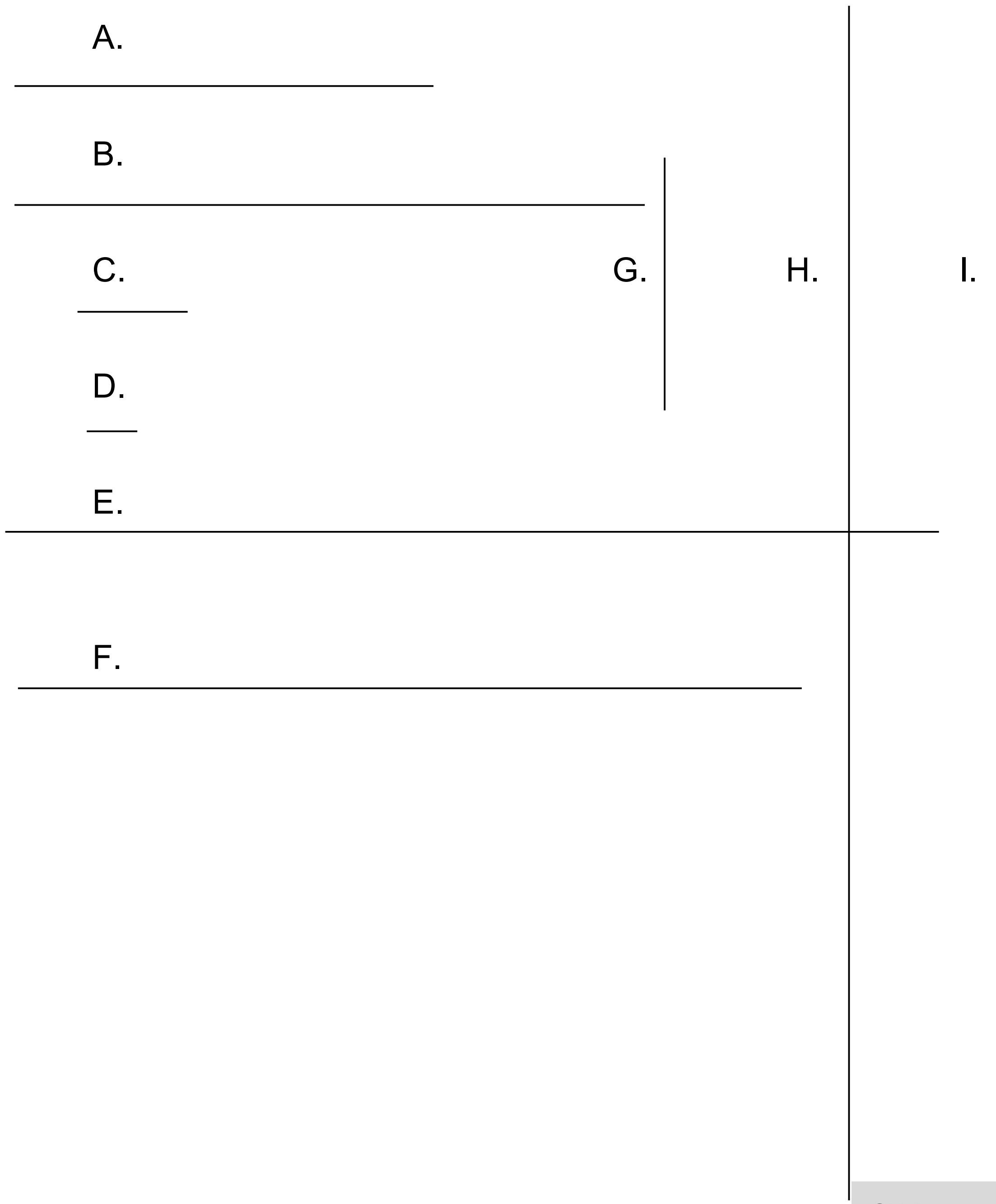
3. Explain why it is important for house builders to measure accurately.

   Sample Response: It is important for house builders to measure accurately because people expect certain parts of the house to be equal in length (congruent). For example, the height of the left and right sides of the house and door are usually equal.

Ruler Without a Ruler

The line segment in the box is 1 inch long. Use that segment to ESTIMATE the lengths of the other segments. (You may cut the segment out and use it as a ruler to help you.)





SMJ page 107

Ruler Without a Ruler
ANSWER KEY

The line segment in the box is 1 inch long. Use that segment to ESTIMATE the lengths of the other segments. (You may cut the segment out and use it as a ruler to help you.)

A. *Acceptable Range: 2-3 inches*

B. *Acceptable Range: 3-4 inches*

C. *Acceptable Range: 1/4-3/4 inch*

D. *Acceptable Range: 0 -1/2 inch*

E. *Acceptable Range: 5-6 inches*

F. *Acceptable Range: 4-5 inches*

G. *Acceptable Range: 1-2 inches*

H. *Acceptable Range: 6 1/2-7 1/2 inches*

I. *Acceptable Range: 3/4-1 1/4 inches*

LIVING ON THE EDGE—
THE ?-INCH RULER

LESSON 14

Big Mathematical Ideas

Sometimes we don't have a ruler handy but need to know the approximate length of something—perhaps a piece of furniture we are trying to squeeze into the corner of the kitchen. We can use an object of known length to get a good estimate of the size.

Lesson Objectives	• Students will be able to use an object of known length to estimate the perimeter of another object.
Materials	• Student Page—*The ?-Inch "Ruler"* [SMJ page 109] • A variety of objects with measurable perimeters or circumferences ○ Examples: Tupperware lids, paper, box tops, etc. (three-dimensional items should be uniform in size so there is no confusion over which part to measure) • Measurement strips labeled by length of various sizes (NO RULERS!)
Mathematical Language	• **Perimeter:** The distance around a figure. • **Circumference:** The distance around a circle. • **Estimate:** An answer that is as mathematically close to the real answer as possible.

Lesson Preview

Students use a variety of measurement strips of given length to estimate the perimeters of objects.

Initiate

1. **A game of charades**
Ask students if they know how to play charades. If anyone does not know, explain that it is a game where you try to get the other players to guess something without using any words. Start with a non-math example and have a student try to get the class to guess an action such as playing the violin or rowing a boat.

Next, ask how many students know what perimeter means. Look around to see if there are students who do *not* know. Ask a student who knows what the term means to define it for the rest of the class *without using any words!* Then, ask another student who did not know the definition if he/she can figure out what the definition is and to define it in words. This is a great opportunity to have students elaborate on one another's comments to construct a definition. The following is an example of a potential conversation:

Teacher:	Now that we have *seen* the definition of perimeter, can somebody define it in words?
Ellen:	It means to go around the object.
Teacher:	Around the object. Would anybody like to add on to what Ellen said?
Toby:	You have to measure it and add up all the sides.
Teacher:	So, you think we should measure and add up all the sides. Does somebody have a different idea?
Ariel:	What if it's a circle? I don't think a circle has sides.
Teacher:	Hmmm. I like your thinking. Does anybody know what the perimeter of a circle is called? *(Class is unsure.)*
Teacher:	The perimeter of a circle is called the circumference. Everybody say circumference. *(Class says the word "circumference.")*
Teacher:	(*Holds up a circular object).* Does anybody have an idea of how we might measure the circumference of this circle?
Robert:	My dad uses a tape measure. I think that will work because it bends around.
Teacher:	That's one good idea. Does anybody have a different idea?
Thomas:	I think you can use string.
Teacher:	How would you use the string, Thomas?
Thomas:	I could put it around the edge and then measure how long the string is.

Investigate

2. **Estimation time**

Assign students to pairs or groups of three based on pre-assessment data and their performances on Check Up #2 measurement items. Direct students to open to *The ?-Inch Ruler* Student Page **[SMJ page 109]**. Give each group one set of varying length strips which are organized according to mathematicians' names. Give each group an object to measure. Provide the following directions:

- Record the name of the object in box #1.
- You have been given cardboard strips of different lengths. Find a way to use these strips to ESTIMATE the perimeter of your object in inches.
- Record the perimeter in the box in the table next to the object under the column labeled "Perimeter."
- In the first row of the third column, describe how you were able to measure the object.
- Once you have finished, I will have you trade objects with another group.

Circulate room and question students about their methods.

- How are you dealing with that 1/2 inch?
- Show me how you measured the perimeter of that one.

Instruct students when it is time to trade objects or have them find their own around the room. It is good to have different groups measure the same object to compare estimates but it is not necessary for all groups to have the same list of objects.

During this lab, students should be discussing whole and fractional pieces of their rulers and trying to agree on good estimates for objects. Probe their thinking by having them explain their methods and results. Encourage students to challenge one another's thinking.

Conclude

3. **Which ruler prevailed?**
Conclude with a discussion of how students used different measurement units to get their estimates. It may be helpful to have students come to the front of the room and show the class how they used the different lengths to obtain the best estimate for a given object. Although students should have similar answers when the object is the same, their methodologies will often produce different estimates. If large discrepancies are found, the class can work together to decide on a good estimate or check using a standard measurement tool, such as a ruler or a measuring tape.

Look Ahead

4. **From estimates to exact measurements**
Students should now have an understanding of how to find the distance around an object. In the next lesson, students look at some exact perimeters measured in "units" on graph paper.

Assess

5. **Connecting perimeter and measurement**
Have students draw a figure that has a perimeter of 24 inches. It can have as many sides as they want, but the total perimeter must be 24 inches. Tell students to label each side with its length and show or explain how they know the perimeter is 24 inches. Increase the level of challenge for students who finish early by asking them to develop a way to make a round object with a 24 inch circumference They may want a piece of string or yarn to complete this task.

Master of the Mystery Ruler: ____________________________

The ?-Inch "Ruler"

Directions: Use the strip of paper provided by your teacher to estimate the perimeter of 4 objects around the room.

- ❖ WRITE the name of the object in the table.
- ❖ ESTIMATE the perimeter of each object using the strip of paper provided.
- ❖ RECORD your estimates in the table.
- ❖ DESCRIBE the process your group used to estimate each perimeter.

MY RULER SIZE IS ________ INCHES.

Object	Perimeter (in inches)	Describe the process used to estimate the perimeter of this object.
1.		
2.		
3.		
4.		

SMJ page 109

The ?-Inch "Ruler"
ANSWER KEY

Directions: Use the strip of paper provided by your teacher to estimate the perimeter of 4 objects around the room.

- ❖ WRITE the name of the object in the table.
- ❖ ESTIMATE the perimeter of each object using the strip of paper provided.
- ❖ RECORD your estimates in the table.
- ❖ DESCRIBE the process your group used to estimate each perimeter.

MY RULER SIZE IS ________ INCHES.

Object	Perimeter (inches)	Describe the process used to estimate the perimeter of this object.
1. *Sample: Sheet of Paper*	*38 inches*	*We estimated one side of the paper at 11 inches and the other at 8 inches. Then, we added them each twice because there are 2 sides of each length. 11 + 11 + 8 + 8 = 38 inches.*
2. *Sample: Small Tupperware Lid*	*16 inches*	*We estimated one side of the lid at 4 inches. Since it looks like a square and therefore has sides equal in length, we did 4 + 4 + 4 + 4 = 16 inches.*

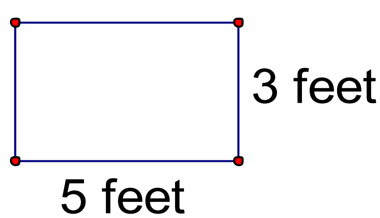
Methods and responses will vary significantly in this activity.

LIVING ON THE EDGE—
SAME PERIMETER, DIFFERENT SHAPE

LESSON 15

Big Mathematical Ideas

You measure the perimeter of a room in your home and find that it is 60 feet around. Given that information, would somebody else be able to figure out the dimensions of that room? Not likely, since a perimeter of 60 might come from a 20 x 10 room or from a 16 x 14 room. This lesson demonstrates that two figures can look different but have the same perimeter.

Lesson Objectives	• Students will be able to find and label dimensions of figures drawn on graph paper. • Students will be able to calculate perimeter using the dimensions of rectangular figures. • Students will be able to draw figures on graph paper with given perimeters.
Materials	• Student Page—*That's Another Dimension!* [SMJ page 111] • Student Page—*Perimeter Seeker* [SMJ page 113] • Check Up #3 [SMJ page 115] • Straight edge (for drawing straight lines, not for measurement) • Graph paper
Mathematical Language	• **Dimensions:** The length and width of a rectangular figure.  The dimensions of this rectangle are 3 feet by 5 feet. This can be written 3 feet x 5 feet. • **Perimeter:** The distance around a figure.

Lesson Preview

Students determine the dimensions of given figures and use those dimensions to find the perimeter. Students then draw a second figure with the same perimeter as the first but with a different shape.

Initiate

1. 30 feet of fence

Pose the following problem:

Create a rectangle that can be made using 30 feet of fence. How long is each side of your rectangle?

For students who get an answer quickly, ask them if that is the only rectangle with that perimeter. Encourage them to come up with multiple answers. Give students sufficient time to work and discuss their solutions. Encourage students to talk about what they tried that did not work.

Some possible answers to the problem:

- 7.5 feet x 7.5 feet (This is the answer students will get if they divide by 4 or use a similar technique to keep the sides equal. Remind skeptics that a square is a rectangle and so this answer is correct.)
- 14 feet x 1 foot
- 13 feet x 2 feet
- 12 feet x 3 feet
- 11 feet x 4 feet
- 10 feet x 5 feet
- 9 feet x 6 feet
- 8 feet x 7 feet
- Other answers are possible, but would have decimals/fractions.

Investigate

2. Same perimeter, different shape intro

Direct students to *That's Another Dimension!* Student Page **[SMJ page 111]**. Tell students that dimensions are the length and width of each rectangle. Students find the dimensions by counting the number of units. Have students find the dimensions of rectangle A. Have students make tick marks to count so that they do not count extra "units."

Students should find the dimensions to be 4 units x 4 units. Remind students that a square is also a rectangle because it meets the definition by having four sides and four right angles. Tell students to calculate the perimeter. They should get 16 units. They can write P = 16 units under Figure A.

Have students try Task 3 for Figure A—draw another rectangle with P = 16 units but with different dimensions. Provide additional graph paper if students would like. Have students label their figures with the same letter and label the dimensions. Discuss students' solutions that may include the dimensions below.

- 7 units x 1 unit
- 6 units x 2 units
- 5 units x 3 units
- All others would include fractional boxes like 6 1/2 units x 1 1/2 units.

3. **Same perimeter, different shape**
Students work on Tasks 1, 2, and 3 for Figures B, C, D and E the same way that they did A. Remind them to label their dimensions and label the second figure with the same letter as the first. As you walk around the room, ensure that students have labeled dimensions and written the perimeter below the original shape. Offer students additional graph paper as needed.

Questions that can be used to challenge students as they work:

- Can you create a third rectangle with the same perimeter as the first two?
- Can you create a non-rectangular shape with that perimeter? *Students may need additional graph paper to try these questions.*

Conclude

4. **Eliciting multiple responses for a single item**
Use one student's solution for Figure E (this is the best figure to choose because it should have a wide variety of answers). When the student gives his or her new dimensions, ask a student with a different answer to verify the first student's results and vice versa. Continue to elicit answers and have students check one another's work for Figure E.

Look Ahead

5. **Relationship to area**
We can have two different figures with the same perimeters, but what is the relationship between perimeter and area? Does the same perimeter guarantee the same area? In the next few lessons, students will look at the concept of area and examine its relationship to perimeter.

Assess

6. **Figures B, C, D**

Since they were not specifically discussed, student work for Figures B, C, and D can be assessed. This extension also can be used as an assessment for perimeter.

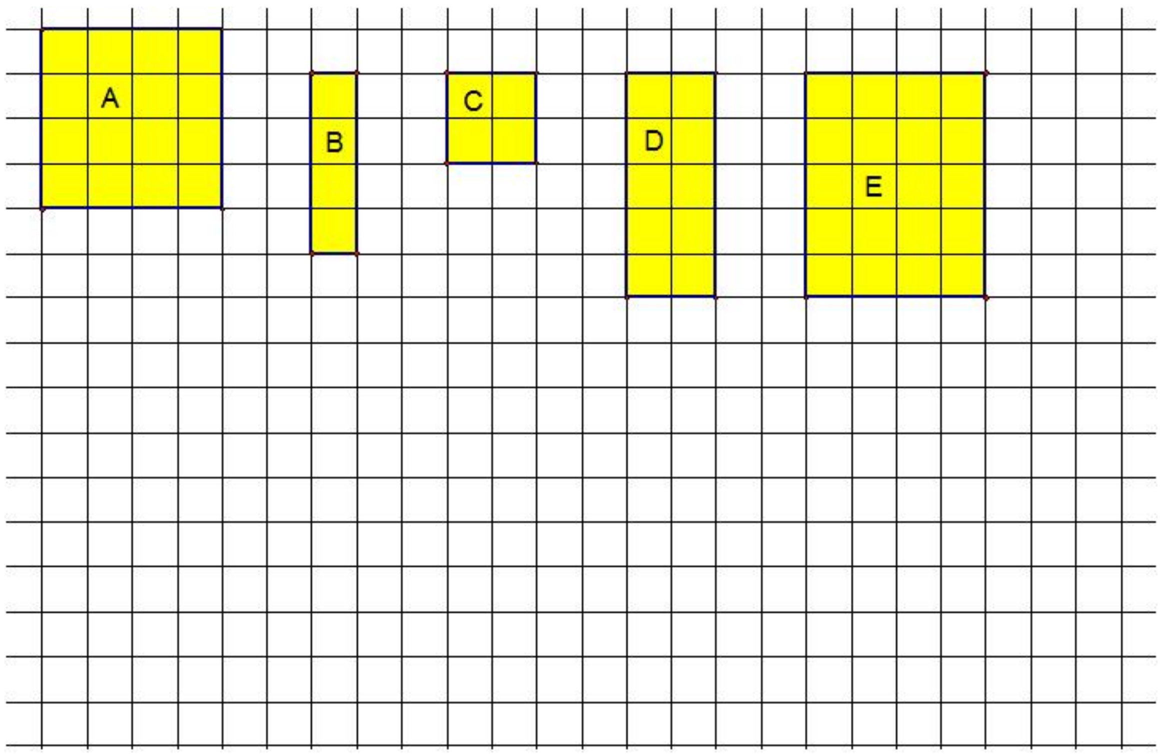
Assess student knowledge on Lessons 10, 11, 12, 13, and 14 by assigning Check Up #3 **[SMJ page 115]**.

7. **Extension to number and operation**

Students complete the *Perimeter Seeker* Student Page **[SMJ page 113]** in which they use counting techniques to find the distance around figures on graph paper.

Shape Wizard: ______________________________________

That's Another Dimension!



1. Label the dimensions of each figure in units.
2. Find the perimeter of each figure.

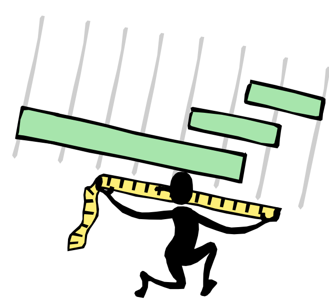
   Perimeter A = ____________________

   Perimeter B = ____________________

   Perimeter C = ____________________

   Perimeter D = ____________________

   Perimeter E = ____________________

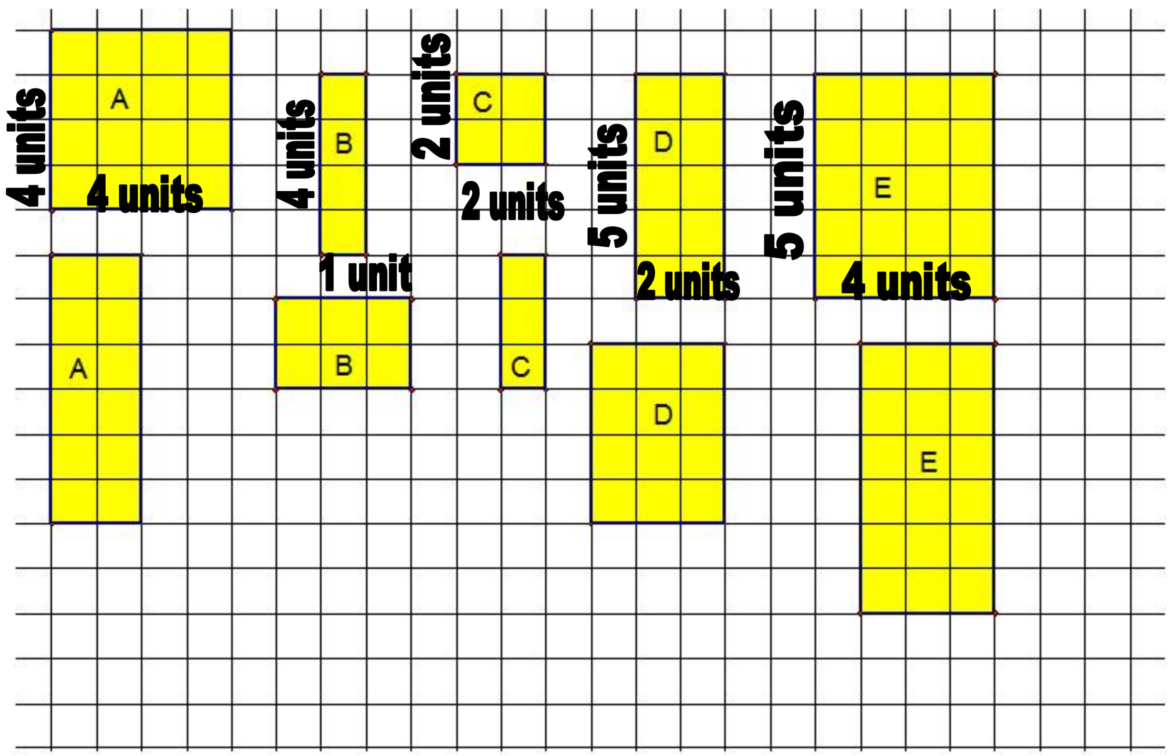


3. Draw another figure that has the SAME perimeter as Rectangle A but DIFFERENT dimensions. Label it A also.
4. Repeat step 3 for Rectangles B, C, D, and E. (If you need more graph paper, ask your teacher.)

SMJ page 111

That's Another Dimension!

ANSWER KEY



1. Label the dimensions of each figure in units.
2. Find the perimeter of each figure.

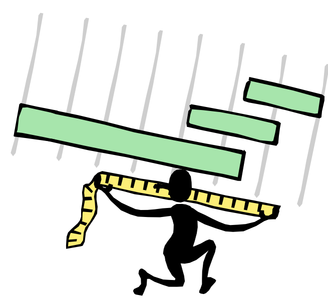
   Perimeter A = *16 units*

   Perimeter B = *10 units*

   Perimeter C = *8 units*

   Perimeter D = *14 units*

   Perimeter E = *18 units*



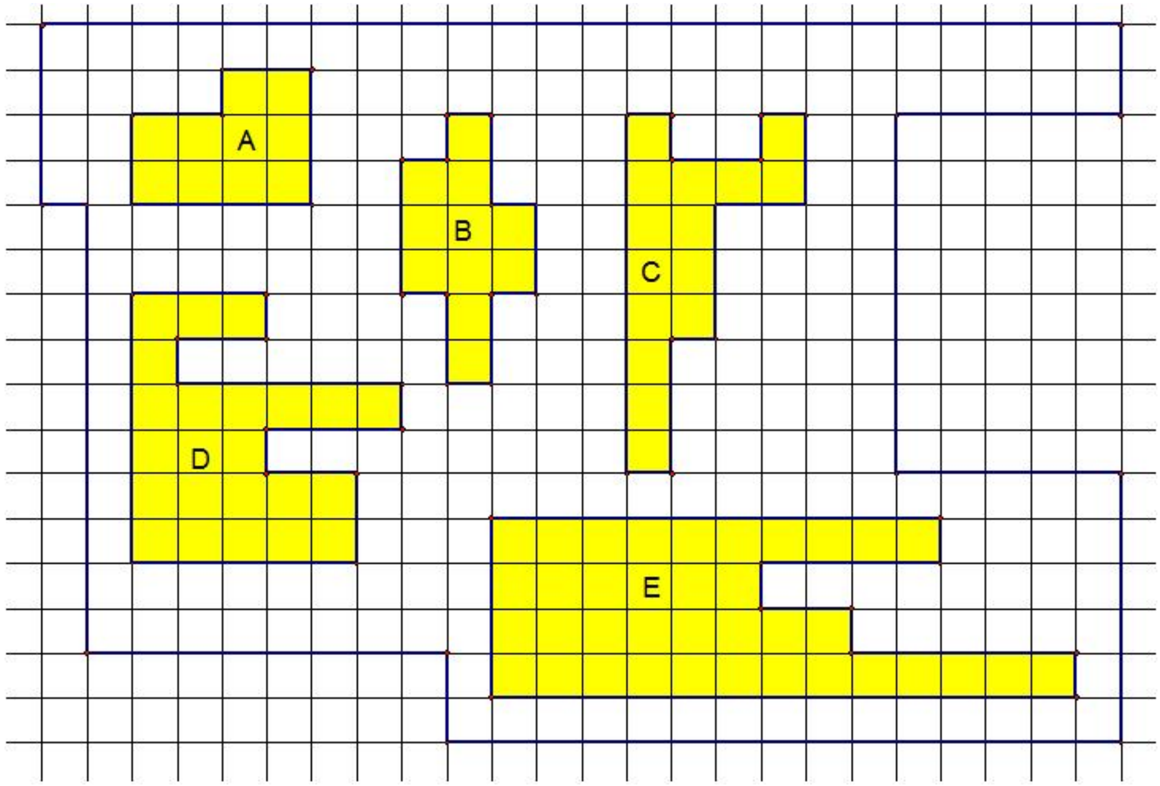
3. Draw another figure that has the SAME perimeter as Rectangle A but DIFFERENT dimensions. Label it A also.

   Sample drawings are provided. Other drawings are possible.

4. Repeat step 3 for Rectangles B, C, D, and E. (If you need more graph paper, ask your teacher.)

Perimeter Seeker: ______________________________________

Perimeter Seeker



Directions: Find the perimeter of Figures A, B, C, D, and E. Then find the perimeter of the shape going around all of those figures.

Perimeter A = ________________

Perimeter B = ________________

Perimeter C = ________________

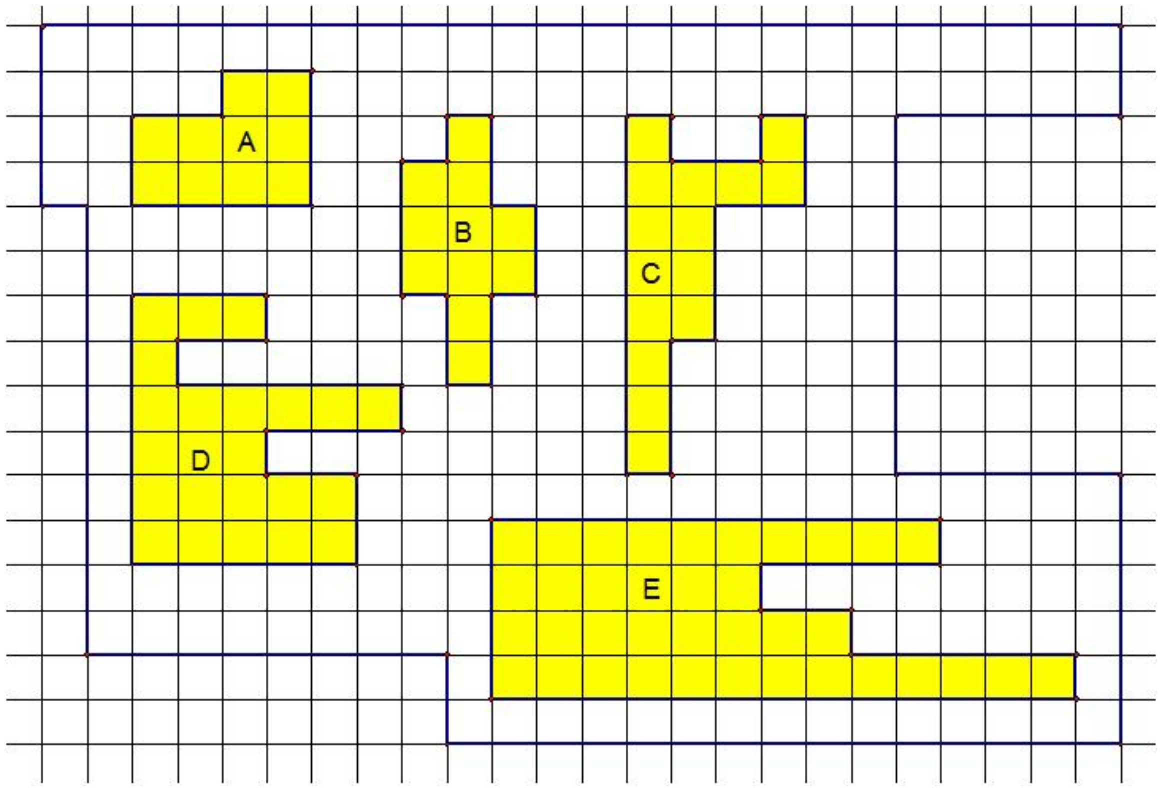
Perimeter D = ________________

Perimeter E = ________________

Outside Perimeter = ________________

SMJ page 113

Perimeter Seeker
ANSWER KEY



Directions: Find the perimeter of Figures A, B, C, D, and E. Then, find the perimeter of the shape going around all of those figures.

Perimeter A = ___*14 units*___

Perimeter B = ___*18 units*___

Perimeter C = ___*26 units*___

Perimeter D = ___*32 units*___

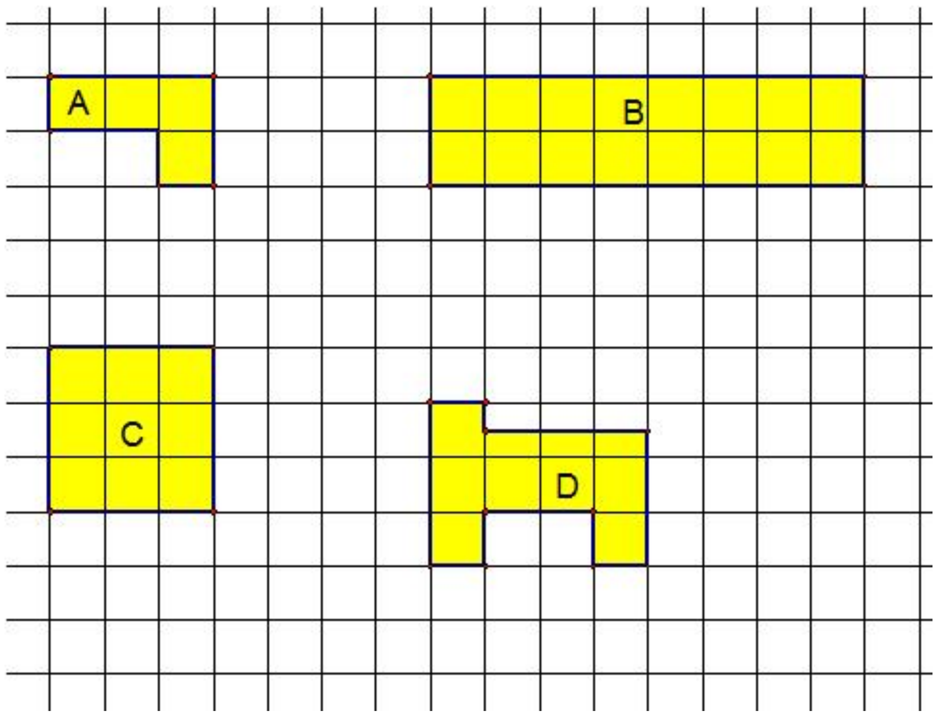
Perimeter E = ___*42 units*___

Outside Perimeter = ___*90 units*___

Name: ________________________________ Date: ___________

Check Up #3

1. Find the perimeter of each figure in the diagram.



Perimeter A = ___________________

Perimeter B = ___________________

Perimeter C = ___________________

Perimeter D = ___________________

2. Draw a line segment each length.

3 inches:

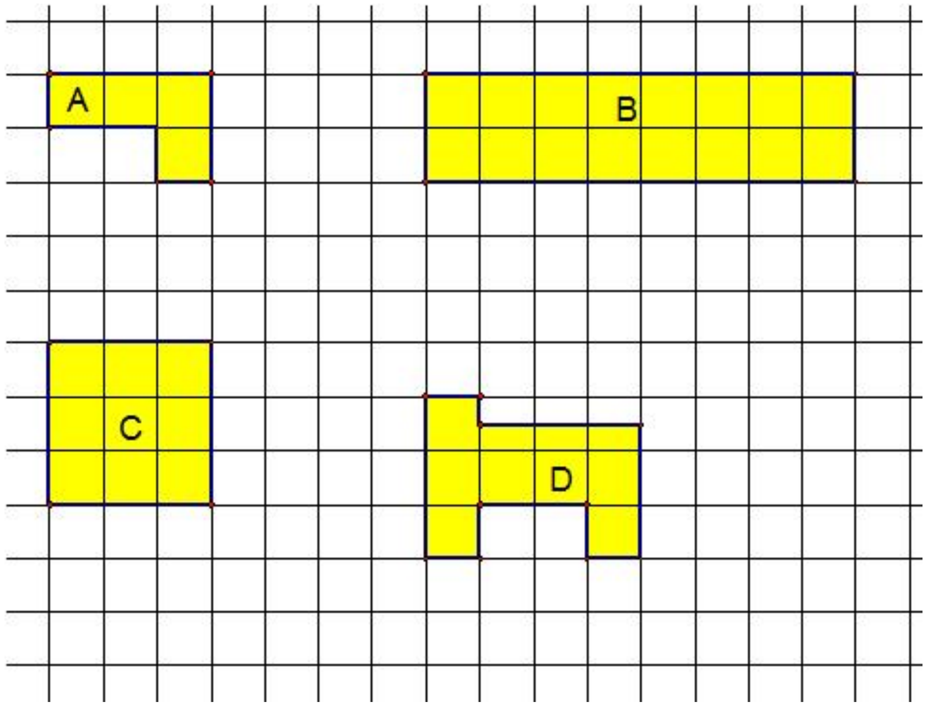
2 1/2 inches:

4 1/4 inches:

SMJ page 115

Check Up #3
ANSWER KEY

1. Find the perimeter of each figure in the diagram.



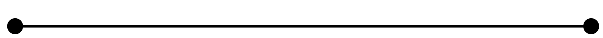
Perimeter A = *10 units*

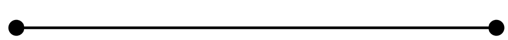
Perimeter B = *20 units*

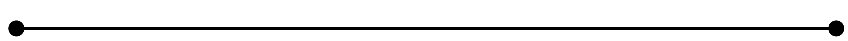
Perimeter C = *12 units*

Perimeter D = *16 units*

2. Draw a line segment each length.

3 inches: 

2 1/2 inches: 

4 1/4 inches: 

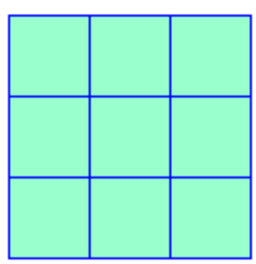
LIVING ON THE EDGE—

A FAIR WAY TO SHADE

LESSON 16

Big Mathematical Ideas

The concept of area is typically driven by formulas, most commonly multiplying the length and width of a square or rectangle. Students are rarely given the opportunity to both view area as something that "covers," and to develop a sense of why multiplying the length and width of a rectangle results in the number of square units "covering" that rectangle.

Lesson Objectives	• Students will be able to divide a region into equal sections on graph paper.
Materials	• Student Page—*A Fair Way to Shade* [SMJ page 117] • Student Page—*Non-counter* (Optional) [SMJ page 123] • Student Page—*Area Estimator (Euclid)* (Optional) [SMJ page 125] • 16 x 17 grid • Colored pencils or crayons • Calculator (1 per group if available)
Mathematical Language	• **Area:** The number of square units covering a surface.  This area is 9 square units. • **Adjacent:** To share a side.

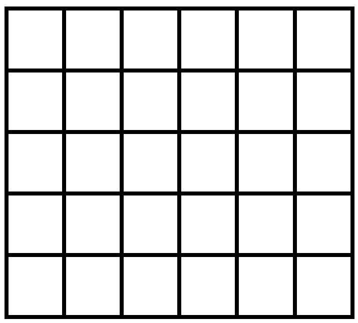
Lesson Preview

Students develop strategies to determine the number of squares a rectangular figure contains without counting one-by-one. In addition, students determine a fair way to shade the paper so that each student gets the same "area."

Initiate

1. **Tell me how many without counting**

Display a rectangle that is 5 x 6 as pictured below. Ask students to determine the number of squares without counting one-by-one.



Discuss the different ways that students determined the number of squares. A discussion similar to the following might occur:

Teacher:	Can somebody explain how he or she found the number of squares without counting one-by-one?
Abel:	I just counted the top and found out there were six. Then I kept adding six plus six plus six plus six plus six. I got thirty.
Teacher:	How did you know how many sixes to add?
Abel:	Each row is another six. So I had to do one, two, three, four, five. Five sixes.
Teacher:	Would you mind coming up to show us what you mean in relation to the picture?

Investigate

2. ***A Fair Way to Shade***

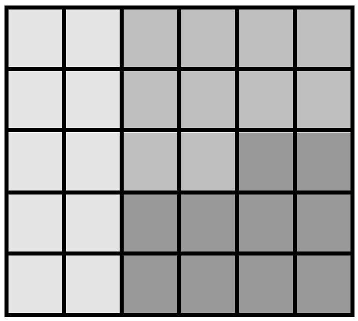
Group students using the following suggestions:

- Groups of 2 make the task much less challenging (Pythagoras).
- Groups of 4 give the task a medium challenge level (Hypatia).
- Groups of 3 make the task challenging (Euclid).

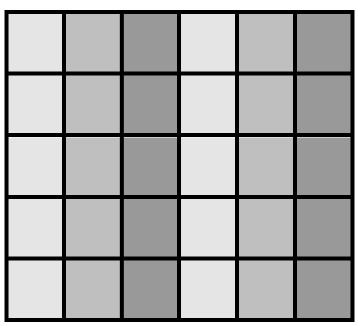
High-ability students are good candidates for a group of 3 since the number of squares is not divisible by 3.

Provide each group with a 16 x 17 grid and four different colored pencils or crayons to use for shading. Tell students that they will be shading the entire paper so that each group member has the same number of squares. Emphasize the rule that each student's boxes must be connected (in Geometry, this is called *adjacent,* which means to share a side). That is, they may not scatter their boxes all over the paper, but rather, each box must share at least one side with another box. Provide a visual example demonstrating what students in Groups of 3 should and should not do, if necessary.

Students should shade "adjacent" boxes as shown below:



Students may not shade "scattered" boxes:



Once students understand that rule, tell them that their first task is again to determine the number of squares without counting just like in the warm-up. Then have students work through the steps and questions on the *A Fair Way to Shade* Student Page **[SMJ page 117]**.

Conclude

3. **Relating the pieces to the whole**

Discuss students' answers to Question 1 on *A Fair Way to Shade*. Find out which methods the groups chose, and ask if there were any disagreements within the groups as to which method is "best."

Next, find out how many squares each person shaded. If there were groups of 3 that had to "split squares" (in Question 2), find out how they split them fairly. Have students draw pictures to explain their answers to the class.

Discuss the relationship between individual areas and the whole area. Use the following questions to guide this discussion:

- What happened when you calculated the sum of your areas in Question 3?
  (The sum is equal to the whole area.)
- Can anybody explain why the sum matched your answer to Question 1?
  (When you combine each of the smaller areas, you get the area of the entire surface or grid.)
- How is this different from what we did yesterday when we were finding perimeter?
  (We are looking at the number of squares inside of a rectangle, not the distance around.)
- If I told you that the answer you got in Question 1 is called the **area**, how would you define **area**?
  (The number of square units that cover a surface.)

4. **Extension to multiplication and division**

If your students know how to multiply and it has not come up already, it is a good time to multiply 16 x 17 (on a calculator if they have not done 2-digit multiplication) to show that this gives the total number of squares on the grid. Have them check if this would have worked on the 5 x 6 rectangle at the beginning of class.

Have students divide 272 ÷ 4. They will be able to see that dividing would have given them the number of squares each student should shade (for smaller groups, divide by 2 or 3).

Even if your students are not using the operations of multiplication and division yet, this lesson is an opportunity to show the efficiency of such operations and provide incentive for students to learn these operations later in the year.

Look Ahead

5. **Transfer to irregular**

It is exciting to learn that area can be found by determining the number of “squares” inside the figure. What happens when the shapes are not perfect rectangles or squares? How can we estimate areas when the boxes are not whole boxes? The next lesson applies these concepts to irregular shapes.

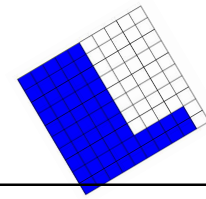
Assess

6. **Create the rectangle**
Give students a sheet of graph paper and ask them to draw a rectangle with each area listed.

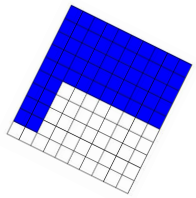
- 4 square units
- 9 square units
- 12 square units
- 18 square units
- 27 square units

* Remind students that a square is a rectangle!

7. **Extension to number and operation**
Students complete the *Non-counter* Student Page **[SMJ page 123]** in which they use counting or multiplication techniques to determine the number of square units in a grid. A Euclid activity, the *Area Estimator* Student Page **[SMJ page 125]**, is available for students who grasp the area concept with little difficulty.



Investigators: ____



A Fair Way to Shade

1. You have been given a grid covered with squares. Without counting one-by-one, figure out how many squares are on the grid.

   How many did you get? ___________

   Explain how your group determined the number of squares.

   ____

   ____

   ____

   ____

   ____

2. Each group member will shade a piece of the grid. You must use the following rules for shading.

   - Each group member must choose his or her own color.
   - The group must determine how many squares each person will shade so that everyone has the same number.
   - Each part that you shade must be connected. You MUST be able to shade your whole section without lifting your pencil.

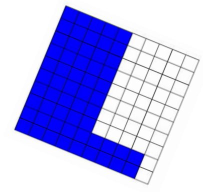
3. How many squares did each student in your group shade?

   Student 1: ______________ Student 2: ______________

   Student 3: ______________ Student 4: ______________

SMJ page 117

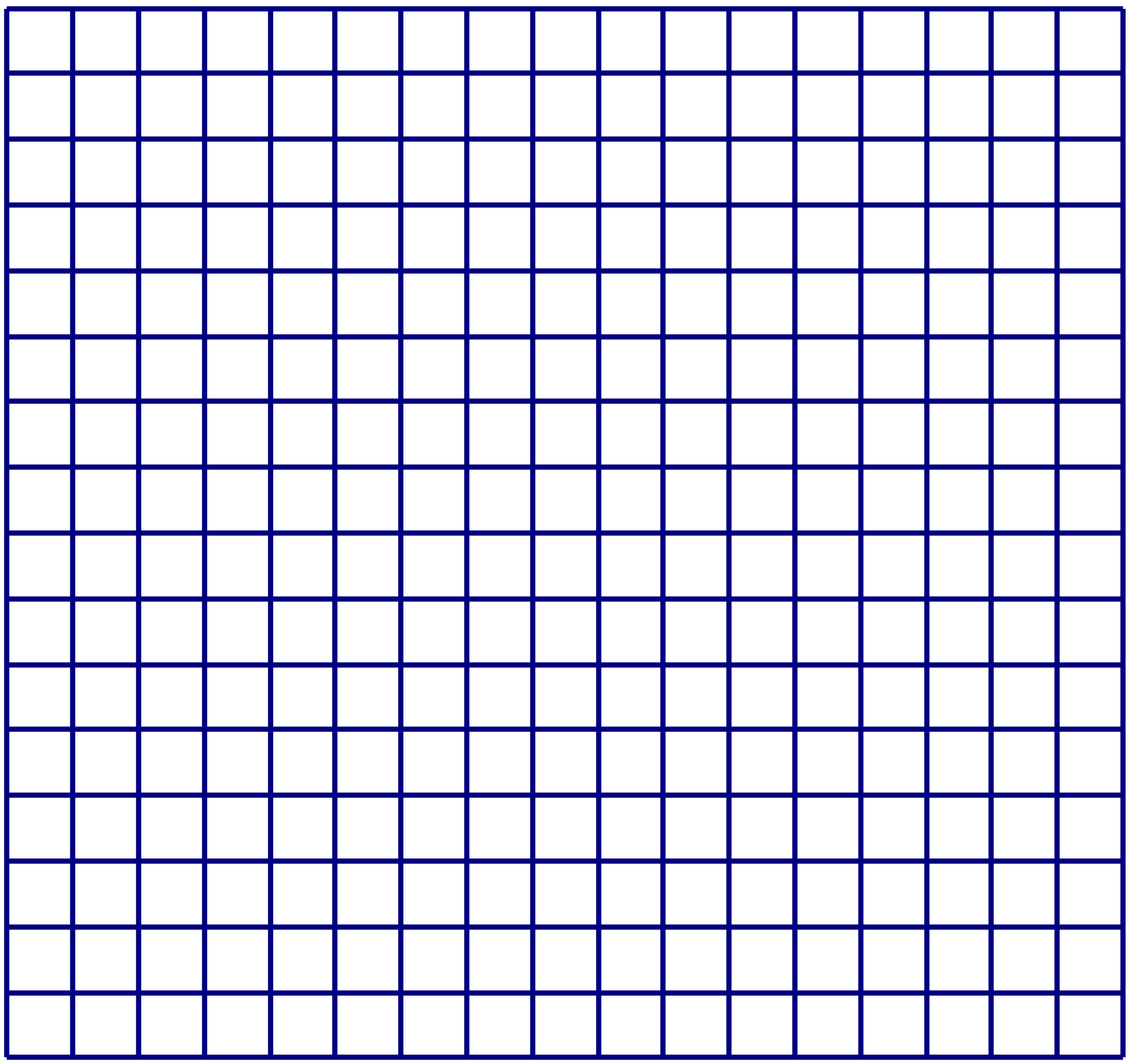
4. Calculate the sum of your answers to #3. Show your work below.



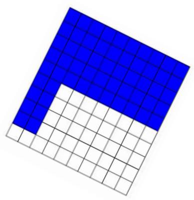
5. What do you notice about the sum you calculated in #4?

SMJ page 119

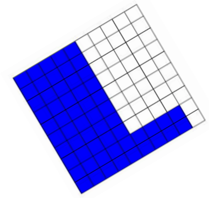
SQUARE GRID: HOW MANY SQUARES?



SMJ page 121



A Fair Way to Shade
ANSWER KEY



1. You have been given a grid covered with squares. Without counting one-by-one, figure out how many squares are on the grid.

   How many did you get? ___________*272*________

   Explain how your group determined the number of squares.

   Sample Answer: We counted one row of squares and got 16. Then, using a calculator we added 16 for every row. There were 17 rows all together, and in total there were 272 square units.

2. Each group member will shade a piece of the grid. You must use the following rules for shading.

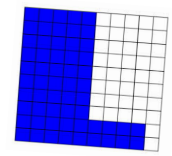
   - Each group member must choose his or her own color.
   - The group must determine how many squares each person will shade so that everyone has the same number.
   - Each part that you shade must be connected. You MUST be able to shade your whole section without lifting your pencil.

3. How many squares did each student in your group shade?

   Students with 4 people in their groups should have shaded 68 squares each. Students with 2 people in their groups should have shaded 136 squares each. Students with 3 people in their groups should have shaded 90 2/3 squares each. They may have explained it as shading 90 squares each and dividing the remaining 2 squares in some equal manner.

4. Calculate the sum of your answers to #3. Show your work below.

   Sample Answer: The sums should equal 272, with the exception of 3-person groups that might be slightly short due to inaccuracy in the fractional portion.



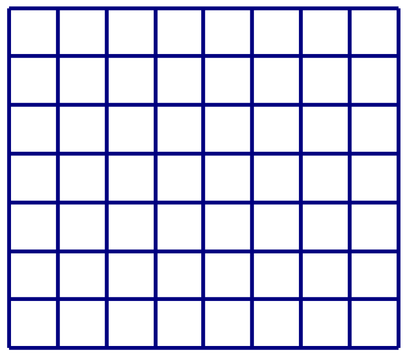
5. What do you notice about the sum you calculated in #4?

   Sample Answer: The sum is equal to the total number of squares in the grid.

Non-counter: ____

Non-counter

1.
   a. Without counting one-by-one, tell how many squares are in this grid.



   b. Explain how you found the number of squares.

2. If you were playing a game with a friend and each of you had to color the same number of squares to fill the grid, how many would each of you color?

3. If 4 people were playing the game, how many squares would each person color?

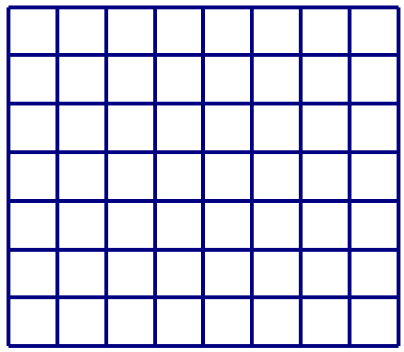
4. If 7 people were playing the game, how many squares would each person color?

5. Explain what would happen if 10 people tried to color the same number of squares each.

SMJ page 123

Non-counter
ANSWER KEY

1.
   a. Without counting one-by-one, tell how many squares are in this grid.



56 squares

   b. Explain how you found the number of squares.

   Sample Answer: I counted the first row which had 8 squares. Then I kept adding 8 for each row. 8 + 8 + 8 + 8 + 8 + 8 + 8 = 56 squares.

2. If you were playing a game with a friend and each of you had to color the same number of squares to fill the grid, how many would each of you color?

   Sample Answer: We would each color 28 squares since 28 + 28 = 56.

3. If 4 people were playing the game, how many squares would each person color?

   Sample Answer: We would each color 14 squares. Since 2 people each had 28, we can split the 28 up into 14 + 14.

4. If 7 people were playing the game, how many squares would each person color?

   Sample Answer: We would each color 8 squares. Since there are 7 rows, we would each get one row, or 8 squares.

5. Explain what would happen if 10 people tried to color the same number of squares each.

   Sample Answer: If 10 people tried to color the same number of squares each, they would each get 5 but there would be 6 left over at the end. I figured this out by using 10 different color crayons and using them in a pattern above.

Area Estimator: ______________________________________ Euclid

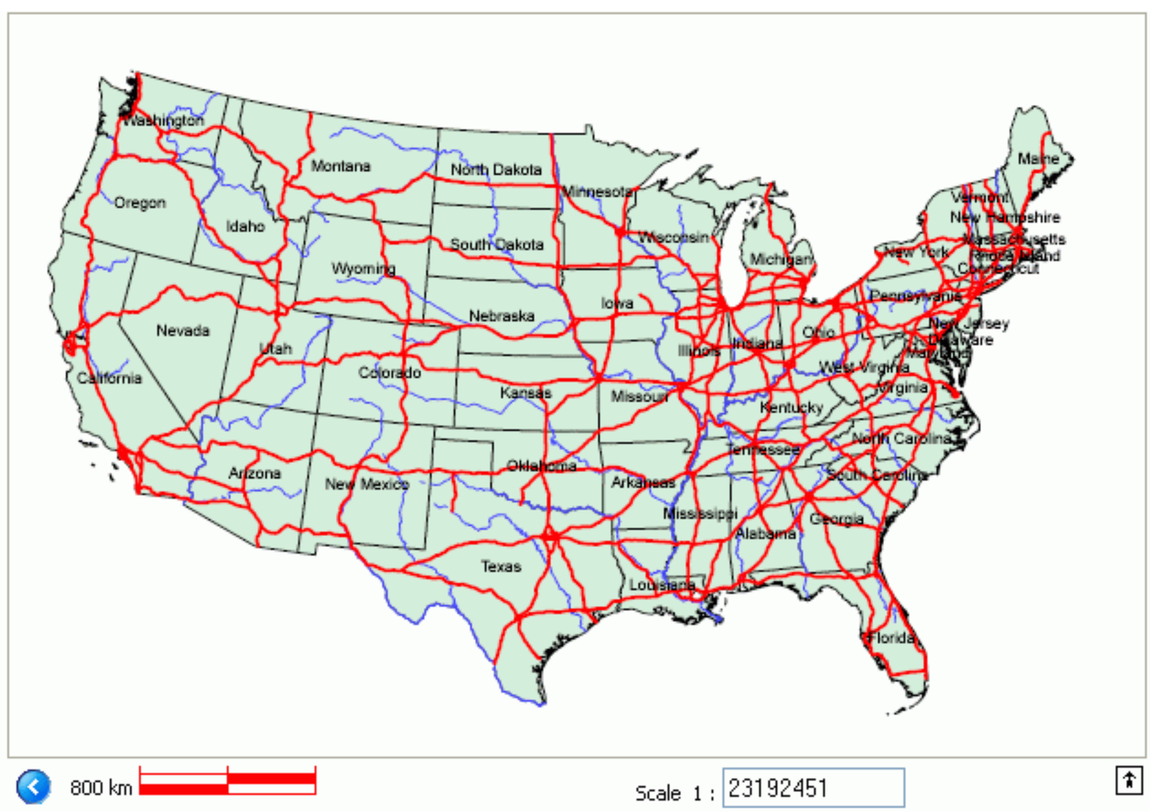
Area Estimator

To complete this activity you may need: A ruler and calculator.

1 inch = 800 kilometers.
Use this information and a ruler to estimate the perimeter of the 48 contiguous United States in kilometers. You may draw on the map to show your work.

1 square inch = 640,000 square kilometers
Use this information to estimate the area of the 48 contiguous United States in square kilometers. 1 inch

(A square inch is a square that is one inch on each side.)
(Contiguous means the States are adjacent, or next to, each other.)



http://gis-web.marylandheights.com/frontdesk/commands/help/helpMap.asp

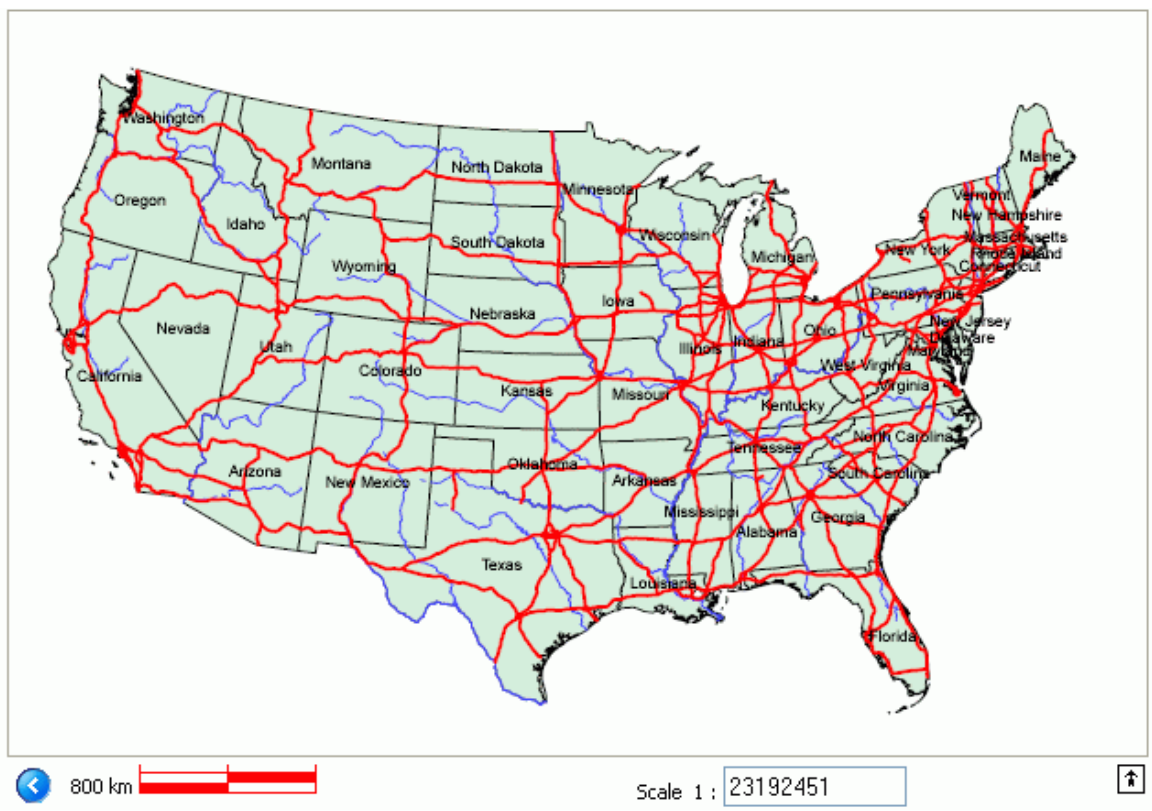
SMJ page 125

Euclid

Area Estimator
ANSWER KEY

To complete this activity you may need: A ruler and calculator.
1 inch = 800 kilometers.
Use this information and a ruler to estimate the perimeter of the 48 contiguous United States in kilometers. You may draw on the map to show your work.

1 square inch = 640,000 square kilometers
Use this information to estimate the area of the 48 contiguous United States in square kilometers.
(A square inch is a square that is one inch on each side.)
(Contiguous means the States are adjacent, or next to, each other.)



http://gis-web.marylandheights.com/frontdesk/commands/help/helpMap.asp

Sample Answers:
For perimeter, the estimate should fall in the range of 11,000 kilometers to 14,000 kilometers.
For area, the estimate should fall in the range of 6,000,000 square kilometers to 12,000,000 square kilometers. (While this range is quite large, third graders may

grasp the concept of area even if they choose a "low estimate" or "high estimate" rather than one in between.)

To help students with the area portion, show them how to draw one-inch squares that cover the picture and then count each as 640,000 square kilometers.

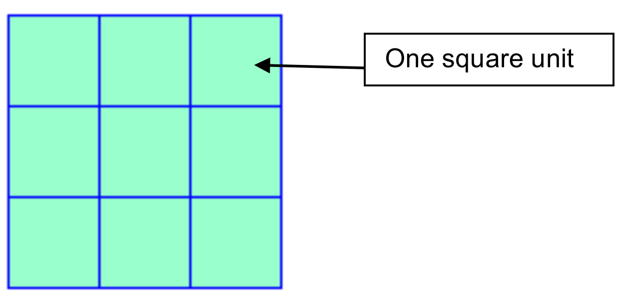
LESSON 17

LIVING ON THE EDGE—

SQUARE UNITS IN AN UNSQUARE WORLD

Big Mathematical Ideas

“Length times width”—it is a formula students hear over and over again from middle school to high school Geometry. However, we do not live in a world of squares and rectangles. In fact, there are shapes that don’t even have conventional names, and yet there are times when it is necessary to calculate the areas of such shapes.

Lesson Objectives	• Students will be able to estimate areas of irregular shapes drawn on graph paper. • Students will be able to apply the concept of “average” to get a group estimate for the area of a given irregular shape. (NOTE: They do not need to know how to calculate an average.)
Materials	• Student Page—*How Many Square Units?* [SMJ page 127] • Student Page—*Area Agreement From the Committees* [SMJ page 131] • Graph paper
Mathematical Language	• **Area:** Number of square units covering a region. • **Square units:** Units used to measure area (two-dimensional). 

Lesson Preview

Students combine pieces of squares to make wholes to estimate areas of irregular shapes in square units. Once they have completed assignments individually, they are asked to convene in groups of 3-4 to create group estimates for each figure. This lesson focuses on oral rather than written communication as students justify their ideas to their groups.

Initiate

1. **Combining pieces to make wholes**
Direct students to the *How Many Square Units?* Student Page **[SMJ page 127]**. Look at the bug problem and explain that students are to find a way to determine the number of square units for this bug. Tell students they must count the "pieces" of squares also because they are part of the bug. Encourage students to come up with ways to turn pieces of squares into whole squares.

Once students have come up with estimates, list all of their answers on the board. Are there any that are unreasonable? Have students think of a way to come up with a class estimate based on the numbers on the board. If there are unreasonable estimates, should they be included in the class estimate? Why or why not? Consider having students share their estimate with a neighbor to see if they can come to an agreement.

Investigate

2. **Irregular shapes**
Students work **independently** to come up with estimates for the areas of shapes A-F in square units. Tell them to think back to the bug problem and any ideas they heard from other students that might work well for them.

3. **A good group estimate**
Once students have completed their independent work, assign them to "committees" of 3 or 4. Students complete the *Area Agreement From the Committees* Student Page **[SMJ page 131]**. In these discussions, students compare the highest and lowest estimates from the group (which may be the same, especially for Figure A), then try to agree on a group estimate. Give students real-life examples of groups that must agree on such items—boy scouts, girl scouts, or voters.

Conclude

4. **The easy decisions and the hard ones, too**
Use these questions to guide a discussion:

- Which area was the easiest to agree on?
(See Answer Key for *Agreement From the Committees* #7.)
- Why was this one the easiest?
(See Answer Key for *Agreement From the Committees* #7.)

- Is your answer to this exact or an estimate? How do you know?
  (The area is exact because there were only whole square units. We did not have to combine pieces of squares to make wholes.)
- Which area was the most difficult to agree on?
  (See Answer Key for *Agreement From the Committees* #8.)
- Can somebody explain what your group discussed when talking about the area of that shape?
  (We talked about counting all of the whole squares first. Then we tried to match up big pieces with small pieces to make whole squares.)
- How did you reach a decision?
  (We decided on a number in between the lowest estimate from our group and the highest.)
- What is the difference between an exact area and an estimated area?
  (An exact area consists of all whole squares that can be easily counted. An estimated area might have pieces of squares that have to be combined to be called "square units.")

Look Ahead

5. **The Unit Project**
Select which unit project the class will do. Briefly explain the project to students.

Assess

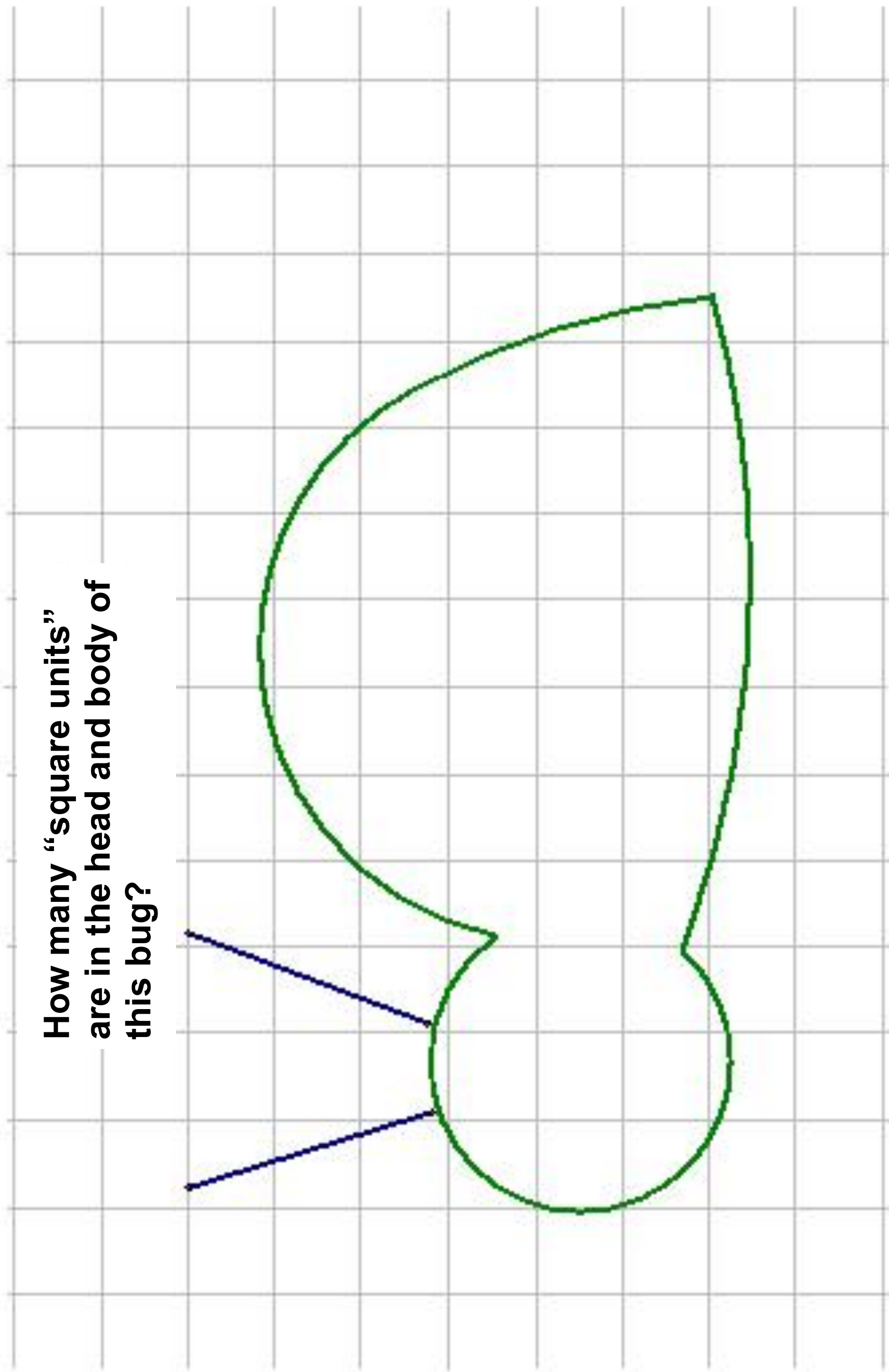
6. **Create a cloud**
Give students a sheet of graph paper. Tell them they are to draw a "cloud" shape on the graph paper based on the instructions below.

- The area of the cloud is greater than 20 square units.
- The area of the cloud is less than 50 square units.
- No rectangular clouds!
- Estimate the area of the cloud.
- Write three or more sentences to convince others that your cloud has an area greater than 20 square units.

Area Analyst: ____

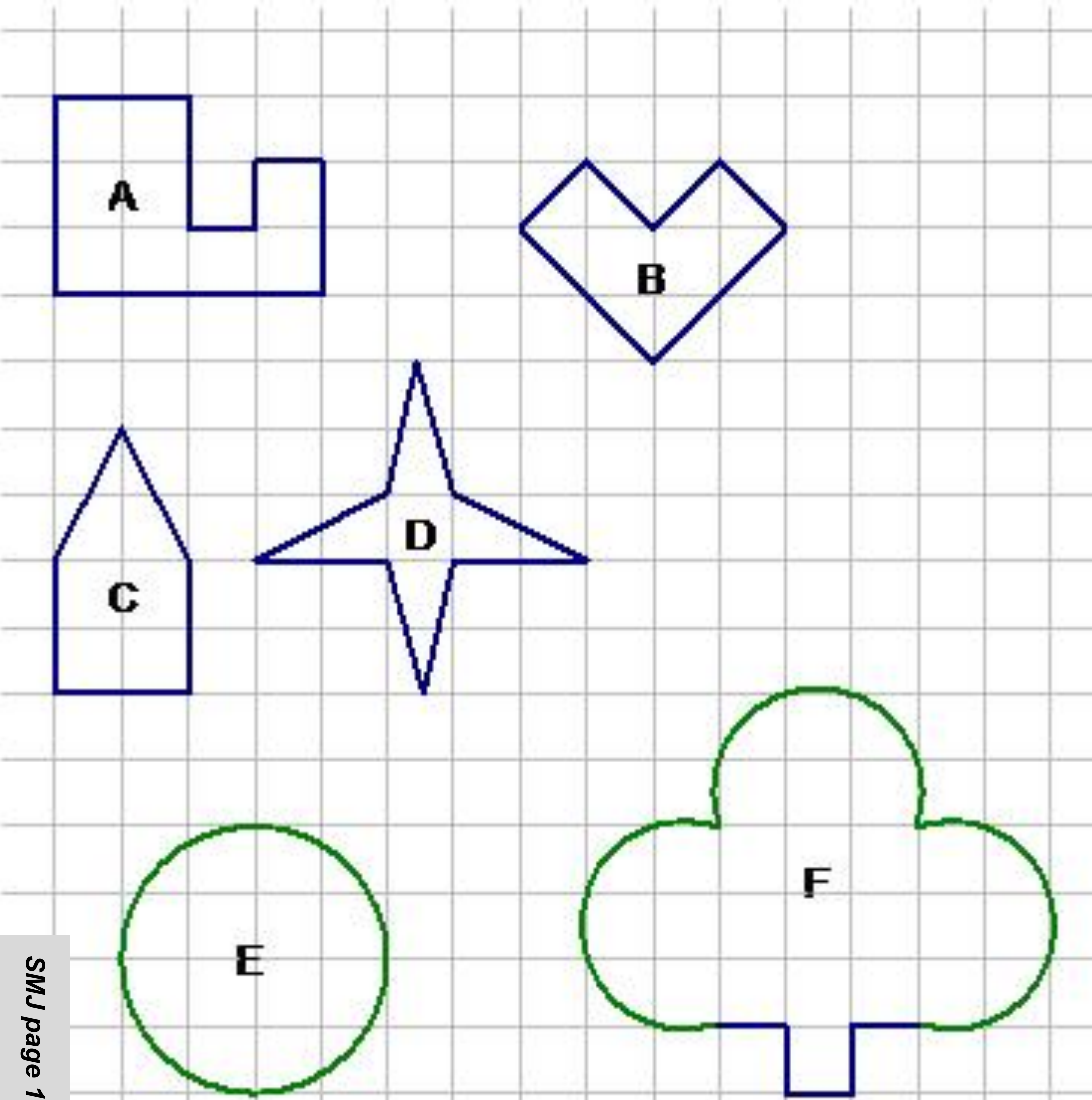
How Many Square Units?

Task #1



SMJ page 127

Task #2



Estimate the area of each figure. Write your answers in “square units.”

Figure A: ________________

Figure B: ________________

Figure C: ________________

Figure D: ________________

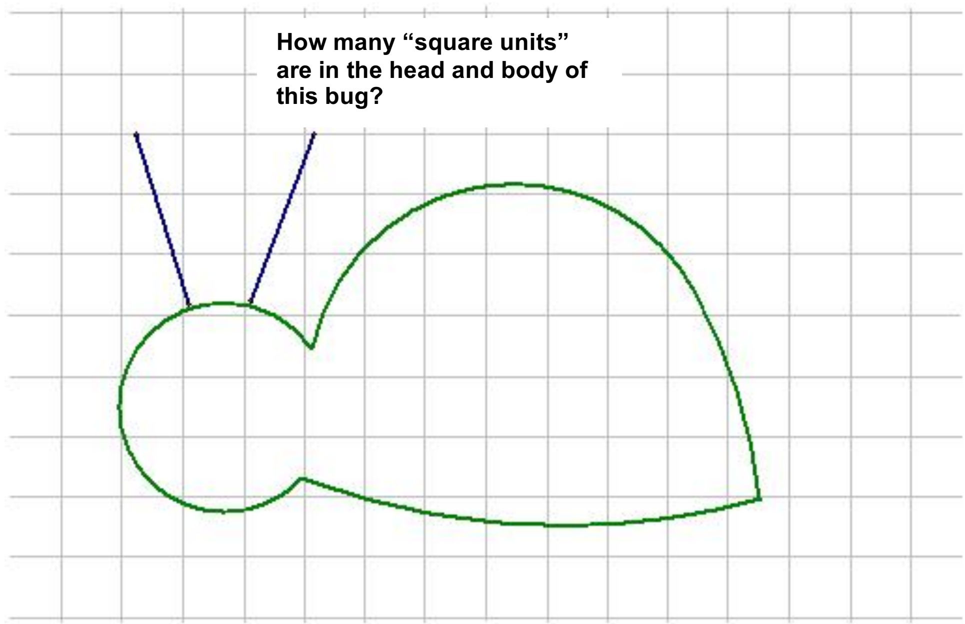
Figure E: ________________

Figure F: ________________

SMJ page 129

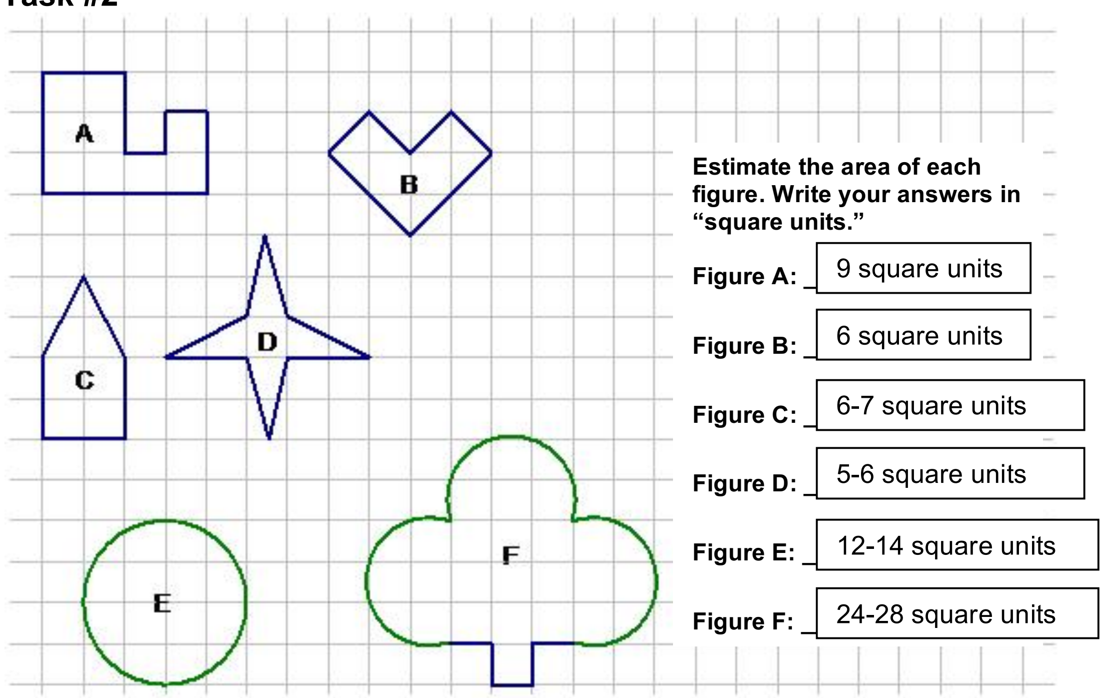
How Many Square Units?
ANSWER KEY

Task #1



Sample Answer: The estimates for this area should fall in the range of 35 square units to 43 square units.

Task #2



Committee Members: ____

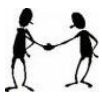
Area Agreement From the Committees

Directions: In your committee, determine which estimate was highest and which was lowest. Record these estimates. Then, come up with a fair estimate for your group and record this on the "Committee's Decision" line.

1. Figure A

High Estimate: ____________________

Low Estimate: ____________________


Committee's Decision: ____________________

2. Figure B

High Estimate: ____________________

Low Estimate: ____________________


Committee's Decision: ____________________

3. Figure C

High Estimate: ____________________

Low Estimate: ____________________


Committee's Decision: ____________________

SMJ page 131

4. Figure D

High Estimate: ____________________

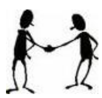
Low Estimate: ____________________

Committee's Decision: ____________________

5. Figure E

High Estimate: ____________________

Low Estimate: ____________________



Committee's Decision: ____________________

6. Figure F

High Estimate: ____________________

Low Estimate: ____________________

Committee's Decision: ____________________

7. Which area was easiest to agree on? Why?

8. Which area was the hardest for your committee to agree on? Why?

SMJ page 133

Area Agreement From the Committees
ANSWER KEY

Directions: In your committee, determine which estimate was highest and which was lowest. Record these estimates. Then, come up with a fair estimate for your group and record this on the "Committee's Decision" line.

Answers will vary depending on estimates within groups. An example is provided for a possible answer for Figure F.

6. Figure F

High Estimate: *28 square units*

Low Estimate: *24 square units*

Committee's Decision: *26 square units*

In each case, students should have a "decision" that is in between the high and the low but not necessarily a mathematical average. Encourage students to use answers such as 5 1/2 when they are trying to agree on an estimate between 5 and 6.

7. Which area was easiest to agree on? Why?

Sample Answer: The first shape was easiest to agree on because there were no parts of squares, only whole squares to count.

8. Which area was the hardest for your committee to agree on? Why?

Sample Answer: The last shape was hardest to agree on because one person had 24 square units and another had 28 square units, so we had to find a number in the middle.

UNIT PROJECT (OPTION 1)

A Geometry Scavenger Hunt

Students have learned several new geometric vocabulary words and concepts throughout the unit. Now, it is time to allow them to see geometry as a concept that surrounds them every day!

Students will be given a list of terms **[SMJ page 135]** that test both vocabulary and understanding of the concepts throughout the unit.

A Geometry Scavenger Hunt

Search your school, home, or other locations to find examples of each of the following:

- A right angle
- An obtuse angle
- An acute angle
- A pentagon
- A hexagon
- An octagon
- Something that is between 3 1/2 and 4 1/2 inches long
- Two congruent rectangles
- A pair of congruent figures that are *not* quadrilaterals
- Something with a perimeter that you might measure in inches
- Something with a perimeter that you might measure in feet
- An object with a circumference
- *Student's choice: Choose your own object related to geometry*
- *Student's choice: Choose your own object related to geometry*

SMJ page 135

Scavenger Hunter: ____

Scavenger Hunt Findings

OBJECT	Where we found it…	How I know what it is…
Right angle		
Obtuse angle		
Acute angle		
Pentagon		
Hexagon		
Octagon		
Object between 3 1/2 and 4 1/2 inches long		
Pair of congruent rectangles		
Pair of congruent figures (*not* quadrilaterals)		
Something with perimeter in inches		
Something with perimeter in feet		
Object with a circumference		
Your choice: ____________________		
Your choice: ____________________		

SMJ page 136

UNIT PROJECT (OPTION 2)

A Shapely Living Room

Day 1 [SMJ pages 137-141]
Assign students to groups with three to four students per group based on performance throughout the unit (Pythagoras and Hypatia groups). Grouping by ability for the unit project enables appropriate opportunities for scaffolding and enrichment as the project progresses.

Give each group materials for Day 1. Explain that the first task is to cut the tiles and place them inside the perimeter to cover the floor. It is not necessary to cut one-by-one, but it may be necessary to use pieces of tile (remind them of the lesson in which partial squares were used to find area). Also, remind students that this is an "estimate," and group members must work together to agree on the number of tiles on the floor.

The second task for Day 1 is to put trim around the edge of the room. Students are required to estimate the perimeter of the room in inches and give this estimate to the teacher who will "cut" the trim (ribbon). Students then glue trim around the border of the room.

Additional time can be used to finish coloring the tiles.

Day 2 [SMJ pages 137-144]
Students use Day 2 to create mini furniture to put in their rooms. Students may use materials provided by the teacher or bring their own. It is important that students focus on using different shapes in the furniture they create as this information will be recorded in a table and used to assess understanding of two dimensional shapes and their properties. Circulate the room to address this issue using guiding questions to focus students' ideas on the mathematics.

- What shapes are you using in your furniture?
    - How do you know?
    - Does this shape have any other names?
- What shapes are others in your group using?
    - How are they similar to and/or different from your shapes?

A Shapely Living Room SCORING RUBRIC

Project Component	**Unsatisfactory (0 - 2 Points)**	**Satisfactory (3 - 6 Points)**	**Outstanding (7 - 10 Points)**	**Maximum Score Possible**
Tiling the floor	Students have estimated above or below the ranges given for a score in the satisfactory range. Tiles may not be colored.	Students have estimated the number of tiles either between 170 and 179 OR 201 and 210, inclusive. Tiles are colored.	Students have estimated the number of tiles between 180 and 200, inclusive. Tiles are colored in alternating, checkerboard style.	10
Trimming the floor	Perimeter outside of ranges given in satisfactory score column.	Perimeter between 20” and 24” OR 28” and 32”.	Perimeter between 24” and 28”, inclusive	10
Furniture design completed table	Students have more than 16 boxes in the table filled out incorrectly.	Students have more than 6 boxes in the table filled out incorrectly.	All furniture has been measured and accurately matches what has been given in the table to the nearest 1/4 inch. (Each box can be scored as 1/2 point.)	10
Overall project	The team did not work well together and the project is not complete.	The project is missing one or two components OR the team’s effort could have been greater.	All components of the project are complete and team put sufficient effort into its completion. Teams scoring 8 may have gone above and beyond what was required in the project.	10
TOTAL SCORE				40

Designers: ____

A Shapely Living Room

You are creating a new room in your house. In this project, you will tile the floor, put trim around the edge, and furnish the room. Follow the directions to create your room.

DAY 1 Materials

- Blueprint of floor (1 per group)
- Small tile sheet (Hypatia level) or Large tile sheet (Pythagoras level)
- 2 different color crayons, markers, or colored pencils
- Inch ruler
- Ribbon or yarn (teacher holds this and cuts to order)
- Glue stick
- Scissors

DAY 2 Materials

- Construction paper
- Scissors
- Glue/Tape
- Inch ruler
- Other materials students bring in to build mini furniture
- Toothpicks or straws

I. Tiling the floor

1. With your group, decide how you will tile the floor using the tiles provided.
2. Use scissors and glue to lay the tile on the floor.
3. Estimate the total number of tiles used.

   - How many tiles are on your floor? ____________________
   - Explain how your group worked together to agree on the number of tiles.

____

____

____

4. Color the tiles using 2 different colors.

SMJ page 137

II. Trimming the room

1. As a group, you must find a way to estimate the perimeter of the room in inches.
2. Once you have an estimate, ask your teacher for border. Tell the teacher how many inches you need to go around your room.

   - Explain the method your group used to find the perimeter.

____

____

____

____

____

3. Glue the border around the perimeter of the room.

III. Mini furniture design (Day 2)

1. Create mini furniture to go in your room. Use these directions:
   - Each person in the group must make at least 1 piece of furniture alone. (Once you have finished your own, you can work with somebody else to create more.)
   - The furniture in the room must contain at least 5 different shapes. Decide as a group what shapes will be used.
   - Look at the pictures for ideas but use the ideas of your group also.

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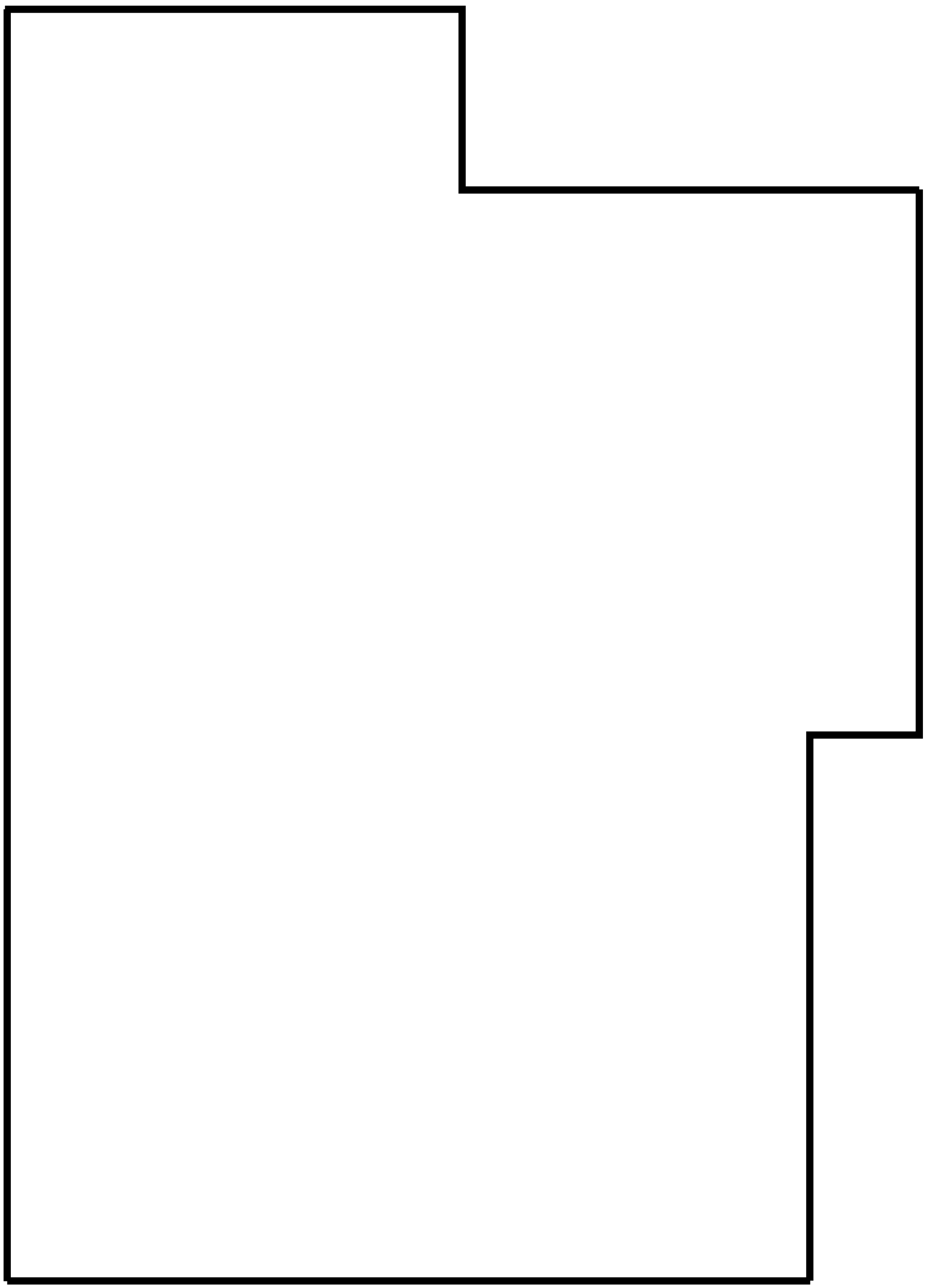
The floor of your room: All furniture must be small enough to fit in this room.



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Designers: ____

Some "Shapely" Furniture Ideas












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Complete the table below about the shapes you used in designing your furniture.

Name of Shape	Which piece of furniture has that shape?	How do you know it is that shape?	How tall (in inches) is the piece of furniture containing that shape?
1.			
2.			
3.			
4.			
5.			

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Appendix A: Written Communication in Mathematics

Young students need guidance when communicating mathematically in writing. Writing is very important in helping student think deeply about a problem to solidify their understanding. The following steps for how to write a good response in math are important to help students develop strong mathematics communication skills.

The following writing process should be displayed in the classroom for continued reference throughout all three units:

How to write a good response in math:

1. **Understand the Question**
   - Read the question. Make sure you know what it is asking you to do.
2. **Make a Plan**
   - Think about how you will solve the problem.
   - Write down your ideas.
   - Think about why you are solving the problem that way.
3. **Come up with an Answer**
   - Use your plan to solve the problem.
4. **Write a Response**
   - Write down your answer.
   - Write down step-by-step how you solved the problem.
   - If you have to explain, write down why you are solving the problem that way.
5. **Reflect and Review**
   - Read your response to yourself. Be sure it makes sense.
   - Sometimes have someone else read your response.
6. **Revise**
   - Make changes to your response if you need to.

Because this strategy can be used with any mathematical problem, the example presented here is not specific to the content of the “What Works” units. Rather, it is intended to demonstrate to students how to break down any mathematical problem in order to write about it effectively. Discuss each step with your students.

Provide each student with one of the following worksheets based on his or her need for more or less support in responding to the prompts. The first worksheet has less support and the second has more.

Student Mathematicians Write About It:

The Problem: Carlos is making trail mix. He buys 14 ounces of nuts, 8 ounces of dried fruit, and 12 ounces of chocolate chips. He and his brother eat 3 ounces of the chocolate chips out of the bag. Carlos mixes all the remaining ingredients to make his trail mix. How many ounces of trail mix does Carlos make?

Understand the Question

____

____

Make a Plan

____

____

____

Come Up With an Answer

____

____

Write a Response

____

____

Reflect and Review

Revise (if necessary). Rewrite or add to your response. Use the back if necessary.

Student Mathematicians Write About It:

The Problem: Carlos is making trail mix. He buys 14 ounces of nuts, 8 ounces of dried fruit, and 12 ounces of chocolate chips. He and his brother eat 3 ounces of the chocolate chips out of the bag. Carlos mixes all the remaining ingredients to make his trail mix. How many ounces of trail mix does Carlos make?

Understand the Question
The question is asking me to

____

____

Make a Plan
I should start solving the problem by

____

____

Come Up With an Answer
I discovered that the answer to the problem is

____

____

Write a Response
The answer to the problem is ___________. I found this by

____

____

Reflect and Review

Revise (if necessary). Rewrite or add to your response. Use the back if necessary.

Appendix B: Talk Moves

Another strategy to extend student's discussion of mathematical problems is **Talk Moves**. This strategy involves asking students to restate and agree or disagree with what another student has said. A video clip of one teacher's use of this strategy can be found on the Awesome Algebra DVD you received with your supplies for the Algebra unit.

A class discussion using **Talk Moves** might proceed like this:

Teacher: How did you find your answer, Sarah?

Sarah: Well, I knew that the two friends wanted to share the 5 cookies, but when they each got two, there was still one cookie left. So the only fair way to split the last cookie was to break it in half. So then each friend got two and a half cookies.

Teacher: That sounds like a good strategy, Sarah. *Paul, could you please restate how Sarah got her answer?*

Paul: Yeah. She said that the only fair way to share the 5 cookies was to divide the remainder into two halves and give an extra half to each friend.

Teacher: I like the way you used the word "remainder" like a mathematician, Paul. *Now, does anyone disagree with how Sarah and Paul solved the problem? Can anyone find a different way to solve the problem?*

Verbal Communication

Discussion is very important in nudging student thinking forward. This is in line with the social constructivist theory of Lev Vygotsky whose research gives credence to the idea that students can be guided to better mathematical understandings as they analyze complex skills and concepts together (Biehler & Snowman, 1993). Vygotsky's research showed that actions involving complex knowledge could be internalized more quickly with the help of guiding questions and discussion (Vygotsky, 1978). One way to conceptualize this type of communication is to compare it to the writing process (Cazden, 2001). Students engaged in this type of communication, known as exploratory talk, are manipulating their ideas much like in the beginning phases of the writing process. Students participating in more elaborate talk, the predominant form of communication in more traditional mathematics classes, tend to express refined ideas that are like those found in the final draft of a written work. Students should have ample opportunities to engage in exploratory talk to help them develop more elaborate ideas.

Students who have had practice in talking at length with their peers and teacher about solving mathematics problems tend to persist longer in trying to understand a new problem. As they get used to the process of explaining their thinking and revising their thinking in light of others' comments, they come to understand that it takes time to think through a problem. As the class becomes more practiced at communicating mathematically, students are motivated to organize, consolidate, and clarify their own thinking to be able to participate with their peers. Students learn to view problems from different perspectives and to appreciate a variety of thinking and problem-solving styles as they listen to their peers' methods of solving problems.

Teachers can utilize particular strategies to foster this type of communication in their classrooms. Chapin, O'Connor, and Anderson refer to such strategies as "talk moves" in their book, *Classroom Discussions: Using Math Talk to Help Students Learn* (2003). In this unit, we use five talk moves:

1. The first move is *re-voicing* in which the teacher restates a student's idea and then verifies whether it was accurate. For example, a teacher might ask, "You said that the area is 24 square inches?" This move can be used when a student's response is unclear. It also allows students to clarify their thinking.

2. A second move is *repeat/rephrase* and is similar to re-voicing. Instead, the teacher asks other students to restate an idea by asking questions such as, "Can you repeat what she said in your own words?" It is important to follow up with the student contributing the original thought to ensure that the idea was heard as intended. This not only validates the idea but also gives the class another version of the idea, allows time to process it, and makes certain that students are following the conversation.

3. The third move, *agree/disagree and why*, is used after the teacher makes sure that students have heard and have had time to process the thought. By posing questions like, "Do you agree or disagree with that idea? Why?" teachers can draw out student thinking by having students apply their own understanding to someone else's thoughts. It is essential that teachers do not offer their positions at this juncture, allowing students to grapple with their own thoughts. Teachers can help students focus on the correct concepts after they have had the opportunity to develop their reasoning.

4. Teachers can prompt students to participate further by using the fourth move, *adding on*. Questions like, "Who would like to add to their ideas?" serve to advance the discussion. Students benefit from this move because original ideas become more comprehensive as more perspectives are considered.

5. Finally, a fifth move, *wait time*, allows students to organize their ideas and serves to encourage all students to contribute, not just those who process their thoughts quickly. **Since students need time to process their responses to complicated questions, teachers should not only wait to call on a student after posing a question but also should wait for a student who has been called on to share his or her idea.** Comments like, "We'll wait for your idea," serve this purpose. If necessary, teachers can ask apprehensive students questions such as, "Should we come back to you later?" It is important to follow up with these students later in the discussion and continue to work with them to encourage them to be more active in the discussions. It is necessary for all students to participate in discussions to benefit.

References

Biehler, R. F., & Snowman, J. (1993). *Psychology applied to teaching* (7th ed.). Boston, MA: Houghton Mifflin.

Cazden, C. B. (2001). *Classroom discourse: The language of teaching and learning.* Portsmouth, NH: Heinemann.

Chapin, S. H., O'Connor, C., & Anderson, N. C. (2003). *Classroom discussions: Using math talk to help students learn.* Sausalito, CA: Math Solutions Publications.

Vygotsky, L. S. (1978). *Mind in society.* Cambridge, MA: Harvard University Press.

Geometry & Measurement for All Shapes & Sizes Mathematical Language

Acute Angle: An angle that is greater than 0° but less than 90°.

Adjacent: To share a side.

Angle: A figure that is formed by two sides of a polygon with a common endpoint.

Area: Number of square units covering a surface.

Asymmetric: Not symmetric.

Circle: Set of points a fixed distance from a center.

Circumference: Distance around a circle.

Clockwise: A rotation in the same direction that the hands of a clock move.

Congruent Figures: Figures that have the same size and shape.

Congruent Sides: Sides of equal length.

Counter clockwise: A rotation in the opposite direction that the hands of a clock move.

Decagon: A polygon with ten sides.

Denominator: The part of a fraction that is below the line and divides the numerator.

Diagonals of a Polygon: Line segments in a polygon connecting non-consecutive vertices.

Diameter: Line segment passing through the center of a circle and has end points on the circle.

Dimensions: The length and width of a rectangular figure.

Endpoints: The points on a line segment that show where it begins and ends.

Equivalent Fractions: Fractions that are equal, line 1/2 and 2/4.

Estimate: An answer that is as mathematically close to the real answer as possible.

Flip (reflect): To turn over.

Fraction: Part of a group, number, or whole.

Heptagon: A polygon with seven sides.

Hexagon: A polygon with six sides.

Horizontal Line Segment: A line segment drawn in the left-right direction.

Inch: A unit of standard measurement.

Line Segment: Part of a line with two endpoints.

Nonagon: A polygon with nine sides.

Numerator: The part of a fraction that is above the line and is divided by the denominator.

Obtuse Angle: An angle that is greater than 90^{o} but less than 180^{o}.

Octagon: A polygon with eight sides.

Pentagon: A polygon with five sides.

Perimeter: The distance around a figure.

Point: An exact location in space, usually represented by a dot.

Polygon: A closed figure formed by three or more line segments.

Quadrilateral: A polygon with four sides.

Radius: Line segment that starts from the center of a circle to any point on the circle.

Ray: Part of a line with one endpoint that goes on forever in one direction. (The sun's rays begin at the sun and go on in one direction.)

Rectangle: A quadrilateral with four right angles.

Regular Polygon: A polygon with all sides and angles congruent.

Rhombus: A quadrilateral with four congruent sides.

Right Angle: An angle that is 90^o.

Sides: The line segments that make up a polygon.

Slide (translate): To move an item in any direction without rotating it.

Square: A quadrilateral with four congruent sides and four right angles.

Square units: Units used to measure area (two-dimensional).

Symmetry: An object is symmetrical when one half is a mirror image of the other half.

Transform: To change the position of something.

Triangle: A polygon with three sides.

Triangle Sum Theorem: The sum of the interior angles of any triangle equals 180^o.

Turn (rotate): To rotate around a point.

Unit: A standard of measurement.

Vertex (vertices): The point where the rays of an angle meet. The point(s) where the sides of a polygon meet.

Vertical Line Segment: A line segment drawn in the up-down direction.

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