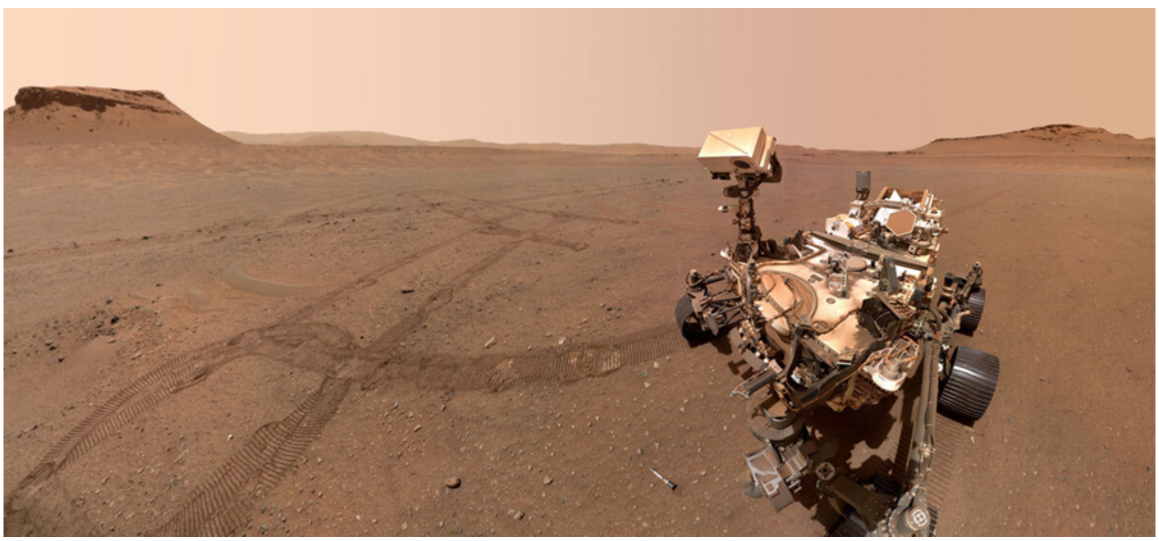
Thinking Like Mathematicians: Challenging All Grade 3 Students

Mission to Mars
Fractions Lessons Collection

Created by the TLM Research Team



Renzulli Center for Creativity, Gifted Education, and Talent Development
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Thinking Like Mathematicians: Challenging All Grade 3 Students

Introduction

In grade 3, students first begin exploring fractions. This is an important period in students' mathematical development because fractions establish an important foundation for algebraic concepts such as percentages, decimals, ratios, and probability, although many students have difficulty understanding fractions (National Mathematics Advisory Panel [NMAP], 2008). Corresponding research has demonstrated that middle school students' ability to place fractions on a number line predicts early algebra skills, including knowledge of algebraic features and skills in solving algebraic equations and word problems (Booth & Newton, 2012). In another study, grade 4 fractions knowledge predicted grade 6 mathematics achievement, even when controlling for other relevant variables such as whole number knowledge and performance on different cognitive tests (Resnick et al., 2019). Acknowledging the role that fractions play in numeracy, these lessons (a) describe pedagogical approaches that support students' mathematical thinking and (b) provide a series of enriched, pre-differentiated, standards-based fractions lessons designed to appropriately challenge the diverse grade 3 student population.

Aligning with the Principles and Standards for School Mathematics (National Council of Teachers of Mathematics [NCTM], 2000), these lessons were designed to help students develop an understanding of fractions, explore the relationships between fractions and whole numbers, represent fractions using number lines and other fraction models, identify and create equivalent fractions, compare the magnitude of different fractions, and fluently perform computations involving fractions. Incorporating other research-supported practices (e.g., Kaplan, 2009; NMAP, 2008; Reinhold et al., 2020), the lessons present pre-differentiated learning activities that develop higher order thinking skills, focus on real-world applications, and provide levels of scaffolding and challenge appropriate to learners with varied readiness and background knowledge.

The majority of students in elementary and middle school grades have difficulty learning about fractions (Roesslein & Codding, 2019). Oftentimes, traditional approaches do not explicitly teach students the similarities, differences, and relationships between fractions and whole numbers, which can hinder students' understanding (Booth & Newton, 2012). Students often transfer mathematical strategies based on natural numbers, which are not always appropriate and can result in ongoing misunderstandings (Reinhold et al., 2020). Fortunately, a

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Sections of the Mission to Mars introduction and supplementary information built upon the *If Aliens Taught Algebra: Multiplication and Division Would be out of This world!* curriculum (2019), authored by Shelbi Cole, Lisa DaVia Rubenstein, Tutita M. Casa, Cindy M. Gilson, Micah N. Bruce-Davis, E. Jean Gubbins, Alexis Melendez, Hannah F. Brown, Siamak Vahidi, Rachel R. McAnallen, and Stacy M. Hayden, University of Connecticut, Storrs, Connecticut.

number of approaches can help address these difficulties, and integrating multiple strategies can improve students' understanding of fractions (Roesslein & Codding, 2019). Practicing ordering, comparing, and modeling fractions contributes to students' fraction knowledge (NCTM, 2000), but the most consistently supported strategy involves using number lines to help students (a) understand the relationship between whole numbers and fractions, which includes understanding whole numbers as fractions themselves, and (b) connect their conceptual understanding with real-world procedures (Booth & Newton, 2012; NMAP, 2008).

This collection of grade 3 fractions lessons aims to simultaneously develop students' conceptual understanding, computational fluency, and problem-solving skills (NMAP, 2008). Based on a Mission to Mars theme, the lessons are arranged according to four core topics: (a) defining and understanding fractions, (b) defining and labeling number lines, (c) equivalent fractions, and (d) comparing fractions. Throughout the lessons, emphasis is placed not only on fractions as distinct number representations, but also on their relation to whole numbers.

Background and Rationale

Despite the influential role students' fraction knowledge plays in their mathematical development, approximately half of grade 8 and 12 students do not have the conceptual and procedural understanding needed to become proficient with fractions and require more targeted support (Shin & Bryant, 2015). Students often demonstrate a "natural number bias" in which they apply whole number procedures to fraction problems, such as when performing arithmetic operations or strategies for comparing sizes (Reinhold et al., 2020). For example, students might incorrectly assume that adding fractions involves adding both numerators *and* denominators, or that when comparing fractions, larger numerators signify greater values irrespective of denominators. Students who have more difficulty understanding fractions also tend to have trouble "placing fractions on a number line, determining equivalent fractions, comparing and ordering fractions, and estimating sums of two fractions" (Jordan et al., 2017, p. 626).

Curriculum Standards

These lessons were designed to align with contemporary standards for mathematics development outlined by both the NCTM (2000) and the Common Core State Standards Initiative (2022). Additionally, lessons emphasize the Partnership for 21st Century Learning's (2019) four learning and innovation skills: creativity, critical thinking, collaboration, and communication.

Differentiation

Differentiation is a useful strategy to support diverse student populations and accommodate their varied readiness to learn about fractions, and studies have shown that differentiated learning increases in students' engagement (Bondie et al., 2019) and academic performance (Deunk et al., 2018). Unfortunately, many teachers do not feel confident that they have the knowledge, skills, and support needed to successfully differentiate in the classroom, and time constraints and standardized testing pressures pose additional barriers to differentiation (Whitley et al., 2019). However, teachers typically hold positive beliefs about differentiation and frequently implement it in practice, particularly when self-efficacy and organizational support are high (Whitley et al., 2019). When differentiating, many teachers have difficulty providing appropriate learning for all students in the class. More often than not, differentiation is used to support learners who are having difficulties with lesson content rather than to challenge students who already have a more complete understanding of lesson content (Rubenstein et al., 2015).

To provide adequate levels of challenge for *all* students, each lesson has three tiered activities that are enriched and pre-differentiated based on three relevant educational models: the Differentiation of Instruction Model (DIM), the Depth and Complexity Model (DCM), and the Schoolwide Enrichment Model (SEM). The DIM specifies that overarching principles of effective teaching and learning are similar regardless of a student's ability level, but students' specific needs can be met by differentiating elements of learning content, processes, products, and/or learning environments (Tomlinson, 2001). In the DCM, learning is organized around themes and big ideas, which intersect with content, processes, and products that are differentiated to provide appropriate levels of depth and complexity (Kaplan & Gould, 2005). Finally, the SEM is a whole-school talent development intervention in which the regular curriculum remains central to learning, but students participate in enrichment opportunities based on their interests (Renzulli & Reis, 1985, 2014). Numerous studies have shown that there are many benefits to integrating these three models (e.g., Callahan et al., 2015; Gubbins et al., 2013).

Altogether, these lessons aim to support teachers by incorporating research-informed differentiation strategies that will guide teachers through the process and reduce preparation time. Each lesson is pre-differentiated, providing three lesson tiers specifically designed to meet the needs of students who require considerable supports and less challenge (Tier 1), a balance of moderate supports and challenge (Tier 2), and greater challenge with fewer supports (Tier 3). The lessons include different diagnostic assessments to help teachers identify students' needs and assign them to appropriate lesson tiers. This approach is based on findings that students learn fractions better through lessons that (a) provide levels of scaffolding specifically designed to support students of varying readiness levels, (b) include relevant visual representations, (c) use number lines to represent fractions and demonstrate their relation to whole numbers, (d) include both direct, teacher-led instruction and opportunities for students to think aloud and interact with the teacher, (e) involve real-world problems, and (f) use formative assessment methods to monitor students' individual progress (Jordan et al., 2017; NCTM, 2000; NMAP, 2008; Reinhold et al., 2020; Shin & Bryant, 2015).

Organization of the Lessons

Lesson Collection Breakdown

The goal of the lessons is to build students’ conceptual understanding of fractions and their ability to use different strategies to model fractions, recognize and produce equivalent fractions, and judge the relative size of fractions. The collection is divided into four sections. The first section includes a variety of lessons that introduce students to the concept of fractions and develop their understanding of fractions as numbers representing parts of a whole. In the second section, students expand their knowledge by understanding and representing fractions as numbers on a number line. Lessons in the third section focus on strategies that help students recognize and generate simple equivalent fractions. The fourth, and final, section features lessons that extend students’ understanding of numerical comparisons to fractions.

Lesson Design

Each of the lessons follow a consistent format. To describe each lesson’s overall focus, the lessons begin by outlining key pieces of information:

- Big ideas
- Lesson objectives
- Common Core State Standards
- Materials
- Mathematical terms
- Selected mathematical practices
- Differentiation guiding questions

Next, the following sections in Table 1 are presented to guide teachers in implementing the lesson.

Differentiation

Each lesson prompts teachers to use different methods to determine students’ level of prior knowledge and readiness, which is used to assign students to pre-differentiated lesson tiers featuring a suitable level of challenge. The lessons contain additional considerations that are highlighted at the beginning of each lesson, which include strategies to differentiate learning content, processes, products, and the learning environment. Each lesson presents differentiated activities that address the same big ideas, content standards, and mathematical practices, while varying the complexity and depth of the tiered student pages to accommodate different readiness levels. Teachers are encouraged to use flexible grouping throughout lesson activities so that students may work together. Lessons shift between (a) whole-class instruction or discussions and (b) differentiated activities better suited to individual work or smaller groupings based on student readiness.

Table 1

Teaching Like Mathematicians: Challenging All Grade 3 Students—Lesson Sections

Lesson Preview	The "Lesson Preview" section provides teachers with a concise overview of the lesson.
Launch	The "Launch" section represents the beginning of the class itself. During this time, teachers will guide a discussion centered on mathematical practices, with the aim of encouraging students to "think like mathematicians."
Explore	The "Explore" section begins with a whole-class activity in which teachers will provide students with an overview of the mathematical concepts covered in the lesson and ask relevant questions to activate prior knowledge and prompt engagement. Formative assessment methods are provided to guide teachers in assigning students to appropriate lesson tiers, and students work on one of three sets of "Student Pages" containing pre-differentiated and enriched activities.
Examine and Elaborate	The "Examine and Elaborate" section prompts students to reflect on their mathematical thinking. The section begins with a "Share and Discuss" activity, where teachers are provided with a sample discussion highlighting the mathematical thinking students have applied. The section also includes a range of differentiation resources to provide students with additional scaffolds and challenges, if needed.
Debrief and Look Ahead	The "Debrief and Look Ahead" section includes another whole-class activity in which teachers stimulate dialogue revisiting the lesson's big ideas and reflecting on how students thought like mathematicians through their application of the selected mathematical practices.
Assess	The "Assess" section describes the different formative and summative assessment methods included in the lesson and how these can be used to support student learning.

The tiered groups are named after famous astronauts who have significantly contributed to humanity's exploration of space (biographies are provided in the supplementary information section).

- Tier 1: Peggy Whitson—least challenging. These Student Pages are designed for students with little-to-no prior knowledge of the topic and who require increased scaffolding.
- Tier 2: Guion Bluford—moderately challenging. These Student Pages are designed for students with some understanding of the topic and who require moderate levels of scaffolding and challenge.
- Tier 3: Ellen Ochoa—most challenging. These Student Pages are designed for students who already understand the topic, require minimal scaffolding, and are ready for more challenging applications.

Most lessons contain additional Hint Cards to support students who are struggling with specific questions. Hint Cards are designed to activate relevant knowledge and stimulate mathematical thinking that will support students in completing difficult questions. In addition to Hint Cards, most lessons include optional Challenge Cards that can extend students' learning if they have completed their assigned work early. Instead of providing extra work on topics students have already mastered, Challenge Cards are designed to capture students' interest and extend their learning by applying critical and/or creative thinking, addressing higher grade-level standards, solving real world and/or multi-faceted problems, communicating their understanding, generalizing their learning, and solving more complex and/or more abstract problems.

Mathematical Practices

Each lesson includes opportunities for direct instruction in mathematical practices that encourage students to think like mathematicians. The eight Common Core State Standards for Mathematical Practice are:

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning (Common Core State Standards Initiative, 2022)

Each lesson focuses on several mathematical practices that align with learning content and processes. Selected mathematical practices are outlined at the beginning of each lesson. Because lesson elements are designed to align with selected mathematical practices, students are encouraged to reflect on and apply these practices during the Launch, Examine and Elaborate, and Debrief and Look Ahead sections.

Instructional Discourse

Each lesson provides multiple features to help teachers anticipate how they might facilitate whole-class discussions in which students think—and communicate—like mathematicians. Throughout the lesson, teachers are encouraged to use focusing questions that push students to justify responses based on their own thinking (Wood, 1998). *It is important to realize that ideas presented by students are plausible ones, and teachers are encouraged to focus on their own students' contributions and needs*. Lessons provide suggestions on how to implement "talk moves" (Chapin et al., 2009) during these discussions, which include:

1. Revoicing—This talk move involves having the teacher restate a student's answer to clarify the teacher's understanding of the student's response. This is especially helpful if the student's response is not clear. An example

would be, “You said that the numbers in this pattern are increasing by threes. Is this right?”

2. Repeat/Rephrase—With this talk move, teachers ask other students to repeat or rephrase what a classmate just stated. For example, “Could someone repeat what Sally just said in your own words?” This allows other students to hear what the first student stated and it gives them a chance to process the information.
3. Reasoning—After a student responds, the teacher might ask: “Do you agree with Sally’s explanation or disagree? Why?” as a way to promote mathematical reasoning among students based on each other’s responses.
4. Adding On—Teachers can encourage participation among more students in the class by asking students “Would someone else like to add more to Sally’s explanation?”
5. Wait Time—In this move, the teacher waits and gives students an opportunity to think about a question that has been asked or think about a classmate’s answer. This gives students a chance to process and formulate a response. The teacher might say, “Take your time, Joe, and we will wait for you to think.”

Conclusion

Although fractions play an important role in numeracy development, many students have difficulty learning about fractions. To support students in this developmental process, we have based these Mission to Mars themed lessons on current standards, evidence-based instructional models, and innovative learning sciences research. Lessons are purposefully organized to guide teachers through the process of delivering differentiated, enriched mathematics lessons, and each lesson includes areas of focus and components that should challenge your students to think like mathematicians.

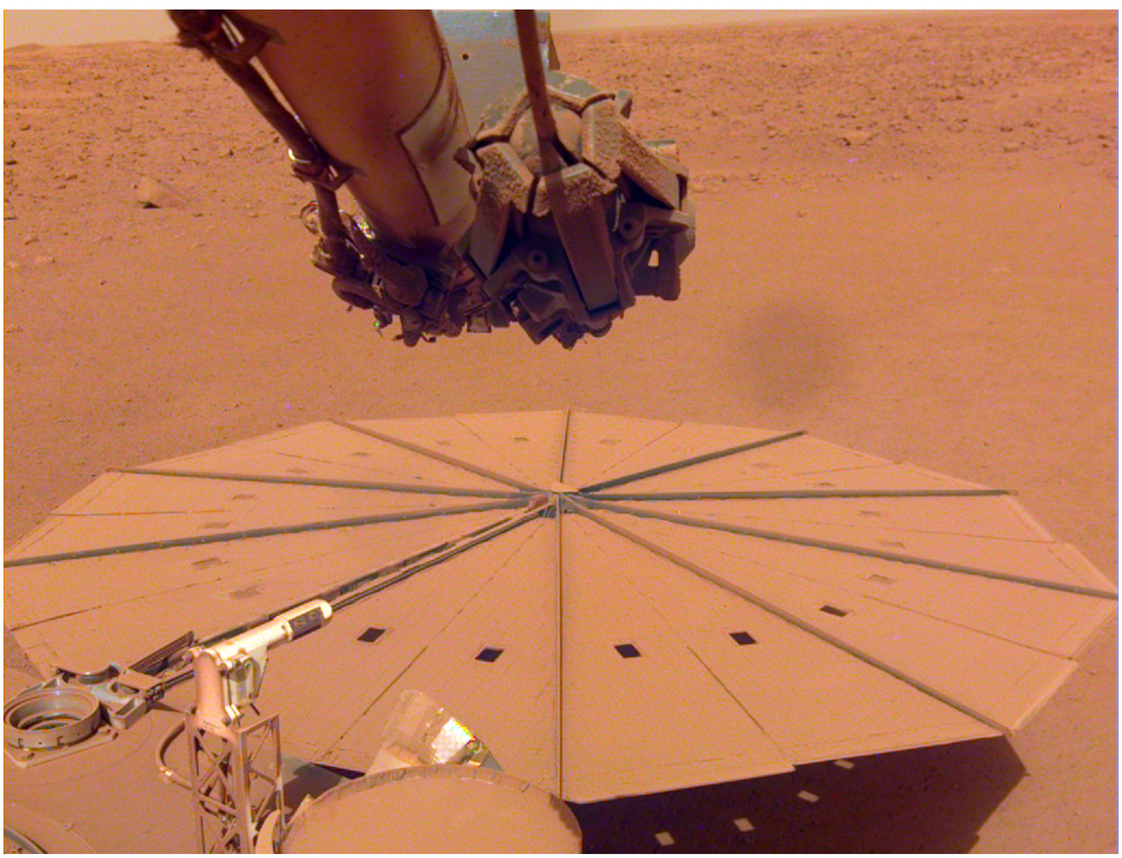
Thinking Like Mathematicians:
Challenging All Grade 3 Students

Common Core State Standard

Defining and Understanding Fractions

CCSS.MATH.CONTENT.3.NF.A.1
Understand a fraction 1/*b* as the quantity formed by 1 part when a whole is partitioned into *b* equal parts; understand a fraction *a*/*b* as the quantity formed by *a* parts of size 1/*b*.

Standard in Plain Language:
Students will need to understand that fractions communicate the relationship between a whole and the parts of a whole. They will recognize that the bottom part of a fraction (i.e., denominator) communicates how many equal parts the whole number is divided into, while the top of the fraction (i.e., numerator) communicates how many equal parts are being represented. Students will use appropriate symbols and models to communicate these relationships with clarity and precision.

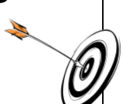


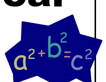
Lesson Designer: Tutita Casa

Lesson 1: Defining a Fraction With Out-of-This-World Pudding—

Big Ideas

Numbers provide consistent methods of communication so everyone can understand the amount that is being considered. Fractions are numbers that precisely describe a situation where a whole has been broken up into *equal* parts. The “denominator” tells you how many equal parts there are in the whole and it is written on the bottom of the fraction. The “numerator” tells you how many of those equal parts you chose, and it is recorded at the top. Being able to distinguish and understand these two mathematical terms is crucial. Fractions provide a clear method to communicate the number of equal parts of a whole and how many of the parts you have, want, or need. Fractions are everywhere and used for measuring (e.g., 3/4 cup of flour, a quarter past 10:00) and sharing (e.g., a sandwich that is half ham and half turkey, splitting a restaurant bill equally among friends). Discussing fractions emphasizes the importance of precision in mathematics and in the world.

Lesson Objectives 	• Students will be able to determine the meaning of the numerator, a, in the fraction a/b where $a < b$. • Students will be able to determine the role of the denominator, b, in the fraction a/b where $a < b$. • Students will be able to distinguish the difference between the numerator and denominator in the fraction a/b where $a < b$. • Students will be able to use fractions in the real world with precision. **Common Core State Standard** Develop understanding of fractions as numbers. <u>CCSS.MATH.CONTENT.3.NF.A.1</u> Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into b equal parts;

	understand a fraction *a*/*b* as the quantity formed by *a* parts of size 1/*b*.
Materials 	• Student Pages—*Out-of-This-World Pudding* • powdered drink, ideally Tang (lemonade, Kool-Aid, and a sports drink are some alternatives); identify or test out the number of scoops (e.g., using a 1/4 cup scoop) of powder and water that are needed to make the drink; make note of the scoops of powder and scoops of water needed • 3-ounce disposable cups (three per student) • manipulatives (e.g., fraction bars, white boards, Unifix cubes) ○ Remind students that manipulatives are available to use strategically and appropriately • Optional: packages of three different flavored pudding powders, such as caramel, chocolate, and vanilla, along with other recipe ingredients, such as milk
Mathematical Terms 	• **Denominator:** bottom number in a fraction that identifies the number of pieces the whole has been divided into equal parts • **Divide:** split into equal parts or groups • **Fraction:** a number that represents part of a whole • **Half:** one of two *equal* parts of a whole • **Numerator:** top number in a fraction that identifies the number of equal pieces considered as part of the whole • **Precision:** the quality, condition, or fact of being exact and accurate • **Whole:** the entire unit that represents one
Selected Mathematical Practices 	• MP3: Construct viable arguments and critique the reasoning of others. *I can explain my math thinking and talk about it with others.* • MP4: Model with mathematics. *I see the math in everyday life, and I can use math to solve everyday problems.*

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- MP6: Attend to precision.
  I can work carefully and check my work.

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Differentiation

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Content
Guiding Questions

- prior knowledge or learner readiness
  What evidence do you have about students' current knowledge and skills?
- tiered activities
  How will you design tiered activities on the same mathematical concept with varied levels of difficulty?
- formative assessment
  What techniques will you use to assess students' prior knowledge and skills?
- "teaching up" (aim high, provide scaffolding)
  How will you increase the depth, breadth, complexity, and abstractness of lessons to challenge and support student learning?
- real-world application
  What real-world connections will you make explicit about mathematical concepts and skills?

Process
Guiding Questions

- questioning strategies
  How will you pose and how will you encourage students to pose open-ended, closed-ended, lower-level, and higher-level questions to promote mathematical discourse?
- 4Cs (21st Century Skills)
  - critical thinking
  How will you promote a learning environment in which students question data and view issues or problems from multiple perspectives?
- 4Cs (21st Century Skills)
  - creative thinking
  How will you encourage students to "think outside the box" and synthesize information in new, different, and useful ways?
- 4Cs (21st Century Skills)
  - collaboration
  How will you encourage students to work with other students and appreciate their contributions

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	to solving problems or making connections to prior work? • 4Cs (21st Century Skills) ○ communication *How will you promote students' opportunities to communicate face-to-face, in large and small groups, in online environments, and with print and non-print resources using their oral, written, and non-verbal skills?* • hands-on activities/manipulatives *How will you incorporate activities promoting the use of manipulatives to clarify or illustrate mathematical concepts?* **Product** ***Guiding Questions*** • oral, visual, and written opportunities *How will you encourage students to represent their thinking and problem solving using different communication strategies?* • summative assessment *How will you assess student learning upon completion of the lesson?* **Learning Environment** ***Guiding Questions*** • whole group/small group/individual instruction *How will you incorporate different grouping plans to address students' learning needs?* • growth mindset *How will you promote the perspective that it is important to view abilities as malleable?* • learning community *How will you support a positive learning community as students are encouraged to think, work, and communicate like mathematicians?*

Lesson Preview

In this lesson, students will be introduced to how astronauts eat food in space and how they rely on powdered substances for most of their food. During the Lesson Launch and Explore sections, students will help determine the fractional representation of the amount of powder and water needed in a recipe to make a good-tasting drink. The teacher will be imprecise in their measurements causing the drink to taste bad and be unhealthy. Students will understand the importance of precision in mathematics and within the real world. Then, students will be placed into different tiered groups based on their response to the entrance ticket and complete an activity involving fractions and pudding recipes. Once students have completed these worksheets, the class will come together for a group discussion debriefing what they learned during the lesson.

Launch

1. **Thinking Like Mathematicians**

As a whole group, discuss why mathematicians use mathematical language. Consider writing a list of students' ideas on a white board or chart paper. Ask the students:

- Why do mathematicians use mathematical language?
- How might mathematical language help mathematicians communicate?

If students struggle to generate ideas, explain that mathematicians use mathematical language to be precise. Using mathematical language allows mathematicians to be clear and specific when they communicate their ideas. Mathematicians might use specific mathematical vocabulary (e.g., numerator, denominator) or provide details in mathematical terms (e.g., giving an answer with the units of measurement).

Tang Got a Boost in Space

Ask the class what they know about the foods and drinks astronauts consume in space. Some may already know or wonder about how the food does not float in space. The following includes some fun facts to share with students to provide the context for the upcoming exploration.

> *One responsibility that food scientists have when creating the astronauts' menu is to ensure that even crumbs do not float off into space because of the minimal amount of gravity in space (Simmons, 2019). Like solid foods, liquids also can float away (Furdyk, 2022).*
>
> *Yet another challenge that food scientists faced during the 1950s and 1960s missions was that the chemical reactions used to create water resulted in a bad taste of water on board NASA missions (Furdyk, 2022). The solution to that problem was for the astronauts to inject water into the pouch holding powdered drinks such as coffee, tea, and juices (Simmons, 2019; Watson, n.d.).*
>
> *The most infamous of those drinks is the Tang that went into the history books on February 20, 1962 during the Friendship 7 mission. Not only did astronaut John Glenn become the first American to orbit the Earth that day, but Tang became the first "soft drink" to do so, too (Hella Entertainment, n.d.)!*

Contrary to popular belief, Tang was not invented by NASA and was not specially created for astronauts. Food scientist William Mitchell already had created the orange-flavored drink for General Foods, and NASA saw this as a solution to their water and gravity challenges. Due to some savvy marketing connecting Tang with the growingly popular space program, it soon began to fly off the shelves (Simmons, 2019)!

Explore

2. First Batch, Odd Taste

In this investigation, students will be working on one of the Student Pages based on their differentiated groups. The groups are based on teacher's observations of students' understanding.

Groups Formed by Student Readiness		
Tier 1: Peggy Whitson	**Tier 2: Guion Bluford**	**Tier 3: Ellen Ochoa**
Student Names	**Student Names**	**Student Names**

Prior to class, make one batch of Tang using the correct, precise measurements and have it ready to serve before the lesson starts. Tang may be a unique drink for most students and this batch will give students the opportunity to taste what Tang is supposed to taste like. Pour and pass out samples of the drink in the 3-ounce Dixie cups, one for each student. While they taste it, write the "recipe" you used on the board using numbers that correspond to your recipe, such as:

two of the eight total scoops are Tang powder
six out of the eight total scoops are water

During this lesson, you are going to make two additional batches of the drink, but now in front of the class. As you make the first batch in front of the class, make sure that you do **not** measure each scoop of the powder and water carefully so that, in the end, the drink is not tasty. As you haphazardly and quickly measure, share with students, for example, that the scoop was "close enough!" Some students might immediately realize that you should be measuring much more carefully. Explain that you think it is fine what you are doing since, for instance, the recipe calls for 2 of the 8 scoops to be powder and 6 of the 8 scoops to be water, which is the number of scoops you are putting in.

Next, ask the class to tell you how well they think you followed the recipe and why by having them test your beverage. Pour and pass out samples of the drink in the 3-ounce Dixie cups for each student. Encourage everyone to wait to drink it at the same time to get their collective reaction. Given that it will be either much too watered down or concentrated, ask them why they think it did not taste good and was different from the initial drink they tested.

Teacher: Jonah, your face clearly showed you did not like the drink! Can you describe the taste?
Jonah: Ugh. It's like it didn't have any flavor.
Teacher: Who else agrees with Jonah?
Students: Yes! Uh-huh! That had way too much water.
Teacher: I heard someone say something about the amount of water. Can someone say more about that observation? *(Adding On talk move)*
Maven: Yeah, I wish it had more of the Tang flavor like the drink from the beginning of class. [Other students nod in agreement.]
Teacher: Huh, I wonder why that happened because I followed the recipe. I knew I was supposed to have 8 total scoops, with 2 being the powder and 6 being the water. This is what I counted. Can someone please explain what they think might have happened?
Danni: Well, you did count, but was it *really* a scoop?
Teacher: Can someone add on to what Danni just said? (*Adding On talk move)*
Xavier: I think what Danni means is that the scoops were different amounts.
Teacher: Can someone repeat what Xavier said in their own words? If you did not hear or understand him, you can ask him to repeat. (*Repeat/Rephrase talk move)*
Jonah: I heard him say that the scoops have to be the same size.
Teacher: Xavier, is that what you meant to say?
Danni: Yeah, you can't have one small scoop then a big one. The scoop has to be the same. I think the drink would then taste good like the batch we tried at the beginning of class!
Teacher: That's a great idea! Let's first make sure we all agree on how to make the recipe again. Who agrees or disagrees with Danni and Xavier's ideas about the scoops and can explain why? *(Reasoning talk move)*
Brandon: I agree with Danni and Xavier because we have to have equal amounts of powder.

Isla: Yes, if the amount of powder in each scoop is different, we are not precise with our measurements.

Second Batch, Good Taste

Apply students' feedback about how to precisely measure the scoops to make the drink. This time, carefully measure out each one and orally emphasize the importance of making them the same size, such as, "I need to make sure the powder goes straight across" and, "This is over the top, so I need to pour some out of the scoop."

As you model, ask students to guide you so they become keenly aware of the size of each scoop. Then, use the same procedure as before to pass out samples so students can try this new drink. They likely will agree that it is much tastier!

Have the class restate why this was the case. As they do so, review or introduce the symbolic notation used to record fractions. For instance, write "two equal-sized scoops out of eight total equal-sized scoops" and record 2/8.

Ask the class to compare and contrast the "top" and "bottom" numbers, which some students might already know respectively are called the "numerator" and "denominator." Have students or the teacher define these terms so that students have a correct understanding of the vocabulary.

Collaborate and Communicate

Have students record their ideas on their individual worksheet. Help them clarify the ideas by asking questions like, "What do you mean here?" and "How might you share that idea with Jalene so she knows exactly what you mean to say?" Point out that mathematicians use various representations to help explain their thoughts and use precise language to do so.

A. [Possible response] *This group . . .*	**B.** [Possible response] *This group . . .*	**C.** [Possible response] *This group . . .*

After the class finishes the Explore section, the students will complete the entrance ticket below and they will be placed into tiered groups based on their responses.

Entrance Ticket

In the fraction 3/8, which number is the numerator?

If Terriesa has a bucket that is 2/6 full of water and then she doubles the amount of water in the bucket, how full is the bucket now? Why?

Draw a picture that represents the fraction 6/8. Can you write the fraction 6/8 in a different way?

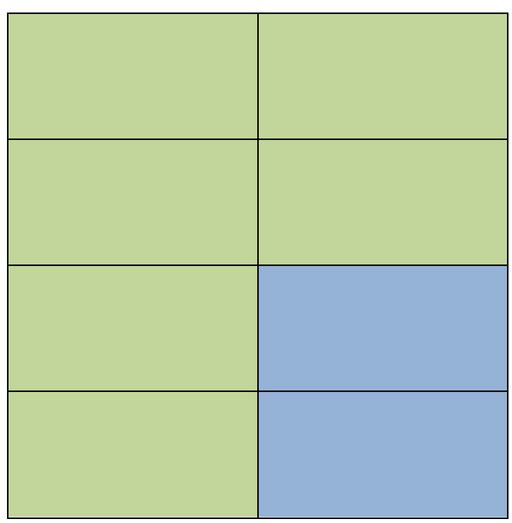
Sample Responses

Entrance Ticket

In the fraction 3/8, which number is the numerator?
3 is the numerator in this fraction.

If Terriesa has a bucket that is 2/6 full of water and then she doubles the amount of water in the bucket, how full is the bucket now? Why?
The bucket is now 4/6 full because 2/6 times 2 equals 4/6.

Draw a picture that represents the fraction 6/8. Can you write the fraction 6/8 in a different way?
Students' answers may vary. The fraction can be written as 3/4 or 12/16.



Examine and Elaborate

3. Highlight Students' Mathematical Thinking

Mathematicians think about possible solutions in a variety of ways. Therefore, it is important for students to realize that they, too, can approach problems using different strategies. Ultimately, students need to understand that a possible solution should be judged by the correctness of the mathematical process, and there might be some valid ideas within a solution when a student has an incorrect answer.

Share and Discuss

It is, therefore, important for students first to clearly share their ideas with others so their validity can be determined by the class. In this sample dialogue, the teacher is discussing the pudding recipe that is discussed within the tiered lesson.

Teacher: I see that you all are helping the astronauts make their pudding. That sounds tasty, but I am not exactly sure how I would make it. Can someone clarify this recipe for us? *(Adding On talk move)*

Liza: I think you need to say how many scoops are supposed to be pudding mix and how many are water ones.

Erik: Oh, yeah. It's supposed to be 3 scoops of pudding mix and 1 scoop of water.

Teacher: All right. How might we write a fraction based on this scenario? *(Adding On talk move)*

Sally: Well, first you need to figure out the total number of scoops you need for the pudding.

Jane: Yep, I figured that out! You need 4 total scoops of ingredients because 3 plus 1 is 4.

Luis: I got that, too! Then, I wrote the fraction 3/4.

Teacher: Can you tell us more about what you mean by that? *(Adding On talk move)*

Luis: Yeah, 3 scoops of the total number of scoops are pudding mix. Like Jane said, there are 4 scoops of ingredients in total.

Teacher: Do you have any advice for the astronauts that are making pudding?

Erik: We need to make sure they know *how* to scoop every time. It won't be like a pudding is supposed to be if they don't do it the right way!

Teacher: So, I heard you say that we need to emphasize to them that the size of the scoops is very important. That is, just because they use a scoop doesn't mean that it is, well, a full scoop. Is that what you said? *(Revoicing talk move)*

Erik: Yeah, we want the pudding to actually taste good!

Differentiate Further as Needed

For Additional Support

- Suggest that students consider what concrete materials they can use to help them with the tiered activity. For example, students using the scoops would be working at a concrete level whereas some math manipulatives, such as linking cubes, are more abstract.
- Point out the importance of recording fractions, which are commonly included with recipes. Remind students of the use of labels with their fractions so that it is clear for someone else to follow their thinking.
- Hint Cards (specifically for Tier 1 Question #2):

Hint Card
How many total scoops are needed for a single batch of pudding?

Hint Card
How many of the total scoops for a single batch are pudding mix?

Hint Card
What do you think of when you hear the word "double"? What does "double" mean?

Sample Responses

Hint Card
How many total scoops are needed for a single batch of pudding?
There are 4 total scoops needed for a single batch of pudding.

Hint Card

How many of the total scoops for a single batch are pudding mix?

3 of the 4 total scoops for a single batch are scoops of pudding mix (3/4).

Hint Card

What do you think of when you hear the word “double”? What does “double” mean?

When I hear the word “double”, I think of twice as many of something. The word “double” means two times as many. For example, if I had 3 toys and the number of toys doubled, I would have 6 toys.

I’m All Done, Now What Do I Do?
Have students select a Challenge Card below or write a new one. They also can create hint cards for a student who might have been absent or is struggling during this lesson.

Challenge Card

If Ellen Ochoa wants to make pudding for all the NASA workers who are helping plan the Mission to Mars, she will need a 458 times batch. What fraction of the ingredients needed are pudding mix? How many ways could you express this fraction?

Challenge Card

Ellen Ochoa wants to follow the recipe that she already has but she wants to create her own flavor of pudding by combining the mixes that she already has. Create a flavor combination that still follows the given recipe. Don’t forget to name your flavor!

Challenge Card

One of the other crewmates wants to help you prepare the pudding for the crew. Explain to them the pudding recipe using fractions, but … you cannot use the numbers 1, 3, or 4 in your explanation.

Sample Responses

Challenge Card

If Ellen Ochoa wants to make pudding for all the NASA workers who are helping plan the Mission to Mars, she will need a 458 times batch. What fraction of the ingredients needed are pudding mix? How many ways could you express this fraction?

The fraction of pudding mix that is needed is 3/4. The recipe says that 3 out of the 4 total scoops of ingredients are pudding mix and when you increase the batch size, the fraction is always equal to 3/4. You can represent 3/4 in an infinite number of ways by increasing the batch size forever.

Challenge Card

Ellen Ochoa wants to follow the recipe that she already has but she wants to create her own flavor of pudding by combining the mixes that she already has. Create a flavor combination that still follows the given recipe. Don't forget to name your flavor!

Students' answers will vary. My flavor combination is of all 3 flavors (chocolate, vanilla, and caramel). The recipe is 3 scoops of chocolate pudding mix, 6 scoops of vanilla pudding mix, 4 scoops of caramel pudding mix, and 13 scoops of water. I call this flavor: Constellation Combo Pudding.

Challenge Card

One of the other crewmates wants to help you prepare the pudding for the crew. Explain to them the pudding recipe using fractions, but … you cannot use the numbers 1, 3, or 4 in your explanation.

Students' answers may vary. To make the pudding recipe, you need pudding mix and water. For every 2 scoops of water, you need 6 scoops of pudding mix to make your pudding. This means that 2/8 of the ingredient scoops is water and 6/8 of the ingredient scoops is pudding mix.

Debrief and Look Ahead

4.

Debrief Content and Skills

Remind students that the mathematical practice for this lesson focused on how mathematicians solve problems and work together. Review some of the ideas that students brainstormed at the beginning of class and have students offer examples of how they acted like mathematicians while they worked together during the lesson.

Remind students that they learned new mathematical terms today and this lesson helped them learn the foundations of fractions. Ask the students to define numerator and denominator in their own words and create their own fraction with the numerator and denominator labeled.

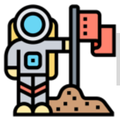
Sample Student Response: The numerator is the top number of the fraction that is above the fraction bar and represents the number of equal pieces from the whole that we are looking at. The denominator is the bottom number of the fraction that is below the fraction bar and represents the number of equal pieces in the total. The fraction I created is 5/6 and the numerator is 5 and the denominator is 6.

Debrief Thinking Like Mathematicians

Remind students that the mathematical practice for this lesson focused on how mathematicians solve problems. Students often need to be precise not just with their words, but also with their numbers. Ask students how precision was important within this lesson and how they will use precision in the world around them.

Sample Student Response: Precision was important because when the teacher measured the drink mix without being precise, the drink did not taste good! When the teacher was more careful and precise with the scoops, the drink tasted much better and the recipe was followed.

Whenever I make something in the kitchen at home, I will have to be precise when measuring so that I follow the recipe accurately and my food turns out right!



Assess

5.

What Students Learned

Formative Assessment

To determine tiered groups, there will be an entrance ticket that students will complete after the Tang launch activity. Based on students' responses to this ticket, they will be placed into the appropriate tier. This serves as a point of formative assessment for the lesson.

Summative Assessment

Students will submit their tiered lesson pages and responses to the teacher. This will serve as a form of summative assessment.

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Pudding Baker ______________________________ Date _________________

Peggy Whitson Tier 1 Lesson

Peggy Whitson, the astronaut, LOVES pudding and she is so excited that she can still have that dessert in space. There are three different flavors of pudding aboard the rocket ship to Mars: vanilla, chocolate, and caramel. The pudding recipe says that for one batch of pudding (any flavor), you need 3 scoops of pudding powder for every 1 scoop of water.

1. Peggy Whitson wants to make a batch of vanilla pudding.
   a. How many scoops of ingredients **in total** would she need? __________

   b. Of those scoops, how many of them are water? _________

   c. Use these numbers to create a fraction that shows how many scoops of water there are. Which number is the numerator? Why?

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   d. Use the square below or use manipulatives to represent your fraction in a visual way.

2. Your classmate, Kaelin, says that for a double batch of pudding, 7/8 of the total scoops are pudding mix.
   a. Do you agree with her?

   b. What is the numerator of Kaelin’s fraction? How do you know?

   c. What is the denominator of Kaelin’s fraction? How do you know?

3. Peggy Whitson wants to create a new flavor of pudding that is a combination of chocolate and caramel. She already measured 4 scoops of water for her recipe.
   a. How many scoops of pudding mix is needed for this batch? How did you get your answer?

   b. What does 1/2 mean? How could you show 1/2 in a picture?

   c. If Peggy Whitson wants the pudding mix to be 1/2 chocolate and 1/2 caramel, how many scoops of **chocolate** pudding mix would she need? Show your reasoning.

4. What does Peggy Whitson need to remember when scooping out her mix? Think back to when your teacher made the drinks for the class.

Pudding Baker ______________________________ Date _________________

Guion Bluford Tier 2 Lesson

Guion Bluford, the astronaut, LOVES pudding and he is so excited that he can still have that dessert in space. There are three different flavors of pudding aboard the rocket ship to Mars: vanilla, chocolate, and caramel. The pudding recipe says that for one batch of pudding (any flavor), you need 3 scoops of pudding mix for every 1 scoop of water.

1. Guion Bluford wants to make a new flavor of pudding that is a combination of vanilla and chocolate. He wants to make a triple batch of pudding.
   a. How many scoops of ingredients will he need **in total**? Show your reasoning.

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   b. How many of those scoops are pudding mix? Write your answer as a fraction.

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   c. Label the numerator and denominator of the fraction you wrote above.

2. Guion Bluford needs help coming up with his new flavor for his triple batch of pudding.
   a. How many scoops of vanilla mix and how many scoops of chocolate mix should Guion Bluford put into his triple batch of pudding?

   b. Your classmate has a different answer than you. Which one of you is correct?

3. What do the astronauts need to remember when scooping out their mix? Think back to when your teacher made the drinks for the class.

4. Guion Bluford's crewmate, Charles Hayes, will only eat vanilla pudding and he wants to make his own vanilla pudding instead of sharing with Guion Bluford. When he opened up the recipe book, it was stained and he could not read it! All that was clean on the page was an empty square in the corner.
    a. Use this square to help Charles Hayes and show him what fraction of the pudding is pudding mix (shade with your pencil) and what fraction is water (leave blank).
    b. How could you express your fraction in a different way?

Pudding Baker ______________________________ Date _________________

Ellen Ochoa Tier 3 Lesson

Ellen Ochoa, the astronaut, LOVES pudding and she is so excited that she can still have that dessert in space. There are three different flavors of pudding aboard the rocket ship to Mars: vanilla, chocolate, and caramel. The pudding recipe says that for one batch of pudding (any flavor), you need 3 scoops of pudding powder for every 1 scoop of water.

1. Ellen Ochoa wants to make one batch of vanilla pudding.
   a. What fraction of her pudding is pudding mix and what fraction of her pudding is water? What do the numerators and denominators of these fractions represent?

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2. Peggy Whitson asks if she can make chocolate pudding for herself. But she is very hungry and wants to make a double batch.
   a. What fraction of Peggy Whitson's pudding is pudding mix and what fraction of her pudding is water?

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b. Your classmate, Luis, says that these fractions are the same fractions you wrote in question #1. Do you agree with Luis? Why or why not? Draw a picture to help support your answer.

3. Ellen Ochoa is making pudding for her and all of her crewmates. She needs a big batch of pudding (3x the original recipe)!
   a. How many scoops of ingredients in total will she need?

   b. Uh oh! Ellen Ochoa went to gather her supplies and noticed that she doesn't have the regular scoop for making pudding. She has a scoop that is 2/3 of the regular scoop. How many scoops of ingredients in total will she need if she only has the 2/3 scoop? Explain your reasoning.

4. What does Ellen Ochoa need to remember when scooping out her mix? Think back to when your teacher made the drinks for the class.

Mission to Mars Student Pages with Answer Keys

Peggy Whitson Tier 1 Lesson

Peggy Whitson, the astronaut, LOVES pudding and she is so excited that she can still have that dessert in space. There are three different flavors of pudding aboard the rocket ship to Mars: vanilla, chocolate, and caramel. The pudding recipe says that for one batch of pudding (any flavor), you need 3 scoops of pudding powder for every 1 scoop of water.

1. Peggy Whitson wants to make a batch of vanilla pudding.
   a. How many scoops of ingredients **in total** would she need?

   4 scoops of ingredients

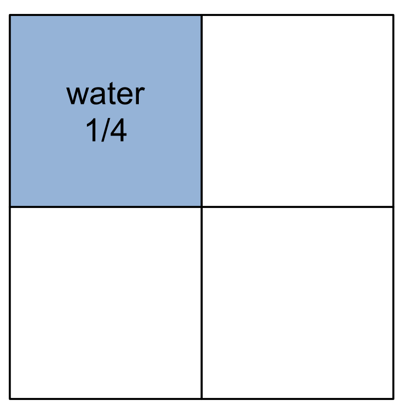
   b. Of those scoops, how many of them are water?

   1 scoop of water

   c. Use these numbers to create a fraction that shows how many scoops of water there are. Which number is the numerator? Why?

   1/4 is the fraction. 1 is the numerator because that is the number on top of the fraction and that represents the part of the total that is water.

   d. Use the square below or use manipulatives to represent your fraction in a visual way.



2. Your classmate, Kaelin, says that for a double batch of pudding, 7/8 of the total scoops are pudding mix.
   a. Do you agree with her?

   No, for a single batch of pudding, 3 of the scoops are pudding mix out of 4 total. So, for a double batch of pudding you would need 3 times 2 out of 4 times 2 and that is 6 scoops of pudding out of 8.

   b. What is the numerator of Kaelin's fraction? How do you know?

   The numerator of Kaelin's fraction is 7. I know this because it is the top number of the fraction and represents the number of scoops of pudding mix needed for a double batch.

   c. What is the denominator of Kaelin's fraction? How do you know?

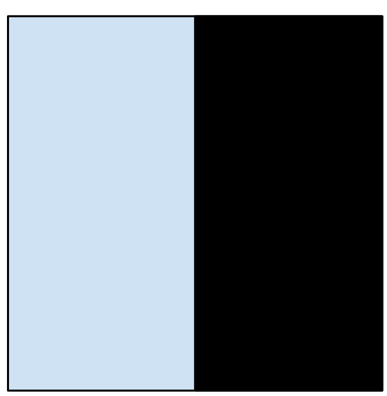
   The denominator of Kaelin's fraction is 8. I know this because it is the bottom number of the fraction and represents the total number of scoops of ingredients needed for a double batch.

3. Peggy Whitson wants to create a new flavor of pudding that is a combination of chocolate and caramel. She already measured 4 scoops of water for her recipe.
   a. How many scoops of pudding mix is needed for this batch? How did you get your answer?

   Peggy Whitson would need 12 scoops of pudding mix. The recipe says for every 1 scoop of water, you need 3 scoops of pudding mix. So, if Peggy Whitson has 4 scoops of water, you need 4 times 3 = 12 scoops of pudding mix.

   b. What does 1/2 mean? How could you show 1/2 in a picture?

   1/2 means one-half or something split into two equal parts and one of those chosen. This square is 1/2 blue and 1/2 black.



c. If Peggy Whitson wants the pudding mix to be 1/2 chocolate and 1/2 caramel, how many scoops of **chocolate** pudding mix would she need? Show your reasoning.

Peggy Whitson needs 6 scoops of chocolate pudding mix. There are 12 scoops of pudding mix and half of 12 is 6.

4. What does Peggy Whitson need to remember when scooping out her mix? Think back to when your teacher made the drinks for the class.

   Peggy Whitson needs to remember to be precise and make sure that her scoops of the pudding mix and water are level. My teacher did not measure their scoops precisely and that made the drink taste bad. Peggy Whitson should measure precisely to make the pudding taste good!

Guion Bluford Tier 2 Lesson

Guion Bluford, the astronaut, LOVES pudding and he is so excited that he can still have that dessert in space. There are three different flavors of pudding aboard the rocket ship to Mars: vanilla, chocolate, and caramel. The pudding recipe says that for one batch of pudding (any flavor), you need 3 scoops of pudding mix for every 1 scoop of water.

1. Guion Bluford wants to make a new flavor of pudding that is a combination of vanilla and chocolate. He wants to make a triple batch of pudding.
   a. How many scoops of ingredients will he need **in total**? Show your reasoning.

   Guion Bluford will need 12 scoops of ingredients in total. For one batch of pudding, you need 3+1=4 scoops of ingredients. For a triple batch, you need 4 times 3=12 scoops of ingredients total.

   b. How many of those scoops are pudding mix? Write your answer as a fraction.

   Of the 12 scoops of ingredients, 9 of them are scoops of pudding mix. There are 3 scoops of pudding mix for each batch and 3 batches, so 3 times 3= 9 scoops of pudding mix.
   9 < numerator

   —

   12 < denominator

   c. Label the numerator and denominator of the fraction you wrote above.

2. Guion Bluford needs help coming up with his new flavor for his triple batch of pudding.
   a. How many scoops of vanilla mix and how many scoops of chocolate mix should Guion Bluford put into his triple batch of pudding?

   Guion Bluford should put 6 scoops of chocolate and 3 scoops of vanilla. Other possible combinations: 8C/1V, 1C/8V, 7C/2V, 2C/7V, etc.

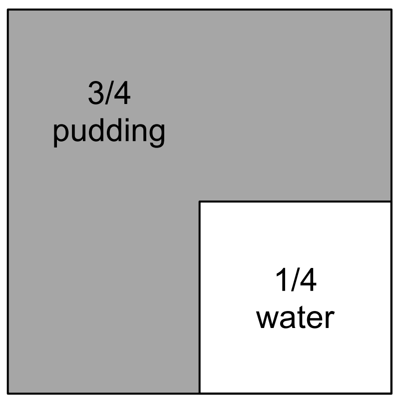
   b. Your classmate has a different answer than you. Which one of you is correct?

   There are multiple correct combinations of vanilla and chocolate scoops. So, as long as the number of scoops of pudding mix adds up to 9, we are both correct!

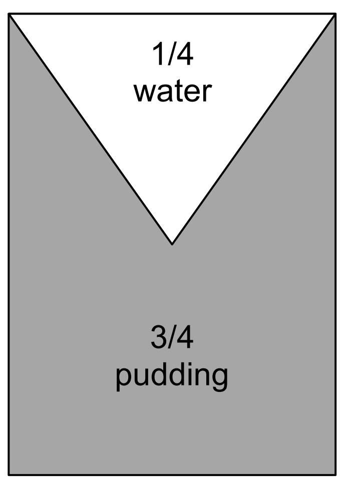
3. What do the astronauts need to remember when scooping out their mix? Think back to when your teacher made the drinks for the class.

   The astronauts need to remember to be precise and make sure that their scoops of the pudding mix and water are level. My teacher did not measure the scoops precisely and that made the drink taste bad. The astronauts should measure precisely to make the pudding taste good!

4. Guion Bluford's crewmate, Charles Hayes, will only eat vanilla pudding and he wants to make his own vanilla pudding instead of sharing with Guion Bluford. When he opened up the recipe book, it was stained and he could not read it! All that was clean on the page was an empty square in the corner.
   a. Use this square to help Charles Hayes and show him what fraction of the pudding is pudding mix (shade with your pencil) and what fraction is water (leave blank).



   b. How could you express your fraction in a different way?



Ellen Ochoa Tier 3 Lesson

Ellen Ochoa, the astronaut, LOVES pudding and she is so excited that she can still have that dessert in space. There are three different flavors of pudding aboard the rocket ship to Mars: vanilla, chocolate, and caramel. The pudding recipe says that for one batch of pudding (any flavor), you need 3 scoops of pudding powder for every 1 scoop of water.

1. Ellen Ochoa wants to make one batch of vanilla pudding.
   a. What fraction of her pudding is pudding mix and what fraction of her pudding is water? What do the numerators and denominators of these fractions represent?

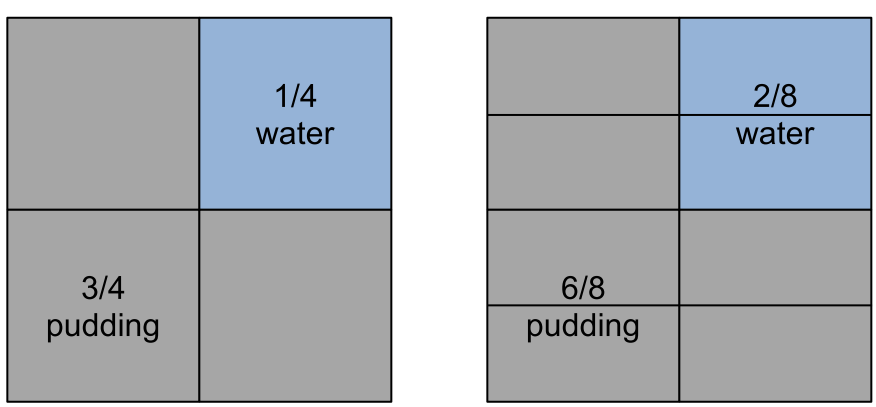
   3/4 of her pudding is pudding mix and 1/4 of her pudding is water. The numerators (3 and 1) are the numbers on top of the fraction bar, and they represent the number that represents the number of mix scoops out of the total number of scoops and the number of water scoops out of the total number of scoops. The denominator (4) is the number below the fraction bar, and this represents the total number of scoops.

2. Peggy Whitson asks if she can make chocolate pudding for herself. But she is very hungry and wants to make a double batch.
   a. What fraction of Peggy Whitson's pudding is pudding mix and what fraction of her pudding is water?

   6/8 of the pudding is pudding mix and 2/8 of the pudding is water.

   b. Your classmate, Luis, says that these fractions are the same fractions you wrote in question #1. Do you agree with Luis? Why or why not? Draw a picture to help support your answer.

   I agree with Luis because, as you can see in the picture below, 6/8 is the same as 3/4 and 2/8 is the same as 1/4.



3. Ellen Ochoa is making pudding for her and all of her crewmates. She needs a big batch of pudding (3x the original recipe)!
    a. How many scoops of ingredients in total will she need?

    Ellen Ochoa will need 4 times 3 = 12 total scoops of ingredients.

    b. Uh oh! Ellen Ochoa went to gather her supplies and noticed that she doesn't have the regular scoop for making pudding. She has a scoop that is 2/3 of the regular scoop. How many scoops of ingredients in total will she need if she only has the 2/3 scoop? Explain your reasoning.

    Ellen Ochoa will need 18 scoops of ingredients using the new scoop that is only 2/3 the size of the normal scoop. I found this by multiplying 12 by 2/3 and got 8. This means that Ellen Ochoa only had 8 full scoops from using 12 scoops. Then, I knew that she needed 4 more full scoops and 4 divided by 2/3 (the scoop size) is 6. Therefore, she needs 6 more scoops of ingredients and 12 plus 6 is 18 scoops total using the new scoop.

4. What does Ellen Ochoa need to remember when scooping out her mix? Think back to when your teacher made the drinks for the class.

    Ellen Ochoa needs to remember to be precise and make sure that her scoops of the pudding mix and water are level. My teacher did not measure the scoops precisely and that made the drink taste bad. Ellen Ochoa should measure precisely to make the pudding taste good!

Lesson Designer: Tutita Casa

Lesson 2: Using Fractions to Make Astronaut Mix—

LESSON 2

Big Ideas

A unit is a critical component of any number, such as realizing that the whole number 3 is made up of three individual units called “ones.” With respect to fractions, the unit can be thought of as the foundation of related fractions because it describes the number of equal parts of a whole. When the unit fraction *(1/b)* is repeated multiple times, it makes related fractional units, *a*/*b,* and eventually a whole (*b/b)*. For example, 1/4 can be repeated three times to obtain the fraction 3/4 and if you added 1/4 four times, you would get the whole (4/4). The denominator in the unit fraction describes the amount of equally partitioned parts in the whole, and the numerator, “1,” defines it as a unit; repeated unit fractions have any number *a* as the numerator. The structure of unit fractions is foundational and is important to be discussed in mathematically realistic scenarios. Unit fractions are the building blocks in everyday measures, such as a 2-cup liquid measuring cup that indicates one-third measures (e.g., 1/3, 2/3, 1 or 3/3, 1 1/3, 1 2/3, 2 or 6/3), and sharing (e.g., if you have 3/4 of a chocolate bar, then that is three of the unit fraction).

Lesson Objectives	• Students will be able to determine the meaning of the numerator, *a*, in the fraction *a*/*b* where *a*<*b* as it relates to the unit fraction 1/*b*. • Students will be able to define the term “unit fraction” and identify unit fractions with different denominators. • Students will be able to identify the structure within unit fractions and apply this knowledge to real world situations. • Students will be able to write a non-unit fraction as a combination of its unit fraction. **Common Core State Standard** Develop understanding of fractions as numbers. <u>CCSS.MATH.CONTENT.3.NF.A.1</u> Understand a fraction 1/*b* as the quantity formed by 1 part when a whole is partitioned into *b* equal parts;

	understand a fraction *a*/*b* as the quantity formed by *a* parts of size 1/*b*.

Materials	• tiered Lesson Student Pages—*Astronaut Mix*
	• student whiteboards (1 per student)
	• student dry-erase markers (1 per student)
	• student whiteboard erasers (1 per student)
	• paper plates (1 per student)
	• student markers (1 per student)
	• measuring cups (1 set per class)
	  ○ liquid 2-cup measuring cup (must have smaller increments labeled as well)
	  ○ 1/4 cup, 1/3 cup, 1/2 cup, 3/4 cup, and 1 cup dry measuring cups
	• Trail Mix (1 serving per student) *optional*
	  ○ mini chocolate chips, M&Ms, raisins, dried apricots, craisins, coconut flakes, peanuts (**if no classroom allergies**)

Mathematical Terms	• **Denominator:** bottom number in a fraction that identifies the number of pieces the whole has been divided into equal parts • **Fraction:** a number that represents part of a whole • **Numerator:** top number in a fraction that identifies the number of equal pieces considered as part of the whole • **Unit:** an individual, single component of a larger or more complex whole • **Unit Fraction:** a fraction that represents one equal part of a whole
Selected Mathematical Practices	• MP1: Make sense of problems and persevere in solving them. *I never give up on a problem and I do my best to get it right.* • MP4: Model with mathematics. *I see the math in everyday life, and I can use math to solve everyday problems.*

Differentiation

Content

Guiding Questions

- prior knowledge or learner readiness
  What evidence do you have about students' current knowledge and skills?
- tiered activities
  How will you design tiered activities on the same mathematical concept with varied levels of difficulty?
- formative assessment
  What techniques will you use to assess students' prior knowledge and skills?
- varied levels of challenge
  How will you vary the level of difficulty for each tiered activity?
- "teaching up" (aim high, provide scaffolding)
  How will you increase the depth, breadth, complexity, and abstractness of lessons to challenge and support student learning?
- real-world application
  What real-world connections will you make explicit about mathematical concepts and skills?

Process

Guiding Questions

- questioning strategies
  How will you pose and how will you encourage students to pose open-ended, closed-ended, lower-level, and higher-level questions to promote mathematical discourse?
- 4Cs (21st Century Skills)
  - critical thinking

  How will you promote a learning environment in which students question data and view issues or problems from multiple perspectives?
- 4Cs (21st Century Skills)
  - creative thinking

  How will you encourage students to "think outside the box" and synthesize information in new, different, and useful ways?
- 4Cs (21st Century Skills)
  - collaboration

  How will you encourage students to work with other students and appreciate their contributions to solving problems or making connections to prior work?

	• 4Cs (21st Century Skills) ○ communication *How will you promote students' opportunities to communicate face-to-face, in large and small groups, in online environments, and with print and non-print resources using their oral, written, and non-verbal skills?* **Product** ***Guiding Questions*** • oral, visual, and written opportunities *How will you encourage students to represent their thinking and problem solving using different communication strategies?* • summative assessment *How will you assess student learning upon completion of the lesson?* **Learning Environment** ***Guiding Questions*** • whole group/small group/individual instruction *How will you incorporate different grouping plans to address students' learning needs?* • growth mindset *How will you promote the perspective that it is important to view abilities as malleable?* • learning community *How will you support a positive learning community as students are encouraged to think, work, and communicate like mathematicians?*

Lesson Preview

In this lesson, students will learn about trail mix and how it is utilized in space travel. Once students have a grasp on the theme of the lesson, they will learn about what a unit fraction is from their teacher. The teacher will use diagrams and measuring cup manipulatives to discuss the concept of unit fractions with the group. Then, the class will learn how to write a non-unit fraction as a combination of its unit fraction. After these lesson sections, the students will complete an entrance ticket (as a type of formative assessment) and students will be separated into tiered groups based on their responses to the entrance tickets. Students will complete a worksheet about unit fractions and Astronaut trail mix at their skill level. These worksheets will serve as a form of summative assessment for the lesson and address the lesson objectives mentioned above.

Launch

1. Thinking Like Mathematicians: Mixing it Up With Astronaut Mix

Explain to the class that there is a lot of meal planning that needs to be done before astronauts go on their space travels. NASA anticipates that spacecrafts for their future missions will have less space inside them than they do now. Calorie-rich foods are necessary for the astronauts and these foods contribute to the total volume and mass on board. Therefore, it is critical that the foods they pack take these points into account (Cranford & Turner, 2021).

Tell students that NASA's Human Research Program further shares that, "Just as a healthy diet can help improve athletic performance, a healthy diet could help astronauts better adapt to the stresses of spaceflight" (Cranford & Turner, 2021, para. 1). Ask them to name some foods that they think are what scientists consider to be "healthy" ones. (Someone will likely mention fruits and vegetables.)

Share that dried fruits and nuts are two common ingredients in trail mix, which could be a snack that astronauts can eat to help them be as physically fit as possible. Ask students to share what they know about trail mixes, which they may have had at home, in the cafeteria, or at a school or community event. If the class is not familiar with trail mix, describe it to them and consider sharing photos or show them a bag. Have students create a list on the board of other ingredients that could be part of a trail mix recipe.

Share with the class that the astronauts need your help with creating and taste testing "astronaut mix" for them. Tell them that it is time for them to put on their aprons and mathematical thinking caps and get to work!

Explore

2. What Really Is a Unit Fraction?

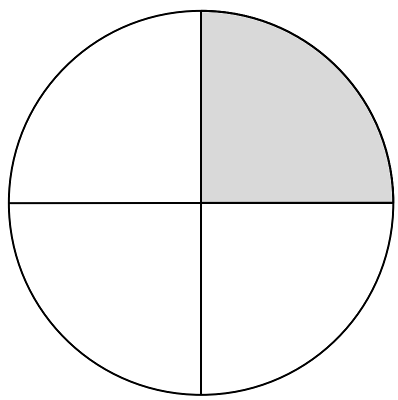
In this investigation, students will be working on one of the Student Pages based on their differentiated groups. The groups are based on teacher's observations of students' understanding. Before splitting the class into differentiated groups, complete this introductory activity together as a group.

Ask the students what they think of when they hear the word "unit." (*Sample responses may include: smaller pieces of something bigger, traveling as a "unit" or a group, units in terms of labeling measurements, etc.)*

Engage in this student-led conversation and if no one brings up units of measurements (specifically in baking) ask students if they ever use units in baking. *(Sample responses may include: I use different measuring cups when baking with my grandma! I have seen my dad measure with tablespoons when cooking!)*

Display the measuring cups for the class (both liquid and dry measuring cups) and ask students what they think about the different units of measurement on the desk.

Reveal the definition of “unit” to the class and ask students if they see a connection between “unit” and “fractions.” Have students complete a Think-Pair-Share where they will think about the connection, discuss their thoughts with a partner, and then share out with the class. *(Sample responses may include: There are fractions on the measuring cups and they are related to the units of measurement that you need for baking/cooking.)*



Highlight the 1/4 cup on the desk and display the image above and ask students how they are related. *(Sample response: They both represent the fraction 1/4.)*

After the fact that these are identical is established, write the fraction 1/4 on the board and describe each identical fourth within the circle being a unit. Label each section 1/4 in the diagram. Then, define the term unit fraction for the class and explain why 1/4 is a unit fraction. Emphasize the structure used when discussing unit fractions and its importance when working on new problems.

Have all students come up to the board and write an example of a unit fraction to solidify their understanding. If a student writes a fraction that is not a unit fraction (e.g., 3/4), use this as an opportunity to have another student categorize it as a non-example and explain why it is **not** a unit fraction. Then, explain to the students that the non-example can be represented as a combination of its unit fractions with the same denominator and write the fraction as such (e.g., 3/4=1/4+1/4+1/4). (If all students write unit fractions on the board, add a non-unit fraction to complete this example.)

Groups Formed by Student Readiness		
Tier 1: Peggy Whitson	**Tier 2: Guion Bluford**	**Tier 3: Ellen Ochoa**
Student Names	**Student Names**	**Student Names**

Collaborate and Communicate

Have students record their ideas on their individual worksheet. Help them clarify the ideas by asking questions like, "What do you mean here?" and "How might you share that idea with Jalene, so she knows exactly what you mean to say?" Point out that mathematicians use various representations to help explain their thoughts and use precise language to do so.

A. [Possible response] *This group . . .*	**B.** [Possible response] *This group . . .*	**C.** [Possible response] *This group . . .*

After the class finishes the Explore section, the students will complete the entrance ticket below and they will be placed into tiered groups based on their responses.

Entrance Ticket

1. Write a unit fraction and justify how you know it is an example of a unit fraction.

2. What is the unit fraction for the fraction 7/6? Can you write 7/6 as a combination of its unit fraction?

Possible Responses

Entrance Ticket

1. Write a unit fraction and justify how you know it is an example of a unit fraction.

   Student answers will vary. A unit fraction example is 1/2, 1/3, 1/4, 1/5, … I know that my fraction is an example of a unit fraction because the numerator is 1 and the denominator is a positive whole number.

2. What is the unit fraction for the fraction 7/6? Can you write 7/6 as a combination of its unit fraction?

   The unit for 7/6 is 1/6. 7/6=1/6+1/6+1/6+1/6+1/6+1/6+1/6

Examine and Elaborate

3. Highlight Students' Mathematical Thinking

Mathematicians think about possible solutions in a variety of ways. Therefore, it is important for students to realize that they, too, can approach problems using different strategies. Ultimately, students need to understand that a possible solution should be judged by the correctness of the mathematics, and there might even be some valid ideas within a solution when a student has an incorrect answer. This concept can be discussed with any section of the lesson (see the example within the Explore section).

Share and Discuss

It is, therefore, important for students first to clearly share their ideas with others so their validity can be determined by the class. In this sample dialogue, the teacher is discussing an Astronaut Mix recipe that is discussed within the tiered lesson.

Teacher: As you can see, for this recipe we need 3/8 cup of coconut flakes for the Astronaut Mix. What do you all notice about this fraction? I will give you time to think about this number *(Wait Time talk move)*

***Ashwani*:** I see that the numerator is 3 and that the denominator is 8 for your fraction.

Teacher: Great work, Ashwani! Thinking back to earlier in today's lesson, do we think that this fraction is a unit fraction?

***Kevin*:** I think that it is a unit fraction because it has a numerator and denominator!

***Maria*:** Kevin, I disagree with you!

***Kevin*:** Why? What did I say that was wrong?

***Maria*:** Just because a fraction has a numerator and denominator does not mean that it is a unit fraction. I think that 3/8 is not a unit fraction.

Teacher: Maria, why do you think that way? *(Reasoning talk move)*

***Maria*:** Well, for a fraction to be a unit fraction, it must represent one unit of the whole. So, the numerator of the fraction must be 1. In the fraction 3/8, the numerator is 3 and not 1.

Teacher: Great explanation, Maria! So, if we are looking at the fraction 3/8 what is the unit that the whole is being divided into equally?

***Ashwani*:** The fraction is 3/8 so the whole is being divided into 8 equal parts. Each piece represents 1/8 of the whole.

***Rachel*:** I agree with Ashwani. I drew a picture to help me picture the fraction (shows a picture of a square cut into 8 equal parts with each piece labeled 1/8 and 3 of them highlighted)!

Teacher: Great job, guys! I love the visual that you used to help you, Rachel. Class, how can we represent the fraction 3/8 by using its unit?

***Jacob*:** I think that you can take 3 of the units and you will get the fraction we need.

Teacher: Nice thinking! Could someone repeat what he said in your own words? (*Repeat/Rephrase talk move)*

***Maria*:** Jacob said that you can add 1/8 three times to get the fraction 3/8. You can see that in Rachel's picture because each piece is worth 1/8 and she highlighted 3 of them. So, we can use addition to add 1/8+1/8+1/8 and get 3/8!

Teacher: Great work, everyone! I appreciate that Maria was able to use Rachel and Jacob's work to help her explain her answer. You all did a great job talking about unit fractions within the recipe today!

Differentiate Further as Needed

If students are struggling with the material and need more scaffolding, present them an appropriate hint card and/or offer them manipulatives so that they can show their thinking in the best way possible for them. If students completed their tiered lesson and need more activities to engage them, you may present them with a challenge card to keep them thinking about the concept of unit fractions.

Hint Card
What do you know about drawing fractions in a diagram?   What does the word "equal" mean to you?   How can you make sure that your paper plate is separated into equal pieces?

Possible Responses

Hint Card

What do you know about drawing fractions in a diagram?

Fractions in a diagram have to start with a whole that is separated into equal pieces.

What does the word "equal" mean to you?

The word "equal" means that two items have the same amount. For example, it would be equal if I had 5 pennies and you had 1 nickel.

How can you make sure that your paper plate is separated into equal pieces?

I could use tracing paper to draw and compare the areas on the plate to make sure that they are equal. For this problem, I need 3 equal pieces.

Hint Card

For the fraction 3/4, what is the unit?

If you had to draw a diagram (using a shape) with that unit in mind, what would it look like?

How many of the units would you need to get 3/4 of the diagram?

What operation do we use to combine numbers?

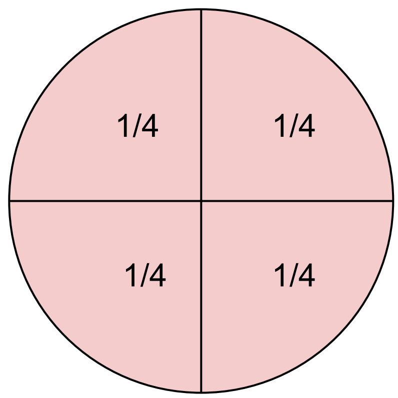
Possible Responses

Hint Card

For the fraction 3/4, what is the unit?

The unit is 1/4.

If you had to draw a diagram (using a shape) with that unit in mind, what would it look like?



How many of the units would you need to get 3/4 of the diagram?

I would need 3 of the sections of that diagram (that each resemble the unit 1/4).

What operation do we use to combine numbers?

To combine numbers, we use addition. So, 3/4=1/4+1/4+1/4.

Challenge Card

How many different units can the whole 1 be split into?

Challenge Card

If Chef Ronniel needs to measure 3/8 cup of dried apricots. But he doesn't have a 3/8 measuring cup. Recommend 3 different measuring cups that he could use instead of a 3/8 cup.

Challenge Card

Can whole numbers be classified as fractions? Why or why not?

Possible Responses

Challenge Card

How many different units can the whole 1 be split into?

The whole "1" can be split into an infinite amount of units. For example, you could split 1 into 12 equal pieces and the unit would be 1/12, or 1 into 2345 pieces and the unit would be 1/2345.

Challenge Card

If Chef Ronniel needs to measure 3/8 cup of dried apricots. But he doesn't have a 3/8 measuring cup. Recommend 3 different measuring cups that he could use instead of a 3/8 cup.

Ronniel could use 3 scoops of a 1/8 measuring cup
Ronniel could use 6 scoops of a 1/16 measuring cup
Ronniel could use 9 scoops of a 1/24 measuring cup

Challenge Card

Can whole numbers be classified as fractions? Why or why not?

Yes, whole numbers can be classified as fractions because the unit for a whole number is 1. If you think about it like 1 is the unit and every whole number is a multiple of 1, each whole number is a fraction. You can write every whole number with a denominator of 1 and it is equal to the whole number itself.

Debrief and Look Ahead

4.

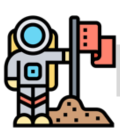
Debrief Content and Skills

Remind students that the mathematical practice for this lesson focused on how mathematicians solve problems and work together. Review some of the ideas that students brainstormed at the beginning of class and have students offer examples of how they acted like mathematicians while they worked together during the lesson.

Remind students of the learning objectives of the day and the Launch and Explore sections from earlier in class. To debrief the concept of unit fractions, ask the students: What is a unit fraction? How can we use unit fractions to create a non-unit fraction? *(Sample Response: A unit fraction is a fraction where the numerator is one. We can add unit fractions together to get a non-unit fraction. For example, 1/5 is a unit fraction and 3/5 is not a unit fraction, but we can do 1/5+1/5+1/5 to get 3/5.)*

Debrief Thinking Like Mathematicians

Remind students that the mathematical practices for this lesson focused on how mathematicians solve problems by looking for and making use of structure. They used the structure of fractions and specifically unit fractions to answer new problems. Ask the students: How did you use structure today within the lesson? Do you think structure is important? Why or why not? *(Sample Response: Today, I used structure because when I had to identify unit fractions, I knew the unit fraction structure and was able to answer the questions. I think that structure is important because it is constant and stable so that everyone who knows about fractions will understand the structure.)*



Assess

5.

What Students Learned

Formative Assessment

To determine tiered groups, there will be an entrance ticket that students will complete after the Launch and Explore group activities. Based on students' responses to this ticket, they will be placed into the appropriate tier. This serves as a point of formative assessment for the lesson.

Summative Assessment
Students will submit their tiered lesson pages and responses to the teacher. This will serve as a form of summative assessment.

Reference

Cranford, N., & Turner, J. L. (2021, May 4). *Enhanced diet may help astronauts adapt to spaceflight*. NASA Human Research Program Strategic Communications. https://www.nasa.gov/feature/enhanced-diet-may-help-astronauts-adapt-to-spaceflight

Mission to Mars Student Pages

NASA Chef ________________________________ Date _________________

Peggy Whitson Tier 1 Lesson

The astronauts and NASA kitchen staff need your help with the trail mix (also known as “Astronaut Mix”) they need for their upcoming mission to provide the astronauts nutrients in small quantities.

1. One of the NASA chefs, Xavier, is making a test batch of Astronaut Mix and needs 1/3 cup of dried apricots for his recipe.
   a. What is the numerator of 1/3?

____

   b. What is the denominator of 1/3?

____

   c. Using a paper plate and a marker (or another available manipulative), create a visual representation of the fraction 1/3. Make sure to label which piece of the plate you are referring to when you say 1/3.

   d. What is a unit fraction?

____

____

   e. Is 1/3 a unit fraction? Why or why not?

____

____

2. Chefs and bakers use liquid measuring cups and dry measuring cups when they make food. Take a look at the **liquid measuring cup** that your teacher has on their desk.
   a. What do you notice about the measurements that are marked on that cup?

____

____

____

____

____

   b. If you look at the markings on the cup, what do you notice about the spaces between the markings? What is the unit on the measuring cup?

____

____

____

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____

   c. Rewrite the fraction 3/4 as a combination of its unit fraction. Draw a picture if you need.

3. After a few test batches, the NASA chefs think that have a good recipe for Astronaut Mix:

Astronaut Mix
2 cups peanuts
1 cup M&Ms
1/2 cup mini chocolate chips
3/8 cup coconut flakes
1/3 cup dried apricots
1/2 cup raisins

a. Which of the numbers within the recipe are unit fractions?

___

b. Circle or highlight one of the numbers that is **not** a unit fraction. Write that number as a combination of its unit fraction.

___

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___

___

4. How did you know which numbers in the recipe are unit fractions? What part of their structure helped you answer question #3?

___

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___

Mission to Mars Student Pages

NASA Chef ________________________________ Date _________________

Guion Bluford Tier 2 Lesson

The astronauts and NASA kitchen staff need your help with the trail mix (also known as “Astronaut Mix”) they need for their upcoming mission to provide the astronauts nutrients in small quantities.

1. One of the NASA chefs, Donna, needs to measure 1/8 cup of raisins for her batch of Astronaut mix. Using a paper plate and a marker, create a visual representation of the fraction 1/8.
   a. Is this fraction a unit fraction?

____

   b. Why or why not?

____

____

   c. How would you define a unit fraction?

____

____

2. In another test batch of the Astronaut Mix, Donna used 1 2/3 cups of mini chocolate chips.
   a. Is the fraction 1 2/3 a unit fraction? Why or why not?

____

____

   b. If not, identify what the unit is and write the fraction as a combination of that unit fraction.

____

____

3. How do you know which numbers are unit fractions? What part of their structure helped you answer question #2a?

4. Here is the recipe that the chefs created for the Astronaut Mix:

Astronaut Mix
1 1/2 cups peanuts
1/4 cup mini chocolate chips
3/4 cup coconut flakes
1 cup M&Ms
1/2 cup raisins

a. Complete the table below. One of the astronauts completed the first row for you!

Ingredient	Recipe Amount	Amount Written as a Combination of Unit Fractions
Peanuts	1 1/2 cups	1 1/2=1/2+1/2+1/2
Mini Chocolate Chips		
Coconut Flakes		
M&Ms		
Raisins		

b. How much Astronaut Mix does this recipe make (in cups)?

c. **CHALLENGE:** If you were making this recipe in your kitchen, could you have used only one measuring cup? If so, which one would you use?

NASA Chef ______________________________ Date _______________

Ellen Ochoa Tier 3 Lesson

The astronauts and NASA kitchen staff need your help with the trail mix (also known as “Astronaut Mix”) they need for their upcoming mission to provide the astronauts nutrients in small quantities.

1. Two of the chefs are arguing in the test kitchen! They both were using the same recipe, but their batches look different. Chef Mark’s mix has many more mini chocolate chips than Chef Maria’s. Here is their recipe:

Astronaut Mix
1 1/2 cups peanuts
1/4 cup mini chocolate chips
3/4 cup coconut flakes
1 cup M&Ms
1/2 cup raisins

   a. Chef Mark said that he measured the raisins and knew that he had to double that amount to get the number of mini chocolate chips he needed. Chef Maria said that you need half the amount of mini chocolate chips as you do raisins. Do you agree with Mark or Maria? Explain why.

____

____

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____

2. One of the other NASA chefs, Jania, is working on a different test batch and needs 2 1/3 cups of peanuts for the recipe. Write this fraction as a combination of its unit fraction.

3. Chef Jania is making her Astronaut Mix recipe but cannot find a 1/3 cup. She has these cups available: 1/6 cup, 1/4 cup, 1/2 cup, and 1 cup.
   a. Can you find a combination of these measurements so that she can measure 2 1/3 cups of peanuts?

   b. Is there another way to do this using the available scoops? List at least 3 combinations.

c. Is there a scoop that you will have to use in every possible combination? If so, which one?

4. Chef Toby was asked to make Astronaut Mix for a large taste testing group. He needs to make a triple batch of Astronaut Mix. Use the recipe in #1 to find the new recipe that Chef Toby will need to use and record it below.

5. How do you classify a number as a unit fraction? What part of their structure helps you recognize them?

Mission to Mars Student Pages with Answer Keys

Peggy Whitson Tier 1 Lesson

The astronauts and NASA kitchen staff need your help with the trail mix (also known as “Astronaut Mix”) they need for their upcoming mission to provide the astronauts nutrients in small quantities.

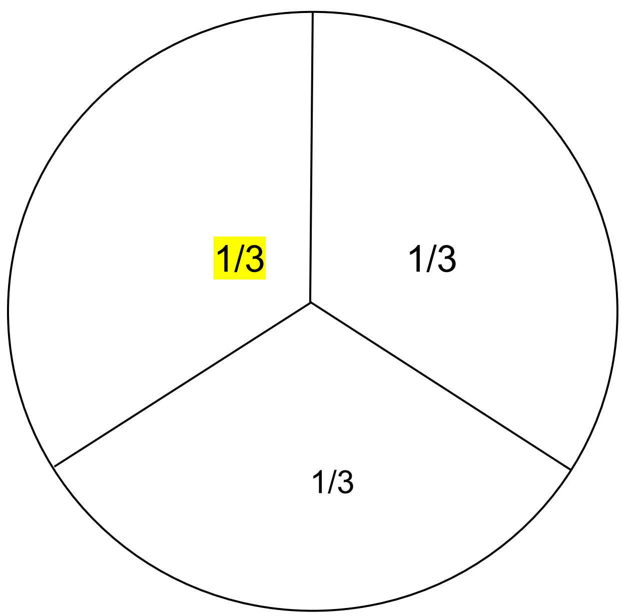
1. One of the NASA chefs, Xavier, is making a test batch of Astronaut Mix and needs 1/3 cup of dried apricots for his recipe.
   a. What is the numerator of 1/3?

   The numerator of 1/3 is 1.

   b. What is the denominator of 1/3?

   The denominator of 1/3 is 3.

   c. Using a paper plate and a marker (or another available manipulative), create a visual representation of the fraction 1/3. Make sure to label which piece of the plate you are referring to when you say 1/3.



   d. What is a unit fraction?

   A unit fraction is a fraction whose numerator is 1. It represents one unit of the whole.

e. Is 1/3 a unit fraction? Why or why not?

1/3 is a unit fraction because it represents one unit of the whole and has a numerator of 1.

2. Chefs and bakers use liquid measuring cups and dry measuring cups when they make food. Take a look at the **liquid measuring cup** that your teacher has on their desk.
   a. What do you notice about the measurements that are marked on that cup?

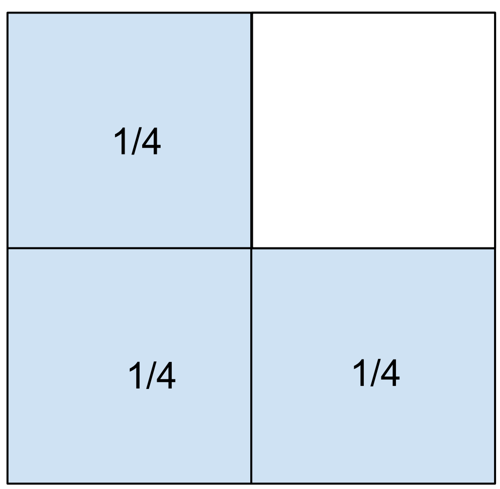
   I notice that the measuring cup has 1/4, 1/2, 3/4, 1, 1 1/4, 1 1/2, 1 3/4, and 2 markings on the measuring cup that represent cups.

   b. If you look at the markings on the cup, what do you notice about the spaces between the markings? What is the unit on the measuring cup?

   The spacing between the markings is the same. This makes sense because the markings represent the cup measurements that the chef is using. The unit here is 1/4 because that is the space between each marking.

   c. Rewrite the fraction 3/4 as a combination of its unit fraction. Draw a picture if you need.

   3/4=1/4+1/4+1/4



3. After a few test batches, the NASA chefs think that have a good recipe for Astronaut Mix:

Astronaut Mix
2 cups peanuts
1 cup M&Ms
1/2 cup mini chocolate chips
3/8 cup coconut flakes
1/3 cup dried apricots
1/2 cup raisins

   a. Which of the numbers within the recipe are unit fractions?

   1/2 (mini chocolate chips), 1/3 (dried apricots), and 1/2 (raisins)

   b. Circle or highlight one of the numbers that is **not** a unit fraction. Write that number as a combination of its unit fraction.

   Students' answers will vary.

   3/8=1/8+1/8+1/8

   1=1/4+1/4+1/4+1/4

   2=1/3+1/3+1/3+1/3+1/3+1/3

4. How did you know which numbers in the recipe are unit fractions? What part of their structure helped you answer question #3?

   I know that numbers are unit fractions if they have a numerator of 1 and this part of their structure helped me identify which numbers on the recipe card are unit fractions.

Guion Bluford Tier 2 Lesson

The astronauts and NASA kitchen staff need your help with the trail mix (also known as “Astronaut Mix”) they need for their upcoming mission to provide the astronauts nutrients in small quantities.

1. One of the NASA chefs, Donna, needs to measure 1/8 cup of raisins for her batch of Astronaut mix. Using a paper plate and a marker, create a visual representation of the fraction 1/8.
   a. Is this fraction a unit fraction?

   Yes

   b. Why or why not?

   1/8 is a unit fraction because the numerator is 1 and this fraction resembles 1 unit when a whole is split into 8 equal parts.

   c. How would you define a unit fraction?

   A unit fraction is a fraction whose numerator is 1. It represents one unit of the whole.

2. In another test batch of the Astronaut Mix, Donna used 1 2/3 cups of mini chocolate chips.
   a. Is the fraction 1 2/3 a unit fraction? Why or why not?

   No, because 1 2/3 does not represent 1 unit of a whole, it is larger than 1. Also, the numerator of the mixed fraction is not 1.

   b. If not, identify what the unit is and write the fraction as a combination of that unit fraction.

   You can rewrite 1 2/3 as 5/3 and the unit fraction is 1/3. So,
   1 2/3=1/3+1/3+1/3+1/3+1/3

3. How do you know which numbers are unit fractions? What part of their structure helped you answer question #2a?

   I know that numbers are unit fractions if they have a numerator of 1 and this part of their structure helped me identify which numbers on the recipe card are unit fractions.

4. Here is the recipe that the chefs created for the Astronaut Mix:

Astronaut Mix
1 1/2 cups peanuts 1/4 cup mini chocolate chips 3/4 cup coconut flakes 1 cup M&Ms 1/2 cup raisins

a. Complete the table below. One of the astronauts completed the first row for you!

Ingredient	Recipe Amount	Amount Written as a Combination of Unit Fractions
Peanuts	1 1/2 cups	1 1/2=1/2+1/2+1/2
Mini Chocolate Chips	*1/4 cup*	*1/4*
Coconut Flakes	*3/4 cup*	*3/4=1/4+1/4+1/4*
M&Ms	*1 cup*	*1*
Raisins	*1/2 cup*	*1/2*

b. How much Astronaut Mix does this recipe make (in cups)?

This recipe (in total) makes 4 cups of Astronaut Mix.
1 1/2+1/4+3/4+1+1/2=4

c. **CHALLENGE**: If you were making this recipe in your kitchen, could you have used only one measuring cup? If so, which one would you use?

Yes, you could have used only the 1/4 measuring cup. For all the fractions listed in the recipe, 1/4 can be the unit.

Ellen Ochoa Tier 3 Lesson

The astronauts and NASA kitchen staff need your help with the trail mix (also known as "Astronaut Mix") they need for their upcoming mission to provide the astronauts nutrients in small quantities.

1. Two of the chefs are arguing in the test kitchen! They both were using the same recipe, but their batches look different. Chef Mark's mix has many more mini chocolate chips than Chef Maria's. Here is their recipe:

Astronaut Mix
1 1/2 cups peanuts
1/4 cup mini chocolate chips
3/4 cup coconut flakes
1 cup M&Ms
1/2 cup raisins

   a. Chef Mark said that he measured the raisins and knew that he had to double that amount to get the number of mini chocolate chips he needed. Chef Maria said that you need half the amount of mini chocolate chips as you do raisins. Do you agree with Mark or Maria? Explain why.

   I agree with Maria because you need 1/4 cup of mini chocolate chips and 1/2 cup of raisins; 1/4 is smaller than 1/2 and 1/4 is half of 1/2. This can be shown by writing 1/2 as 2/4 and 2/4=1/4+1/4. So, Maria was right because you need half the amount of mini chocolate chips as you do raisins.

2. One of the other NASA chefs, Jania, is working on a different test batch and needs 2 1/3 cups of peanuts for the recipe. Write this fraction as a combination of its unit fraction.

   2 1/3=1/3+1/3+1/3+1/3+1/3+1/3+1/3

3. Chef Jania is making her Astronaut Mix recipe but cannot find a 1/3 cup. She has these cups available: 1/6 cup, 1/4 cup, 1/2 cup, and 1 cup.
   a. Can you find a combination of these measurements so that she can measure 2 1/3 cups of peanuts?

   Student answers may vary.

   Jania could use 2, 1 cup scoops and then 2 1/3 cup scoops.
   2 1/3=1+1+1/3+1/3

b. Is there another way to do this using the available scoops? List at least 3 combinations.

Students' answers may vary.

2 1/3=1/6+1/6+1/6+1/6+1/6+1/6+1/6+1/6+1/6+1/6+1/6+1/6+1/6+1/6

2 1/3=1/2+1/2+1/2+1/2+1/6+1/6

2 1/3=1/4+1/4+1/4+1/4+1/4+1/4+1/4+1/4+1/6+1/6

2 1/3=1+1/2+1/4+1/4+1/6+1/6

c. Is there a scoop that you will have to use in every possible combination? If so, which one?

Yes, you will always need the 1/6 measuring cup because 1/6 is the only cup available that can be a unit for 1/3.

4. Chef Toby was asked to make Astronaut Mix for a large taste testing group. He needs to make a triple batch of Astronaut Mix. Use the recipe in #1 to find the new recipe that Chef Toby will need to use and record it below.

3x Astronaut Mix
4 1/2 cups peanuts
3/4 cup mini chocolate chips
2 1/4 cups coconut flakes
3 cups M&Ms
1 1/2 cups raisins

5. How do you classify a number as a unit fraction? What part of their structure helps you recognize them?

   I know that numbers are unit fractions if they have a numerator of 1 and this part of their structure helped me identify which numbers on the recipe card are unit fractions.

Lesson Designer: Lisa DaVia Rubenstein

LESSON 3

Lesson 3: Defining Fractions—

Preparing for Take-Off: Designing the Perfect Flag Let's Meet in the Middle

Big Ideas

Numbers provide consistent methods of communication so everyone can understand the precise quantity being considered. Fractions are numbers that precisely describe a situation where a whole has been broken up into *equal* parts. The "denominator" tells you how many equal parts there are in the whole, and it is written on the bottom of the fraction. The "numerator" tells you how many of those equal parts you chose, and it is recorded at the top. Fractions provide a clear method to communicate the number of equal parts of a whole and how many of the parts you have, want, or need. Fractions are everywhere, used for providing precise measurements (e.g., 3/4 cup of flour, a quarter past 10:00) and helpful for fair sharing (e.g., a pizza that's 1/2 pepperoni and 1/2 cheese, splitting a restaurant bill equally among friends).

Lesson Objectives

- Students will recognize and communicate that a fraction represents when a whole unit is divided into equal parts.
- Students will define "denominator" as the bottom number in the fraction and recognize a denominator communicates how many total parts make up the whole.
- Students will define "numerator" as the top number in the fraction and recognize the numerator communicates how many parts of the whole are present.
- Students will construct a mathematical argument that includes definitional clarity, examples, non-examples, and concrete evidence.

Common Core State Standard
Develop understanding of fractions as numbers.

	CCSS.MATH.CONTENT.3.NF.A.1 Understand a fraction 1/*b* as the quantity formed by 1 part when a whole is partitioned into *b* equal parts; understand a fraction *a*/*b* as the quantity formed by *a* parts of size 1/*b*.
Materials	Students should have access to graph paper, rulers, scissors, scrap paper to support their own reasoning. Students may not choose to build their argument using these tools, but in general, mathematicians have various tools at their disposal to test their hypotheses.
Mathematical Terms	• **Denominator:** bottom number in a fraction that identifies the number of pieces the whole has been divided into equal parts • **Equal:** shows the same amount • **Fraction:** a number that represents part of a whole • **Half:** one of two *equal* parts of a whole • **Numerator:** top number in a fraction that identifies the number of equal pieces considered as part of the whole
Selected Mathematical Practices	• MP1: Make sense of problems and persevere in solving them. *I never give up on a problem and I do my best to get it right.* • MP2: Reason abstractly and quantitatively. *I can solve problems in more than one way.* • MP3: Construct viable arguments and critique the reasoning of others. *I can explain my math thinking and talk about it with others.* • MP5: Use appropriate tools strategically. *I know how to choose and use the right tools to solve a math problem.* • MP6: Attend to precision. *I can work carefully and check my work.*

Differentiation

Content
Guiding Questions

- prior knowledge or learner readiness
  What evidence do you have about students' current knowledge and skills?
- tiered activities
  How will you design tiered activities on the same mathematical concept with varied levels of difficulty?
- formative assessment
  What techniques will you use to assess students' prior knowledge and skills?
- varied levels of challenge
  How will you vary the level of difficulty for each tiered activity?
- "teaching up" (aim high, provide scaffolding)
  How will you increase the depth, breadth, complexity, and abstractness of lessons to challenge and support student learning?

Process
Guiding Questions

- questioning strategies
  How will you pose and how will you encourage students to pose open-ended, closed-ended, lower-level, and higher-level questions to promote mathematical discourse?
- 4Cs (21st Century Skills)
  - creative thinking

  How will you encourage students to "think outside the box" and synthesize information in new, different, and useful ways?
- 4Cs (21st Century Skills)
  - collaboration

  How will you encourage students to work with other students and appreciate their contributions to solving problems or making connections to prior work?
- 4Cs (21st Century Skills)
  - communication

  How will you promote students' opportunities to communicate face-to-face, in large and small groups, in online environments, and with print and non-print resources using their oral, written, and non-verbal skills?

	• hands-on activities/manipulatives *How will you incorporate activities promoting the use of manipulatives to clarify or illustrate mathematical concepts?* **Product** ***Guiding Questions*** • oral, visual, and written opportunities *How will you encourage students to represent their thinking and problem solving using different communication strategies?* • multiple ways to demonstrate knowledge, understanding, and skills *How will you encourage students to share their understanding of mathematical concepts and skills?* • multiple models and representations *What techniques of lesson design will you include to support students' deep understanding and the ability to apply mathematical concepts and skills?* • summative assessment *How will you assess student learning upon completion of the lesson?* **Learning Environment** ***Guiding Questions*** • flexible grouping *How will you use your tiered lesson to support flexible grouping?* • whole group/small group/individual instruction *How will you incorporate different grouping plans to address students' learning needs?*

Lesson Preview

The content focus of this lesson will be promoting student understanding that fractions require **equal** parts. This will be demonstrated by exploring the concept of one-half, using only 2 parts. Subsequent lessons will build on this concept by incorporating equivalent fractions and more complex designs to reinforce this concept. Students will be designing "fair flags" that perfectly represent two countries, equally. The process emphasis of this lesson will be constructing viable arguments.

Launch

1.

Thinking Like Mathematicians: Situating the Lesson Processes

Developing Viable Arguments

The mathematical practice emphasized in this lesson is the development of mathematically sound arguments that include definitions, examples/non-examples, and specific evidence to justify conclusions. Importantly, this evidence may be gathered using a variety of tools and approaches. The whole group should discuss how mathematicians support their answers.

Explain: Mathematicians often need to provide support for their answers. Imagine a teacher decided to cancel recess. You would want to know why, right? The same thing happens in math. Mathematicians propose an answer, but all the other mathematicians want to know why.

Ask: How could a mathematician explain their responses?

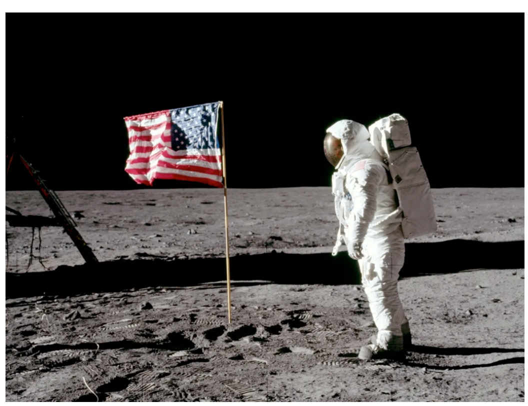
If students struggle to generate ideas, suggest mathematicians share the steps they used. They use precise mathematical language/definitions. They provide examples and non-examples, and they provide evidence using manipulatives or tools.

Situating the Lesson Content

Flying the Flag: Historical Background

Explain. In this lesson, we will be using our ability to construct a mathematical argument to help our astronauts design and choose their crew's flag. We will explore the creation of a flag for our astronauts to place on the surface of Mars, and first, we will discuss the background of planting flags in outer space.

When Apollo 11 landed on the moon, they planted a United States of America flag.



(Source: Platoff listed below in Reference)

Years before, President John F. Kennedy explained the meaning of the flag in space exploration:

> "We mean to lead [the exploration of space], for the eyes of the world now look into space, to the moon and to the planets beyond, and we have vowed that we shall not see it governed by a hostile flag of conquest, but by a banner of freedom and peace."
> -President John F. Kennedy, 1962

In 1967, the United Nations (U.N.) adopted the "Treaty on Principles Governing the Activities of States in the Exploration and Use of Outer Space, Including the Moon and Other Celestial Bodies." Quite a long name! Basically, this treaty stated that nations cannot claim ownership of planets or moons. The goal of space exploration is to learn, discover, and grow, so when the Apollo 11 mission placed the United States' flag on the moon, it symbolized national support and team pride in this historic achievement, not ownership of the moon. Now, many different countries have assembled teams of astronauts to explore outer space, and they continue to represent their home countries using their flags.

Reference

Platoff, A. M. (1993, August). Where no flag has gone before: Political and technical aspects of placing a flag on the Moon. *NASA Johnson Space Center.* https://historycollection.jsc.nasa.gov/JSCHistoryPortal/history/flag/flag.htm

Current Task

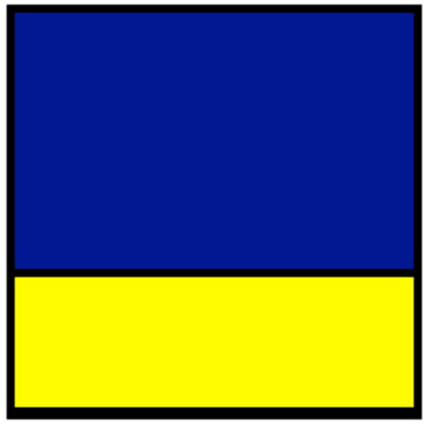
Explain: For today's mission to Mars task, we are going to consider potential flags our crew could plant on the surface of Mars when they land. Importantly, half the crew is from the United States, and half the crew is from China. While they could each bring their countries' flags, they wanted to design a single flag that demonstrated their partnership. The US astronauts chose blue from their flag, and the Chinese astronauts chose yellow from their flag. We are going to consider multiple ways we might design this flag to perfectly honor the partnership, the fair flag.

Explore

2. Designing a Fair Flag

Before splitting students into their differentiated groups, explore this introductory task together. Observe students as they consider this task; use your observations to place students in appropriate groups.

Ask: Would this be a fair flag?
(Alternative wording: Is this flag fair to both countries on the mission? Is it fairly divided in half? Does the blue space equal the yellow space?)



Encourage students to write down ideas in their journals and share with their neighbors. After students have this entry discussion, start to probe their thinking by asking for evidence.

Possible process questions include:

- How do you know if it is fair? (Promotes mathematical reasoning and proof-MP3.)
- What tools could you use to make your conclusion? (Encourages appropriate use of tools-MP5.)
- Is there another way you could determine whether the flag is fair? (Promotes fluency of thought, which is a component of creativity.)

To emphasize core content, ask students: Can we say this flag was *divided* in half? Connect this question to the core concept that a fraction must be composed of equal parts. One half communicates a perfectly fair flag, so this flag is not fair. It is not split in half. Continue to discuss one half is written as 1/2, so can we say the blue section is 1/2 of the flag? Why or why not?

Differentiated Examination of Additional Options
Break students into groups based on readiness levels. They will be receiving different flags to discuss and different levels of available scaffolds. Explain that given the first flag was not fair, each of their groups is going to be given a new option, and they will need to determine if it is fair.

In this investigation, students will be working on one of the Student Pages based on their differentiated groups. The groups are based on teacher's observations of students' conceptual understanding and mathematical practice acumen.

Groups Formed by Student Readiness		
Tier 1: Peggy Whitson	**Tier 2: Guion Bluford**	**Tier 3: Ellen Ochoa**
Student Names	**Student Names**	**Student Names**

Collaborate and Communicate

Have students record their ideas on their individual worksheets or one for the small group. Help them clarify their ideas by asking questions like, "What do you mean here?" and "How might you share that idea with the rest of the class?" Point out that mathematicians use various representations to help explain their thoughts and use precise language to do so.

A. [Possible response]	**B.** [Possible response]	**C.** [Possible response]
This group . . .	*This group . . .*	*This group . . .*

Examine and Elaborate

3.

Highlight Students' Mathematical Thinking

Mathematicians think about possible solutions in a variety of ways. Therefore, it is important for students to realize that they, too, can approach problems using different strategies. Ultimately, students need to understand that a possible solution should be judged by the correctness of the mathematics, and there might even be some valid ideas within a solution when a student has an incorrect answer.

Share and Discuss

After the groups have an opportunity to explore their assigned flags, bring the class back together for a full group discussion. In this discussion, it is important to stress that mathematicians explain their thoughts using mathematical definitions, examples/non-examples, and concrete evidence using tools. In this case, students may have used graph paper, folding, and/or cutting to demonstrate how the colors are taking up equal space on the flag, although some halves might not be congruent. After the groups share their approaches to their assigned flag, invite students to share how they created their own assigned flag. Tier 1 created an unequal flag, Tier 2 created an equal flag, and Tier 3 created many versions of an equal flag.

Throughout their responses, students should be incorporating the concept of equality into their discussion points, and again, students should demonstrate their ability to construct arguments using definitions, examples, and different representations. During this discussion, start to connect the language of fairness and equality to fractions and fraction notation. Fractions only describe when a whole is divided into EQUAL parts. As they share, connect back to fraction notation, demonstrating the blue sections are 1/2 of the flag.

Teacher:	Let’s start with the Whitson group. You had a table to help organize your thoughts on whether your flag was fair. Can someone describe this table? How did you complete it?
Monroe:	We used the table to help us decide if the astronauts would think the flag was fair.
Teacher:	What were the components?
Monroe:	It asked us to define and provide evidence.
Teacher:	What type of evidence? (Probing the mathematical practice.)
Monroe:	We used graph paper, scissors, and tiles.
Teacher:	All other groups, look at your first response. Did any of you also use these forms of evidence?
Simone:	We used graph paper!
Teacher:	Can someone add more details to Simone’s approach? *(Adding On talk move)*
Luna:	First, we traced the flag onto the graph paper, and then we counted how many squares were colored in with each color.
Teacher:	Someone from the Whitson group, can you repeat how Luna described her process? *(Repeat/Rephrase talk move)*
Joel:	They traced the flag onto the graph paper and counted.
Teacher:	So, is that how your group did it also?
Joel:	Sort of. Our flag did not always fit in perfect squares in the graph paper, so we needed to also count the half squares.
Teacher:	Interesting approach! Now when we want to describe the blue parts in this flag, because it is equally divided into two pieces, we can use a fraction. The blue fraction is 1/2 or one half. Can anyone describe why the fraction is 1/2?
Eva:	There are 2 equal parts and one is colored in blue.

Debrief and Look Ahead

4. **Debrief Content and Skills**

Remind students that the mathematical practice for this lesson focused on how mathematicians explain their responses using definitions, examples/non-examples, and different pieces of evidence. Further, mathematicians use a variety of tools and approaches to explain their thinking. In the next lesson, students will be expanding their conceptual understanding of halves when the flag is split into more than 2 pieces. They will start to write fractions with various denominators, including 4, 6, and 8. They may begin to connect 1/2 with 2/4, 3/6, and 4/8. Review all the ways students acted like mathematicians throughout this lesson.

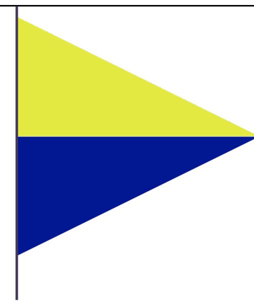
Assess

5. **What Students Learned**

Use the following exit card to assess what students learned from this lesson.

Exit Card

The astronauts are still considering their crew's flag. Astronaut Li tried this new design. Is it a fair flag? How do you know?



What fraction would describe the yellow part? ____

What fraction would describe the blue part? ____

Exit Card Answer Key

This is a fair flag. Students could respond to the prompt in a variety of ways, but here are the key concepts: (a) Fair means equal parts/portions (The definition must include equality.) (b) Examples of fairness may vary, but the response should include the concept of equally distributed. Everyone receives the same amount. Students may or may not describe how they may be able to provide evidence of the fairness, like graph paper, scissors, folding, using a ruler, tiles, or any other method that demonstrates equal area.

Yellow Part: 1/2

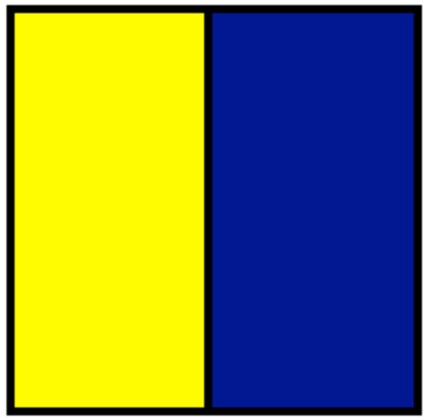
Blue Part: 1/2

Mission to Mars Student Pages

Flag Designer ______________________________ Date _________________

Peggy Whitson Tier 1 Lesson

The crew proposed the following flag:



1. Is this a fair flag? Why or why not? Explain to the crew how you know.

Use these prompts to guide your response.	Your Thoughts
Define: What does "fair" mean?	
Example: What would be an example of something that is fair?	

Use these prompts to guide your response.	Your Thoughts
Evidence: How could you use graph paper to demonstrate your thoughts?	
Evidence: How could you use scissors to demonstrate your thoughts?	
Evidence: Is there another way to show how you know?	

2. What would be an example of an **unfair** flag? Draw below.

3. How did you make your flag? How do you know your flag is NOT fair?

4. Can you provide additional evidence that it is an unfair flag?

Flag Designer ________________________________ Date __________________

Guion Bluford Tier 2 Lesson

The crew proposed the following flag:

1. Is this a fair flag? Why or why not? Explain to the crew how you know.

____

____

____

____

____

2. Joe doesn't believe you. How could you explain it using different evidence?

____

____

____

____

____

3. What would be an example of a flag that looks unfair at first but really is a fair flag that perfectly represents the two countries?

4. Explain how you know it is a fair flag.

Flag Designer ______________________________ Date _________________

Ellen Ochoa Tier 3 Lesson

The crew proposed the following flag:

1. Is this a fair flag? Explain to the crew how you know.

____

____

____

____

____

2. How might you demonstrate if the flag is fair in a different way?

____

____

____

____

____

3. The crow wondered how many other options exist for a fair flag. Using the squares below, how many different ways can you design a fair flag? (Hint: There are more than 4 ways to do this.)

(Add more squares on an extra piece of paper if needed.)

4. Explain your approach. How many options are possible? How can you prove that they are fair?

Mission to Mars Student Pages with Answer Keys

Throughout their responses, students need to incorporate the concept of equality. Fractions are only fractions when the portions are divided equally. Students should also demonstrate their ability to construct arguments using definitions, examples, and different representations. The final full class discussion will bring their experiences together to describe fractions only describe when a whole is divided into EQUAL parts.

Peggy Whitson Tier 1 Lesson

1. *This is a fair flag. Students could respond to the prompt in a variety of ways, but here are the key concepts: (a) Fair means equal parts/portions (The definition must include equality.) (b) Examples of fairness may vary, but the response should include the concept of equally distributed. Everyone receives the same amount. (c) Graph paper provides smaller equal boxes to demonstrate each side of the flag occupies the same number of boxes on the graph paper. (d) Scissors could be used to cut the square into blue and yellow pieces. Then the pieces could be placed on top of each other to demonstrate equal area. (e) Other ways to show fairness could be folding, using a ruler, tiles, or any other method that demonstrates equal area.*

2. *Students' responses may vary, but all responses should describe the division of the flag into unequal parts.*

3. *Students' responses may vary, but the explanation should describe that the flag was not divided into equal parts. They may demonstrate this inequality using graph paper, scissors, folding, tiles, or any other tool.*

4. *Students may use any of the additional methods they did not use in #3, such as graph paper, scissors, folding, tiles, or any other tool.*

Guion Bluford Tier 2 Lesson

1. *This is a fair flag. Students could respond to the prompt in a variety of ways, but here are the key concepts: (a) Fair means equal parts/portions (The definition must include equality.) (b) Examples of fairness may vary, but the response should include the concept of equally distributed. Everyone receives the same amount. (c) They should construct an argument with a definition and specific types of evidence, including but not limited to graph paper, scissors, folding, using a ruler, tiles, or any other method that demonstrates equal area.*

2. *Any additional piece of evidence previously unused: graph paper, scissors, folding, using a ruler, tiles, or any other method that demonstrates equal area.*

3. *Students' responses may vary, but all responses should include the division of the flag into equal parts. The added challenge of "looks unfair" encourages students to parse out the difference between "eye balling" equality of fractional parts and really empirically demonstrating that equality.*

4. *Students' responses may vary, but the explanation should describe that the flag was divided into equal parts. They may demonstrate this equality using graph paper, scissors, folding, tiles, or any other tool.*

Ellen Ochoa Tier 3 Lesson

1. *This is a fair flag even though it may not appear as a traditional division into halves. Students could respond to the prompt in a variety of ways, but here are the key concepts: (a) Fair means equal parts/portions (The definition must include equality.) (b) Examples of fairness may vary, but the response should include the concept of equally distributed. Everyone receives the same amount. (c) They should construct an argument with a definition and specific types of evidence, including but not limited to graph paper, scissors, folding, using a ruler, tiles, or any other method that demonstrates equal area.*

2. *Any additional piece of evidence previously unused: graph paper, scissors, folding, using a ruler, tiles, or any other method that demonstrates equal area.*

3. *Students' responses will vary. They simply need to all have equal portions of blue and yellow. Students may struggle after dividing horizontally, vertically, and both diagonals, so you may want to encourage them to look at how the first square in their worksheet task was divided.*

4. *Students' responses will vary, but the area occupied by blue and yellow should be equal. There are infinite options, which is why this is a challenging question. These options will be explored more in the next two lessons. "Proving" fairness can be done using the definition, examples, and representations. Again, the representations could be developed using any number of tools: graph paper, scissors, folding, using a ruler, tiles, or any other method that demonstrates equal area.*

Hint Cards* for Developing an Argument

Hint 1 Is the blue section 1/2 of the flag? How might you add a definition of "one-half" into your explanation?	**Hint 2** How could you use scissors to test your thoughts?
Hint 3 How might graph paper help demonstrate your thinking?	**Hint 4** How might folding the paper help to demonstrate your thoughts?

*These are already embedded within the Tier 1 page, but the other students may also find them useful. They do not necessarily need to be distributed in order.

Challenge Cards* for Fraction Understandings

Challenge 1 Draw five other examples of fair flags. Are there more options?	**Challenge 2** Astronaut Joe created a flag with 2 blue sections and 2 yellow sections. Could this be a fair flag? How? Could this also be an unfair flag? How?

*The first card is already embedded within the Tier 3 page, but other students may find it interesting after they finish their work.

Lesson Designer: Lisa DaVia Rubenstein

LESSON 4

Lesson 4: Patterns & Fractions—

Preparing for Take-Off: Designing the Perfect Flag Let's Get Creative!

Big Ideas

Being able to recognize patterns supports efficient problem solving by supporting the development and use of generalizations to make informed predictions. Mathematicians look for patterns to better understand and model different situations. Patterns happen in tile designs, in mathematical operations, in weather forecasting, among many other examples. In this lesson, students will explore the pattern of halving halves. This pattern illustrates when both halves are halved, the overall relationship remains the same. In the future, students may recognize this pattern in solving equations: if/when the same operation is done on both sides of the equal sign, the relationship remains intact.

Lesson Objectives	• Students will recognize and communicate that a fraction represents when a whole unit is divided equally. • Students will be able to recognize and design patterns using the halving pattern to ensure the overall relationship remains the same. **Common Core State Standards** Develop understanding of fractions as numbers. <u>CCSS.MATH.CONTENT.3.NF.A.1</u> Understand a fraction 1/*b* as the quantity formed by 1 part when a whole is partitioned into *b* equal parts; understand a fraction *a*/*b* as the quantity formed by *a* parts of size 1/*b*.

Materials	Students should have access to graph paper, rulers, scissors, scrap paper to support their own reasoning. Students may not choose to build their argument using these tools, but in general, mathematicians have various tools at their disposal that they can use to test their hypotheses.
Mathematical Terms	• **Denominator:** bottom number in a fraction that identifies the number of pieces the whole has been divided into equal parts • **Equal:** shows the same amount • **Fraction:** a number that represents part of a whole • **Half:** one of two *equal* parts of a whole • **Hypothesis:** an idea that might be true but needs to be tested • **Numerator:** top number in a fraction that identifies the number of equal pieces considered as part of the whole • **Pattern:** predictable, repeated way that something is done or presented
Selected Mathematical Practices	• MP1: Make sense of problems and persevere in solving them. *I never give up on a problem and I do my best to get it right.* • MP2: Reason abstractly and quantitatively. *I can solve problems in more than one way.* • MP3: Construct viable arguments and critique the reasoning of others. *I can explain my math thinking and talk about it with others.* • MP5: Use appropriate tools strategically. *I know how to choose and use the right tools to solve a math problem.* • MP7: Look for and make use of structure. *I can use what I know to solve new problems.*

Differentiation

Content

Guiding Questions

- learning objectives
  What do you want students to know, understand, and be able to do?
- prior knowledge or learner readiness
  What evidence do you have about students' current knowledge and skills?
- tiered activities
  How will you design tiered activities on the same mathematical concept with varied levels of difficulty?
- formative assessment
  What techniques will you use to assess students' prior knowledge and skills?
- varied levels of challenge
  How will you vary the level of difficulty for each tiered activity?
- "teaching up" (aim high, provide scaffolding)
  How will you increase the depth, breadth, complexity, and abstractness of lessons to challenge and support student learning?

Process

Guiding Questions

- questioning strategies
  How will you pose and how will you encourage students to pose open-ended, closed-ended, lower-level, and higher-level questions to promote mathematical discourse?
- 4Cs (21st Century Skills)
  - critical thinking

  How will you promote a learning environment in which students question data and view issues or problems from multiple perspectives?
- 4Cs (21st Century Skills)
  - creative thinking

  How will you encourage students to "think outside the box" and synthesize information in new, different, and useful ways?
- 4Cs (21st Century Skills)
  - collaboration

  How will you encourage students to work with other students and appreciate their contributions

	to solving problems or making connections to prior work? • 4Cs (21st Century Skills) o communication *How will you promote students' opportunities to communicate face-to-face, in large and small groups, in online environments, and with print and non-print resources using their oral, written, and non-verbal skills?* • hands-on activities/manipulatives *How will you incorporate activities promoting the use of manipulatives to clarify or illustrate mathematical concepts?* **Product** ***Guiding Questions*** • oral, visual, and written opportunities *How will you encourage students to represent their thinking and problem solving using different communication strategies?* • multiple ways to demonstrate knowledge, understanding, and skills *How will you encourage students to share their understanding of mathematical concepts and skills?* • multiple models and representations *What techniques of lesson design will you include to support students' deep understanding and the ability to apply mathematical concepts and skills?* • summative assessment *How will you assess student learning upon completion of the lesson?* **Learning Environment** ***Guiding Questions*** • flexible grouping *How will you use your tiered lesson to support flexible grouping?* • whole group/small group/individual instruction *How will you incorporate different grouping plans to address students' learning needs?*

Lesson Preview

In this lesson, the mathematical content knowledge is secondary to the mathematical practice of recognizing and using patterns. The specific pattern that will be explored is the pattern of halving in the context of a fair flag, equally divided between two colors; however, the patterns will become more complicated as students explore different methods to ensure the outcome remains the same.

Launch

1. Thinking Like Mathematicians: Situating the Lesson Processes

Look for and Make Use of Structure

The mathematical practice emphasized in this lesson is the recognition of patterns to predict what may come next in the series. These patterns are embedded within the fair flag context and increase content knowledge surrounding the concept of halves.

Ask: Where do you see patterns in your life? What is a pattern?

Explain: A few examples might be (a) the seasons (e.g., they can predict what follows spring), (b) their daily activities (e.g., they can predict when lunch will occur during the day), and (c) their family traditions (e.g., they can predict when they will get gifts throughout the year). In general, patterns are predictable. They follow a specific repeating rule. Mathematicians are always looking for patterns. Mathematicians use patterns to predict what might happen next or how certain numbers will function. For example, we know every time we multiply by 10, the number in the ones place will be 0. That is a pattern.

Ask: How do mathematicians find patterns?

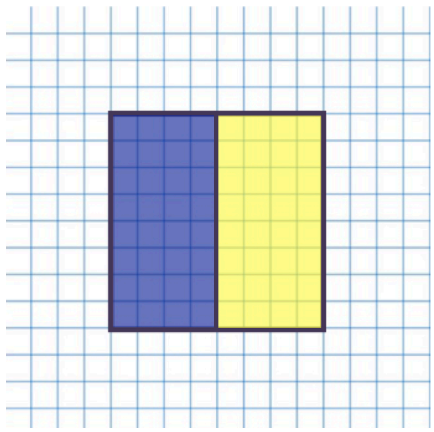
Support: If students struggle to generate ideas, suggest mathematicians analyze the features (like the order of numbers, operations, even/odd numbers) of an example and then they see how those features change or stay the same in the subsequent example and how those changes affect the outcomes. They might then form a hypothesis. A hypothesis is a guess at what might happen next. Mathematicians may make a hypothesis and then, examine another example to test their hypothesis. They may repeat this process many, many times.

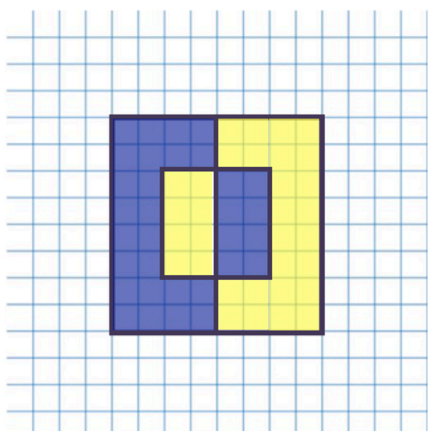
Situating the Lesson Content

Flying the Flag: Background

For today's mission to Mars task, we are going to continue to explore potential flags that a crew could plant on the surface of Mars when they land. As a reminder, half the crew is from the United States, and half the crew is from China. While they could each bring their countries' flags, they wanted to design a single flag that demonstrated their unified partnership. The US astronauts chose blue from their flag, and the Chinese astronauts chose yellow from their flag. We are going to consider multiple ways we might design this flag to perfectly honor the partnership, the fair flag.

Introduce the Pattern: Here are two additional flag designs. We have seen the first flag design in our previous lessons, but once again, our thoughtful astronauts decided to play with the design. Li proposed the second option as well. Take a few minutes and consider what you notice about these two options. Record your ideas in your notebook.



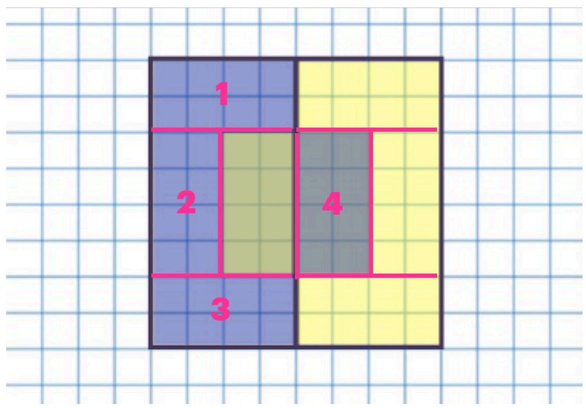


After providing a few minutes of independent thinking time, ask students to turn and talk to their partners about what they notice. This is a purposefully ambiguous question, providing low entry point for all students. There are many potential observations. After students have an opportunity to consider what is different or similar about these two options with a partner, bring students back together for a mini discussion.

This is the most challenging pattern, and the Ochoa group will explore it in more detail; however, at the most basic level, all students should recognize the same number of blue squares were changed to yellow (8) as the number of yellow squares changed to blue (8). Thus, it is still a fair flag.

Here is a potential series of questions:

- What was the most interesting thing your partner noticed? (This question prompts students to recognize the importance of listening to their partners' ideas.)
- What is the difference between the first and second option?
- Are both options fair? Could we use the fraction of 1/2 to communicate the portion of the flag that is yellow in both cases? Is there any other way you could *communicate* it? (Responses vary, but one alternative approach is drawn below. You can divide the square into 8 equal pieces, and the blue pieces are 4/8 of the total.)



Connect: Mathematicians often think about patterns. Patterns have a general rule that helps mathematicians "hypothesize" about what happens next. A hypothesis is an educated guess, something you think might be true but you still need to test it. Mathematicians make hypotheses all the time! Sometimes, they think they see a pattern, they create a rule to describe the pattern, and then, they realize they were wrong. That is still helpful information; they figured out one idea to rule out! Today you are going to be a pattern sleuth just like these mathematicians. Some of you will work on this pattern, and others of you will explore different patterns!

Before placing students in groups, provide a few instructions on patterns. Specifically, patterns are governed by a rule. You can determine rules by thinking about what is different between the first and second example. Then, you can hypothesize what might come next based on your rule.

Explore

2. Investigation: Designing a Fair Flag—Let's Get Creative!

In this investigation, students will be working on one of the Student Pages based on their differentiated groups. The groups are based on teacher's observations of students' understanding.

Groups Formed by Student Readiness		
Tier 1: Peggy Whitson	**Tier 2: Guion Bluford**	**Tier 3: Ellen Ochoa**
Student Names	**Student Names**	**Student Names**

The purpose of this task is to recognize and describe visual patterns. These patterns reinforce the concept of halves.

Collaborate and Communicate

Have students record their ideas on their individual worksheets or one for the small group. Help them clarify their ideas by asking questions like, "What do you mean here?" and "How might you share that idea with the rest of the class?" Point out that mathematicians use various representations to help explain their thoughts and use precise language to do so.

A. [Possible response] *This group . . .*	**B.** [Possible response] *This group . . .*	**C.** [Possible response] *This group . . .*

Examine and Elaborate

3.

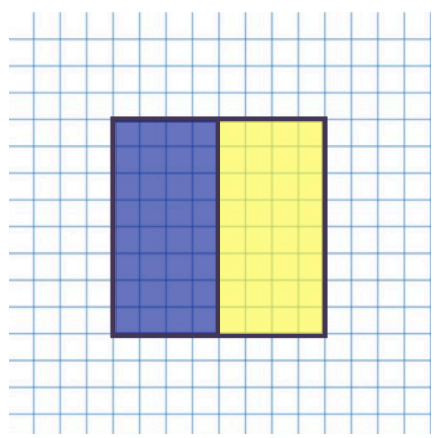
Highlight Students' Mathematical Thinking

Mathematicians think about possible patterns in a variety of ways. Therefore, it is important for students to realize that they, too, can approach take many approaches for identifying patterns, and further, many patterns may be present.

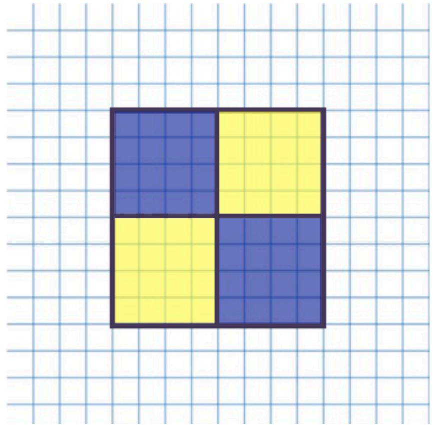
Share and Discuss

After the groups have an opportunity to explore these flag patterns, bring the class back together for a full group discussion. First, go back to Li's original pattern. Then, provide the opportunity for groups to share their groups' own original patterns and allow other students to hypothesize what will come next. As they share, continue to emphasize *patterns are governed by a consistent rule*, and *this rule allows mathematicians to predict what will happen next*. The following general pattern will emerge when starting with the first flag: whatever they do to the right half-they must do to the left half to maintain equality.

Teacher: Let's all gather and discuss the patterns your groups saw within your flags. Each group explored a slightly different pattern. Everyone started from this pattern:



Teacher: So, Whitson group had this as their second flag:



Teacher:	Everyone take a look and make a hypothesis of a possible rule to describe this pattern. If you were in the Whitson group, think about an additional possible rule to test. [Give students 2-3 minutes to think about their rule. (*Wait Time talk move)*] What rules do you see?
Avelyn:	Both sides are halved.
Teacher:	Anyone in the Whitson group want to add on to Avelyn's response? *(Adding On talk move)*
Ivy:	I agree with Avelyn, but also when they were halved using a horizontal line, and the top part stayed their original color, but the bottom halves switched to the other color.
Teacher:	Interesting details. Did any of the other groups see a similar pattern? [*Critical Thinking*: Analysis and application to a new situation.]
Robert:	Yes, well, sort of. In the Bluford group, ours was also halved, but ours was halved vertically instead of horizontally.

The teacher can repeat similar conversations with each group's patterns, and finally ask: What do all of these patterns have in common? Students should recognize all of them are still fair flags. They all can be represented by the fraction 1/2, which can be demonstrated by rearranging tiles.

As a challenge:

Teacher:	So, what if you said the rule was to half the original parts, how many different second flags could you get? What are the ones you can think of? [*Creative Thinking:* Fluency question.] [Give students 2-3 minutes to think about their options. (*Wait Time talk move)*]
Bingam:	I got three different ways, halving vertically, horizontally, and diagonally.
Teacher:	So, Bingam says three different ways. Can anyone agree or disagree with Bingam's response? *(Reasoning talk move)*

Ivy:	I agree Bingam described three different correct ways. But I also think there are more ways. You could half each individual square. You could half each square vertically, horizontally, and diagonally. I can think of so many different ways!

Debrief and Look Ahead

4. **Debrief Content and Skills**

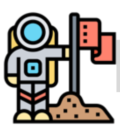
Remind students this lesson focused on mathematical patterns, specifically, how mathematicians use patterns to predict what might happen next. Ask students to consider where else they see patterns in their lives. A few examples might be (a) the seasons (e.g., they can predict what follows spring), (b) their daily activities (e.g., they can predict when lunch will occur during the day), and (c) their family traditions (e.g., they can predict when they will get gifts throughout the year).

Summary of the Flag Series

Say: We have examined a multitude of different flag options. Now is your opportunity to decide on which one the crew should use. Design your favorite perfect flag.

- What is a way you could create it that no one else would think of? (This question promotes originality of thought.)

The class may want to vote on which flag they will collectively use by hanging all the options the board and giving every student a sticky note to vote on their favorite. (Stipulate they can’t vote for themselves, if necessary.)



Assess

5. **What students learned**

The following exit card reviews all three lessons in the fair flag series of lessons. It serves as a mini-review of this series.

Exit Card

1. Define one half.

2. What does 1/2 communicate? Is this one half?

3. What does 3/6 communicate? Is this one half?

4. How can you ensure a flag is fair using any design?

5. What is an example of a pattern?

Exit Card Answers

1. *Students' responses may vary, but "one half" communicates when a whole is equally divided into two pieces. It is when a whole is equally shared among two individuals.*

2. *Yes, this is one half. 1/2 communicates one of two equal pieces. Students may also choose to draw a representation or describe this relationship in a variety of ways.*

3. *Yes, this is also one half. 3/6 communicates three of six equal pieces. Students may also choose to draw a representation or describe this relationship in a variety of ways.*

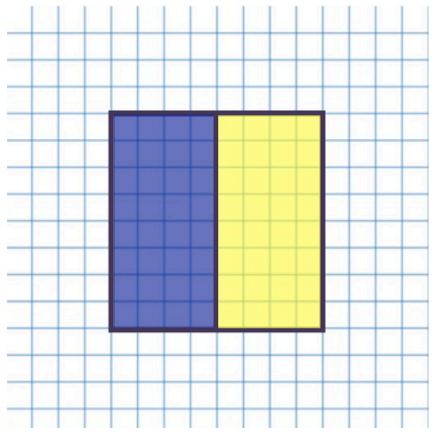
4. *A flag is fair if the two colors occupy equal areas. You can demonstrate this using graph paper, cutting or folding the flag, using tiles, or any other way to show the same amount of area is occupied by the two colors.*

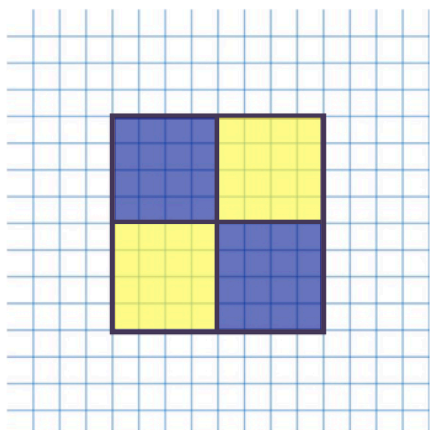
5. *A pattern is predictable. Often, there is a specific rule governing a pattern. Examples could include the seasons/holidays, halving of flags, the school day, among many others.*

Pattern Explorer ______________________________ Date __________________

Peggy Whitson Tier 1 Lesson

Examine the fair flag pattern:





1. Now, assuming there is a pattern, what might be the next flag? Draw your idea below. Explain how you designed your flag to follow your hypothesized pattern. (Include your pattern's rule.)

Explanation:

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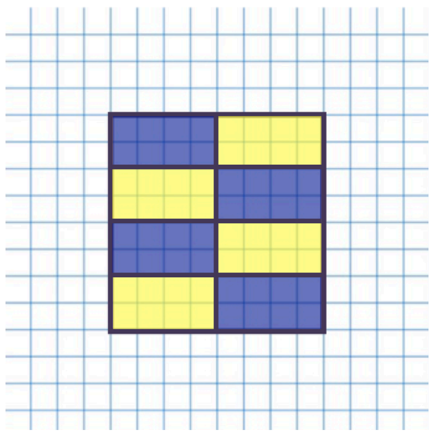
Li stated this was the third flag in her pattern.



2. Explain how her next flag built on the previous two. What pattern do you notice? Was this similar to your hypothesis?

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3. What fraction of the flag is yellow? ___________

4. Is this a fair flag? ___________

5. Again, assuming there is a pattern, what might the next flag look like? Draw your idea below. **Explain** how you designed your flag to follow your hypothesized pattern.

Explanation:

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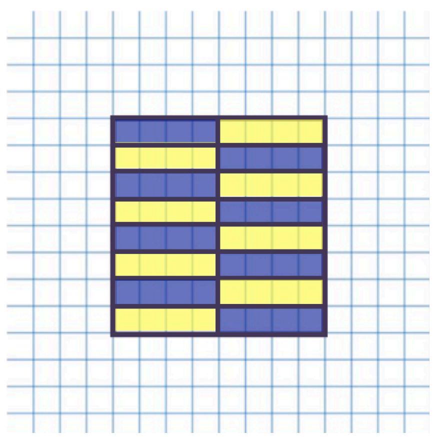
Li proposed this as the fourth flag in her pattern.



6. What fraction of the flag is yellow? ___________

7. Is this a fair flag? ___________

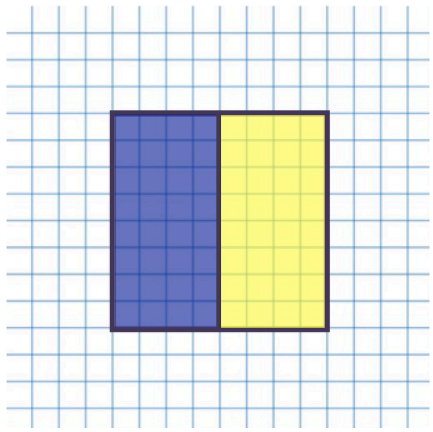
8. Explain how her next flag built on the previous two. What pattern do you notice now?

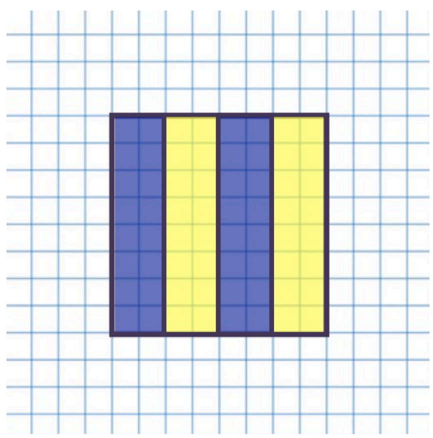
9. Is this the last possible flag in the pattern? Explain.

Pattern Explorer ______________________________ Date __________________

Guion Bluford Tier 2 Lesson

Examine the fair flag pattern:

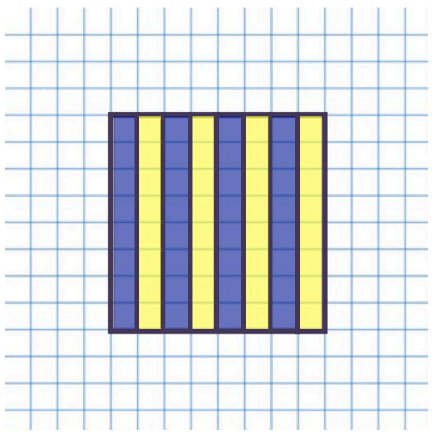




1. Now, assuming there is a pattern, what might be the next flag? Draw your idea below. Explain how you designed your flag to follow your hypothesized pattern.

Explanation:

Li stated this was the third flag in her pattern:

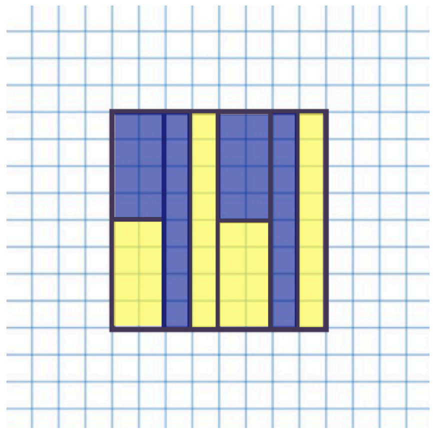


2. Explain how her next flag built on the previous two. What pattern do you notice? Was this similar to your hypothesis?

3. What fraction of the flag is yellow? ___________

4. Is this a fair flag? ___________

Joe looked at Li's third flag and disagreed. He argued that this should be the next flag in the series:



5. What fraction of the flag is yellow? ___________

6. Is this a fair flag? ___________

7. Explain what pattern Joe recognized.

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8. Who is correct: Joe or Li? Explain.

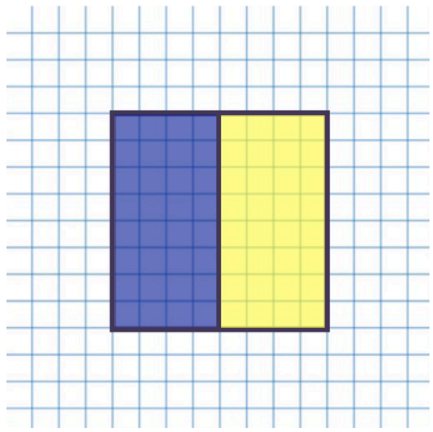
9. Is this the last possible flag in the pattern? Explain.

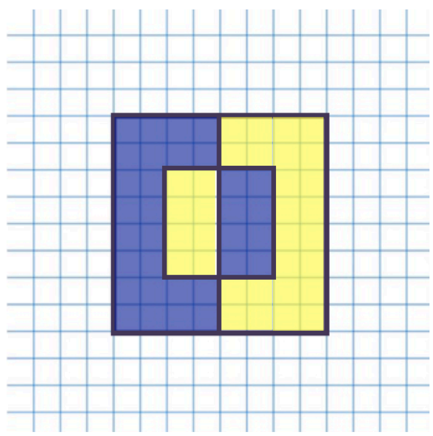
Mission to Mars Student Pages

Pattern Explorer ______________________________ Date __________________

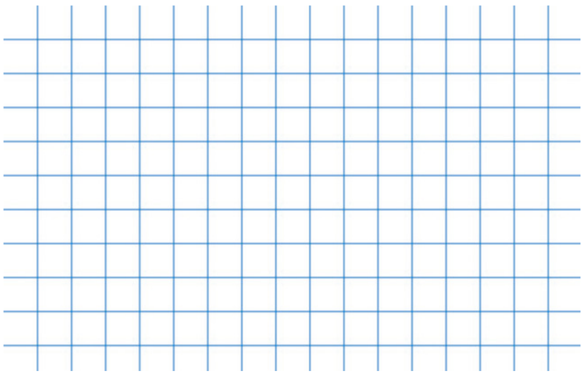
Ellen Ochoa Tier 3 Lesson

Examine this fair flag pattern:



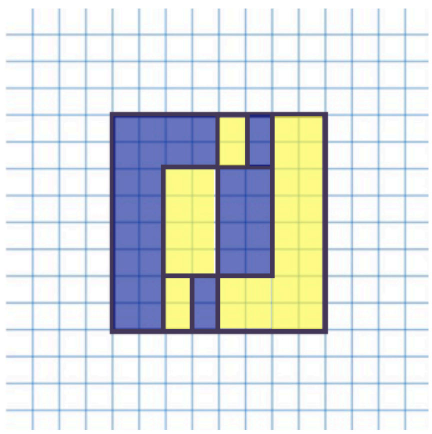


1. Now, assuming there is a pattern, what might be the next flag? Draw your idea below. Explain how you designed your flag to follow your hypothesized pattern.



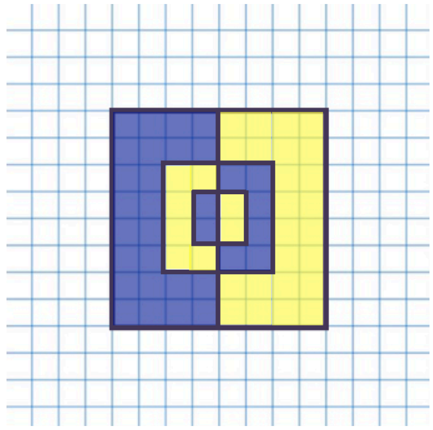
Explanation:

Joe suggested that this would be the third flag in the pattern:



2. Explain what pattern Joe recognized.

Li looked at Joe's hypothesis and told him she did not think he was correct. Li drew this as the third flag in her pattern:



3. Explain how her next flag built on the previous two. What pattern do you notice? How is this flag alike or different from your hypothesis and Joe's hypothesis?

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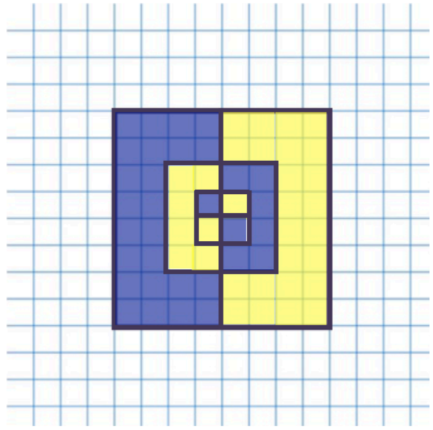
4. What fraction of the flag is yellow? ___________

5. Is this a fair flag? ___________

6. What would Li's fourth flag look like? Draw your idea below. **Explain** how you designed your flag to follow your hypothesis of Li's pattern.

Li proposed this as the fourth flag in her pattern.



7. What fraction of the flag is yellow? ___________

8. Is this a fair flag? ___________

9. Is this the last possible flag in the pattern? Explain.

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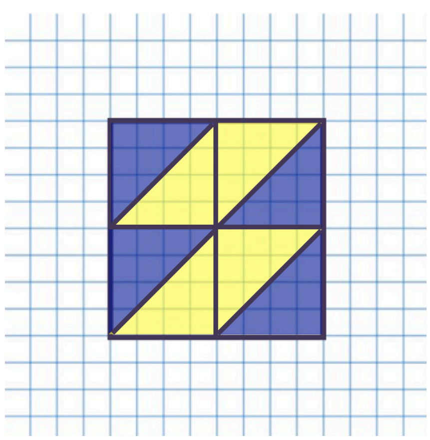
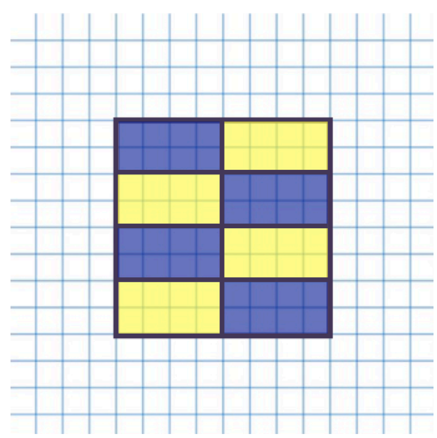
There are several differences among the tiers. The patterns themselves differ and the complexity of comparisons differ. However, students should demonstrate their ability to recognize and construct patterns throughout all tiered responses. Patterns are governed by a consistent rule. In this lesson, the following general pattern will emerge—whatever they change to one color, they must change the exact same amount to the other color to maintain fairness. Throughout their responses, students need to incorporate the concept of equality and equivalence. Fractions are only fractions when the portions are divided equally. If the total number of parts are divided perfectly in half, equivalent fractions can be used to describe the flag.

Establishing patterns requires more than 2 examples, so when the students are only provided the first two flags, there are multiple ways they could defend.

Peggy Whitson Tier 1 Lesson

1. *Students' responses will vary. There are many potential hypotheses. They may describe the pattern visually or they may use numbers. Establishing patterns requires more than 2 examples, so when the students are only provided the first two flags, there are multiple ways they could defend their responses. The essential idea is to recognize what changed between the first and second flag and then apply it to their own drawing. For example, the difference between Flag 1 and Flag 2 is that each part was split in half again, and within those parts, half were blue and half were yellow. If this were the specified rule, there are multiple next flag options. For example, both of these would be correct:*





2. *Li's rule is more specific; her rule is to horizontally divide each existing part in half, and then color the top portion of the first row using the original colors and then subsequently alternate colors. Students may recognize this rule was more specific than their own or perhaps, they used the same rule.*

3. *Students' responses may vary, but all will be equivalent to 1/2.*

4. *Yes, it is a fair flag.*

5. *The essential idea is that students recognize and describe a rule and follow that rule in their design. At this point, they will likely have Li's specific rule: horizontally divide each existing part in half, and then color the top portion of the first row using the original colors and then subsequently alternate colors. However, it is possible they have a less specific version of the rule still, and that is appropriate.*

6. *Students' responses may vary, but all will be equivalent to 1/2.*

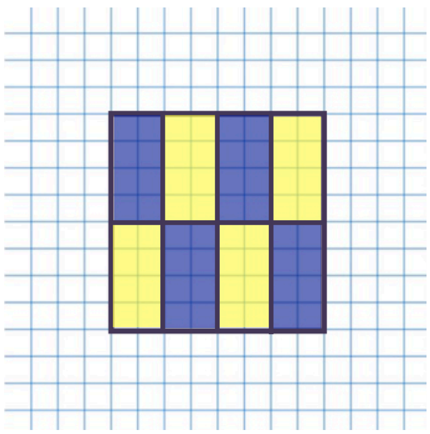
7. *Yes, it is a fair flag.*

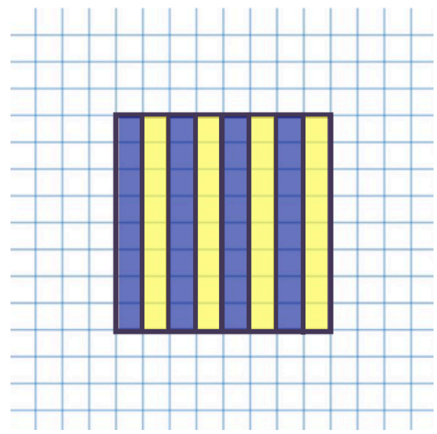
8. *Again, the essential idea is that students recognize and describe a rule. At this stage, the rule should be getting more nuanced to describe how the boxes will be colored and where.*

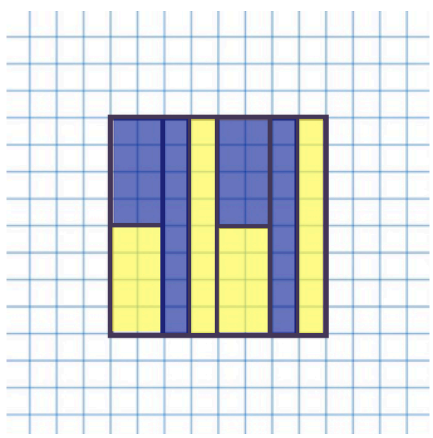
9. *This is the last flag in the series that can be made without dividing the graphing squares; however, the pattern can continue if the squares themselves are halved.*

Guion Bluford Tier 2 Lesson

1. *Students' responses will vary. There are many potential hypotheses. They may describe the pattern visually or they may use numbers. Establishing patterns often requires more than 2 examples, so when the students are only provided the first two flags, there are multiple ways they could defend at different levels of rule specificity. The essential idea is to recognize what changed between the first and second flag and then apply it to their own drawing. For example, students may describe a general difference between Flag 1 and Flag 2: each part was split in half again, and within those parts, half were blue and half were yellow. If this were the specified rule, there are multiple next flag options. For example, any of these would be correct using this generic rule:*







2. *Li's rule is to vertically divide each existing part in half, and then color the first column blue and then subsequently alternate colors. Students may recognize this rule was more specific than their own or perhaps, they used the same rule.*

3. *Students' responses may vary, but all will be equivalent to 1/2.*

4. *Yes, it is a fair flag.*

5. *Students' responses may vary, but all will be equivalent to 1/2.*

6. *Yes, it is a fair flag.*

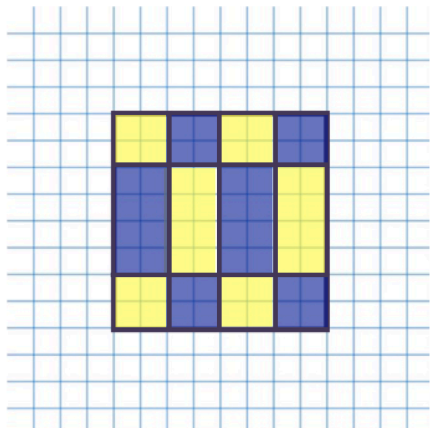
7. *Joe's response fits a more general pattern with flag 1 and 2: each part was split in half again, and within those parts, half were blue and half were yellow. However, Li's pattern more accurately accounts for more of the changes. Li's rule is to vertically divide each existing part in half, and then color the first column blue and then subsequently alternate colors.*

8. *Li's rule is more accurate. Li and Joe developed different rules to explain the relationships between Flag 1 and 2. Joe's response fits a more general pattern with Flag 1 and 2: each part was split in half again, and within those parts, half were blue and half were yellow. Joe's detailed rule is to horizontally divide the first part in half, vertically divide the second part in half, horizontally divide the third part in half, and vertically divide the fourth part in half. He also always started with blue, and alternated colors going down or across. However, Li's detailed rule more accurately accounts for more of the changes. Li's rule is to vertically divide each existing part in half, and then color the first column blue and then subsequently alternate colors.*

9. *Technically, this is the last flag in the series that can be made without dividing the graphing squares; however, the pattern can continue if the squares themselves are halved.*

Ellen Ochoa Tier 3 Lesson

1. *Students' responses will vary. There are many potential hypotheses. They may describe the pattern visually or they may use numbers. Establishing patterns often requires more than 2 examples, so when the students are only provided the first two flags, there are multiple ways they could defend at different levels of rule specificity. The essential idea is to recognize what changed between the first and second flag and then apply it to their own drawing. For example, students may describe a general difference between Flag 1 and Flag 2: 8 boxes were changed their color on each half of the flag. If this were the specified rule, the next flag option could be:*



2. *At the most specific level, Joe recognized that 32 original blue boxes were divided by four, and then 8 boxes were colored blue on the other side. Similarly, 32 yellow boxes were divided by 4, and 8 boxes on the other side were colored yellow. Then he took 8 and divided by 4 to get 2 and colored 2 boxes blue on the yellow side and 2 boxes yellow on the blue side. (There are other ways students could describe this more generally. This is a very specific explanation.)*

3. *Li's rule is to create a square that is half the length and half the height of the original. Then, color vertically one half of that square yellow and one half blue. Students may recognize this rule was more specific than their own or perhaps, they used the same rule. They should notice that Li and Joe simply applied different rules, and Li's more accurately accounts for the color and box placement.*

4. *Students' responses may vary, but all will be equivalent to 1/2.*

5. *Yes, it is a fair flag.*

6. *Students' responses should not vary if they follow Li's specific rule. Although, they may be able to make a case for their drawing using a more generic version of the rule.*

7. *Students' responses may vary, but all will be equivalent to 1/2.*

8. *Yes, it is a fair flag.*

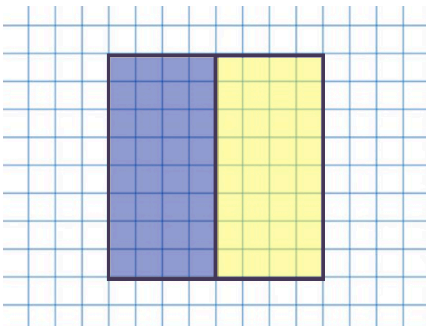
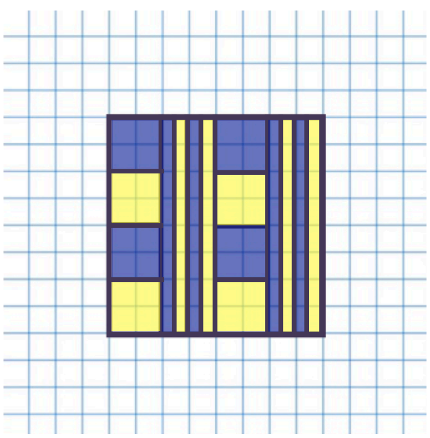
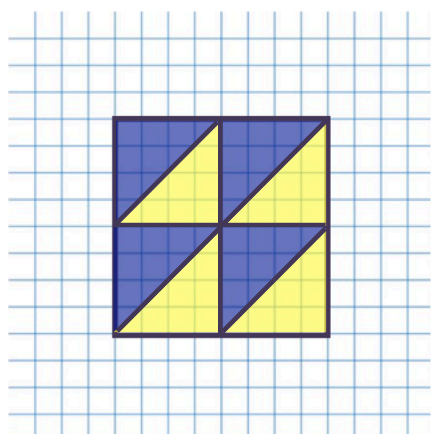
9. *Technically, this is the last flag in the series that can be made without dividing the graphing squares; however, the pattern can continue if the squares themselves are divided.*

Hint Cards* for Recognizing and Describing Patterns

Hint 1 What might a general rule be to explain the difference between the first and second flag?	**Hint 2** What is different between the first and second flag in the pattern?
Hint 3 How do the colors change between flags?	**Hint 4** How does the placement of colors change?

*All students may find these prompts helpful.

Challenge Cards

Challenge 1	Challenge 2
Challenge 1 Design your own flag pattern progression, start with the flag below as your first flag. Each flag must remain fair. The blue part of the flag should be 1/2 of the flag. The yellow part of the flag should also be 1/2 of the flag. 	**Challenge 2** Are some rules more specific than others? Can a rule describe only part of the changes? Are specific rules more helpful? Explain.
Challenge 3 This was the third flag in a series. What might the second flag have been? 	**Challenge 4** Can you create a flag pattern that uses this flag anywhere in the pattern? 

Lesson Designer: Rachael Cody

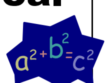
Lesson 5: Defining and Understanding Fractions—

The Grand Partition of Martian Rocks

Big Ideas

We have already worked to develop an understanding of the meanings of multiplication and division of whole numbers. We can extend this understanding by learning to recognize fractions as numbers that represent parts of a unit whole. Fractions show us the relationship between the parts out of a whole (numerator) and the whole itself (denominator). Fractions are parts of a unit whole, parts of a collection, divisions of whole numbers, and/or locations on number lines. So, we can write fractions as 1/*b,* where *b* represents the number of equal parts that have been used to divide the whole number. This means that 1 teacup in a collection of 10 teacups can be written equivalently as 1/10, where the number 10 is equivalent to the entire collection of teacups and 1 is equivalent to the single teacup within the collection. This can be applied to ANY fraction, not only even denominators such as ten. A whole could be divided into 9 parts (1/9) or 17 equal parts (1/17). It is important for us to understand fractions as equal partitions of whole numbers so that we can effectively communicate how we should use these parts. We need to be able to understand fractions as equal parts of the whole if we want to express our ideas using mathematical reasoning. In real life, if you were asked to complete a chore list that had 3 items on it with 2 other people, you would want to know that each item represents 1 equal part out of the whole. This would help make sure you were splitting the work fairly with everyone!

Lesson Objectives	• Students will recognize various examples of fractions as parts of a unit whole. • Students will understand a fraction 1/*b* as the quantity of 1 part when a whole (a single unit) is partitioned into *b* equal parts. • Students will understand a fraction *a*/*b* as the quantity formed by parts of size 1/*b*. • Students will reason abstractly and quantitatively when describing parts of whole numbers.

	Common Core State Standards Develop understanding of fractions as numbers. CCSS.MATH.CONTENT.3.NF.A.1 Understand a fraction 1/*b* as the quantity formed by 1 part when a whole is partitioned into *b* equal parts; understand a fraction *a*/*b* as the quantity formed by *a* parts of size 1/*b*.
Materials 	• colored pencil sets (one per student) • pencils (at least one per student) • dry-erase marker • whiteboard eraser • smartboard or similar projector with computer and audio/visual setup • Interactive Rover Game (https://spaceplace.nasa.gov/explore-mars/en/) • Mars Organic Materials Video (https://www.jpl.nasa.gov/videos/mars-in-a-minute-why-is-curiosity-looking-for-organics) • entrance ticket (one per student) • tiered student pages (with answer keys)
Mathematical Terms 	• **Denominator:** bottom number in a fraction that identifies the number of pieces the whole has been divided into equal parts • **Equal:** shows the same amount • **Equivalent:** equal in value • **Fraction:** a number that represents part of a whole • **Number Line:** a line with numbers placed in their correct position • **Numerator:** top number in a fraction that identifies the number of equal pieces considered as part of the whole • **Partitioned:** divided into parts • **Unit:** an individual, single component of a larger or more complex whole • **Whole:** the entire unit that represents one • **Whole Number:** the set of numbers that include zero and natural numbers

Selected Mathematical Practices	• MP1: Make sense of problems and persevere in solving them. *I never give up on a problem and I do my best to get it right.* • MP3: Construct viable arguments and critique the reasoning of others. *I can explain my math thinking and talk about it with others.* • MP4: Model with mathematics. *I see the math in everyday life, and I can use math to solve everyday problems.* • MP7: Look for and make use of structure. *I can use what I know to solve new problems.*
Differentiation	**Content** ***Guiding Questions*** • prior knowledge or learner readiness *What evidence do you have about students' current knowledge and skills?* • tiered activities *How will you design tiered activities on the same mathematical concept with varied levels of difficulty?* • formative assessment *What techniques will you use to assess students' prior knowledge and skills?* • varied levels of challenge *How will you vary the level of difficulty for each tiered activity?* • "teaching up" (aim high, provide scaffolding) *How will you increase the depth, breadth, complexity, and abstractness of lessons to challenge and support student learning?* • real-world application *What real-world connections will you make explicit about mathematical concepts and skills?* **Process** ***Guiding Questions*** • questioning strategies *How will you pose and how will you encourage students to pose open-ended, closed-ended, lower-level, and higher-level questions to promote mathematical discourse?*

	• 4Cs (21st Century Skills)
	  ○ critical thinking
	  How will you promote a learning environment in which students question data and view issues or problems from multiple perspectives?
	• 4Cs (21st Century Skills)
	  ○ creative thinking
	  How will you encourage students to “think outside the box” and synthesize information in new, different, and useful ways?
	• 4Cs (21st Century Skills)
	  ○ collaboration
	  How will you encourage students to work with other students and appreciate their contributions to solving problems or making connections to prior work?
	• 4Cs (21st Century Skills)
	  ○ communication
	  How will you promote students’ opportunities to communicate face-to-face, in large and small groups, in online environments, and with print and non-print resources using their oral, written, and non-verbal skills?
	• connections
	  How will you use “big ideas” to emphasize connections between and among mathematical concepts and skills and their connections to real-world situations?
	
	Product
	Guiding Questions
	• oral, visual, and written opportunities
	  How will you encourage students to represent their thinking and problem solving using different communication strategies?
	• multiple models and representations
	  What techniques of lesson design will you include to support students’ deep understanding and the ability to apply mathematical concepts and skills?
	• summative assessment
	  How will you assess student learning upon completion of the lesson?

	Learning Environment ***Guiding Questions*** • whole group/small group/individual instruction *How will you incorporate different grouping plans to address students' learning needs?* • growth mindset *How will you promote the perspective that it is important to view abilities as malleable?* • learning community *How will you support a positive learning community as students are encouraged to think, work, and communicate like mathematicians?*

Lesson Preview

In this lesson, students will learn about fractions as equal parts of the same whole and how to represent them as such. They will do this while engaging with information about the Mars Mission, the Curiosity Rover, and the search for organic materials in Martian rocks. As a class, students will develop a basic knowledge about fractions as numbers and learn about what role the Curiosity Rover serves in the Mars Mission, as well as why scientists at the National Aeronautics and Space Administration (NASA) are searching for organic materials in samples of Martian rocks. Then, the class will discuss how to represent one part of a whole number within a mini lesson. Once the discussion has concluded, students will complete a student entrance ticket. Their performance on this student entrance ticket will determine their assignment into a tiered group, at which point students will complete student pages appropriate for their assigned tier. The teacher will collect these assignments as a form of summative assessment for this lesson and will lead the class in a discussion about the lessons learned and the skills that the students were able to acquire and demonstrate throughout the lesson. This lesson will conclude with a brief discussion about how students thought like mathematicians within the lesson.

Launch

1. Thinking Like Mathematicians

As a whole group, discuss why mathematicians use mathematical language. Consider writing a list of the students' ideas on a whiteboard or chart paper. Ask the students:

- How do mathematicians use mathematical language?
- How might mathematical language help mathematicians communicate?

If students struggle to generate ideas, explain that mathematicians use mathematical language to be precise. Using mathematical language allows mathematicians to be clear and specific when they communicate their ideas. Mathematicians might use specific mathematical vocabulary (e.g., numerator, denominator) or provide details in mathematical terms (e.g., giving an answer with the units of measurement).

The Grand Partition Launch

Ask students what they know about robotic rovers and Mars. Explain to students that the first-ever robotic rover to land on Mars was in 1997 and was a part of the Mars Pathfinder Mission. Currently, Curiosity is one of the rovers exploring Mars, carrying the biggest and most advanced instruments for scientific studies ever sent to the Martian surface with the purpose of studying the history of the Martian climate through the chemistry and structure of its rocks and soil.

Scientists sent the Curiosity rover to Mars because they had a research question. Scientists wanted to answer the following question, “Did Mars ever have the right environmental conditions to support small life forms called microbes?” Tell students that Curiosity explores the Gale Crater, collecting rock, soil, and air samples for onboard analysis. Scientists are interested in finding organic material in these rocks. Students can hear about why scientists are interested in organic material in this video (https://www.jpl.nasa.gov/videos/mars-in-a-minute-why-is-curiosity-looking-for-organics). For more information about Gale Crater, follow this link (https://mars.nasa.gov/msl/timeline/prelaunch/gale-crater/).

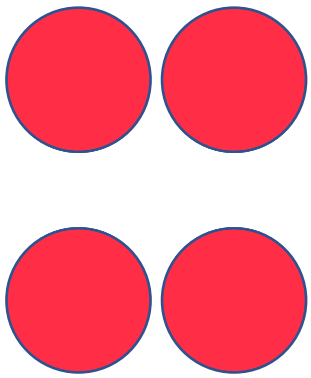
Tell students that Curiosity and its advanced kit of 10 science instruments, 17 cameras, drill, and laser (which it uses to vaporize and study spots of rocks at a distance) continues the hunt for Martian rocks that formed in water. For more information, teachers can visit this link (https://mars.nasa.gov/msl/mission/overview/). You should allow students to visit this link (https://spaceplace.nasa.gov/explore-mars/en/) for the opportunity to cruise the Martian surface as Curiosity and collect information about Martian rock samples. Note that the Curiosity interactive experience is a single game divided into eight “expeditions,” which is an example that you may use to describe fractions as partitions of a whole. Tell the students that today they are going to use their knowledge about fractions and numbers to describe Martian rock samples.

Explore

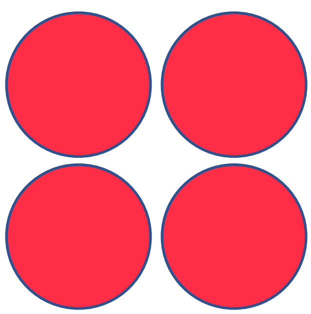
2. **The Grand Partition of Martian Rocks**

Begin by asking students to think about a time in their life when they have had to separate one thing into multiple, equal parts. Allow students to share their examples. Tell students that they have provided examples of times where they behaved like mathematicians! Let them know that in this lesson they will learn how to recognize fractions as equal parts of a whole, which will help them to communicate mathematical ideas like mathematicians.

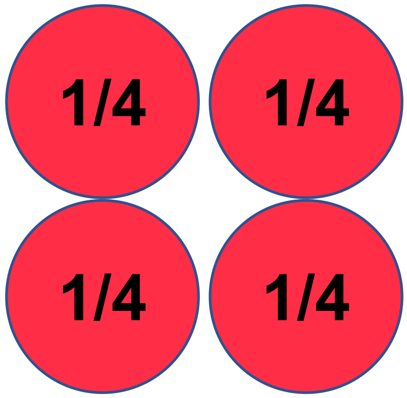
Continue to guide students through a review of fractions as a mathematical concept. Explain that fractions are representations of numbers. Remind students that fractions are numbers that mathematicians use to represent parts of a whole. Write this definition of a fraction (a number that represents parts of a whole) on the board. Share the following images with students (you may project onto a screen or draw these images on a whiteboard). Guide your students through a discussion about fractions using these images.



Ask students to count how many images they see. Wait for students to respond. Tell students that this number refers to the total number of symbols on the board. Explain that each circle represents one equal part of the whole number. Draw a box around the symbols to demonstrate that each of the four circles are part of the same group. Ask students to write down how they might represent each circle using fractions. Allow 1 minute for students to independently think and write before sharing.



After 1 minute has passed, allow students to share their answers. Once they have shared, write 1/4 next to each circle symbol. Explain that each symbol is one part of the four parts that make up the whole group.



= 4/4

Point to each of the four symbols and ask students to give the mathematical term for the numbers written in the middle of the symbol. At this point, at least one student should be able to say that the correct term is “fraction.” Ask if the number 4/4=1 is also a fraction. Students may disagree. If they do, you can say that the number 4/4=1 is a fraction because it is referring to all four parts in the whole group.

Explain to students that the denominator of all of the fractions within the group should be four. Tell them that the denominator identifies the number of equal parts that divide the whole number. Explain that the top number of the fraction identifies the number of equal pieces that are considered as part of the whole. This is called the numerator. Explain that, in the fraction 1/4, 1 would be the numerator and 4 would be the denominator.

Ask a student to come up and write a different number. Then, have the student draw circles that are equivalent in number to the number written down. Then, have the student draw a box around these circles. Ask students if they can write a fraction that represents one of these circles. Remind students that, although the number of total parts is different from before, the same rules apply.

Recall the Curiosity Rover activity that students explored at the beginning of this lesson. In this activity, students directed the rover using a variety of commands. Students could use up to 6 commands during each “expedition.” Students could complete up to 8 expeditions in the interactive experience.

Tell students that each command they used represents 1 out of 6 (or 1/6) possible commands for the total expedition. If students used 4 out of 6 possible commands, they would be using 4/6 of the possible commands in the expedition. Similarly, each expedition made up 1/8 of the total available activities in the entire activity.

Explain that each student may have completed the activity in a different way and that fractions can help them communicate how they completed the activity. Ask students to share an example of how they used the commands in a single expedition. Ask them to share the numerator and denominator in their examples.

(***Sample Responses:*** *5 commands = 5/6, where 5 is the number of commands and 6 is the number of available commands*)

Explain to students that if they used all the commands (6/6, or 1), they would have used all available commands. This would be all six parts of the same whole, where the "whole" refers to a single expedition.

After this class discussion, share entrance tickets to be completed independently. The teacher will use their observations of students' understanding, based on student entrance tickets and student responses during class discussion, to separate the students into 3 differentiated groups. In the following investigation, students will be working on one of the Student Pages based on their differentiated groups.

Groups Formed by Student Readiness		
Tier 1: Peggy Whitson	**Tier 2: Guion Bluford**	**Tier 3: Ellen Ochoa**
Student Names	**Student Names**	**Student Names**

Collaborate and Communicate
Have students record their ideas on their individual worksheets or one for the small group. Help them clarify their ideas by asking questions like, “What do you mean here?” and “How might you share that idea with the rest of the class?” Point out that mathematicians use various representations to help explain their thoughts and use precise language to do so.

A. [Possible response]	**B.** [Possible response]	**C.** [Possible response]
This group . . .	*This group . . .*	*This group . . .*

Entrance Ticket
National Aeronautics and Space Administration (NASA) scientists have outlined four goals for the Mars Exploration Program. Information about these goals can be found through this link (https://mars.nasa.gov/msl/mission/science/goals/). Tell students that the first goal, and the focus of the following entrance ticket, is to determine whether life ever existed on Mars. Explain that the Mars Science laboratory looks for six chemical elements that compose the building blocks of life on Earth. These include carbon, hydrogen, nitrogen, oxygen, phosphorus, and sulfur.

Tell students that the following entrance tickets will further their understanding of fractions as parts of a whole and their understanding of why Curiosity is exploring Mars.

There are four questions in the following entrance ticket. Ask students to choose only *one* of the following questions to answer (**A**, **B**, or **C**). These questions represent varying degrees of challenge: **A** is the most challenging (corresponds to Tier 3), **B** is moderately challenging (corresponds to Tier 2), and **C** is the least challenging (corresponds to Tier 1). Assign students to Tier 1, 2, or 3 based on the questions that they have attempted to solve. If you notice a mismatch between student responses and the problems they are attempting to solve, encourage students to attempt a different problem. If a student answers more than one question, assign the student to the tier that corresponds with the most challenging question.

Entrance Ticket

Choose ONE of the following questions (**A**, **B**, or **C**) to answer:

A) Scientists are analyzing 50 samples from Martian rocks to find the chemical elements necessary for life on earth. They find that 5 of these samples contain at least one of the desired chemical elements. Take one of these 5 samples and represent it using at least two different fractions.

Answer 1) __________________ Answer 2) ___________________

Explain why you chose these fractions

____

____

B) Scientists in two different labs are analyzing 8 samples each from Martian rocks to find the chemical elements necessary for life on earth. Scientists from the first lab only found hydrogen in 3 of their 8 samples. Scientists from the second lab did not find hydrogen in any of their samples but found carbon in 2 of their 8 samples.

How many samples were used in both labs combined? ______________

How many samples in total contained hydrogen? ______________

How many samples in total contained carbon? ______________

How many samples in total contained one of the chemical elements necessary for life on earth? ______________

C) The Curiosity rover delivered a sample from a Martian rock to scientists, who separated the sample into six equal parts. They found that each of the parts had exactly one of the following chemical elements that are necessary for life on earth: carbon, hydrogen, nitrogen, oxygen, phosphorus, and sulfur.

How many equal parts were in the whole sample? ______________

Use fractions to depict how much of each element was in the rock sample:

Carbon: ______________ Hydrogen: ______________

Nitrogen: ______________ Oxygen: ______________

Phosphorus: ______________ Sulfur: ______________

Entrance Ticket Key

Choose ONE of the following questions (**A**, **B**, or **C**) to answer:

A) Scientists are analyzing 50 samples from Martian rocks to find the chemical elements necessary for life on earth. They find that 5 of these samples contain at least one of the desired chemical elements. Take one of these 5 samples and represent it using at least two different fractions.

Answer 1) ________1/5________ Answer 2) ________1/50________

Explain why you chose these fractions

One sample would be 1 of 50 samples, or 1 part of 50 parts in the whole sample collection

One sample would be 1 of 5 samples that contained the needed chemical elements

B) Scientists in two different labs are analyzing 8 samples each from Martian rocks to find the chemical elements necessary for life on earth. Scientists from the first lab only found hydrogen in 3 of their 8 samples. Scientists from the second lab did not find hydrogen in any of their samples but found carbon in 2 of their 8 samples.

How many samples were used in both labs combined? ______16______

How many samples in total contained hydrogen? ______3/16______

How many samples in total contained carbon? ______2/16______

How many samples in total contained one of the chemical elements necessary for life on earth? ______5/16______

C) The Curiosity rover delivered a sample from a Martian rock to scientists, who separated the sample into six equal parts. They found that each of the parts had exactly one of the following chemical elements that are necessary for life on earth: carbon, hydrogen, nitrogen, oxygen, phosphorus, and sulfur.

How many equal parts were in the whole sample? _______6_______

Use fractions to depict how much of each element was in the rock sample:

Carbon: ______1/6______ Hydrogen: ______1/6______

Nitrogen: ______1/6______ Oxygen: ______1/6______

Phosphorus: ______1/6______ Sulfur: ______1/6______

Examine and Elaborate

3. Highlight Students' Mathematical Thinking

Mathematicians think about possible solutions in a variety of ways. Therefore, it is important for students to realize that they, too, can approach problems using different strategies. Ultimately, students need to understand that a possible solution should be judged by the correctness of the mathematics, and there might be some valid ideas within a solution when a student has an incorrect answer.

Share and Discuss

It is therefore important for students first to clearly share their ideas with others so their validity can be determined by the class.

Teacher: Today, we learned that fractions represent equal parts of a whole. Learning about the Curiosity Rover helps us to understand a little bit more about fractions and why mathematicians use them to communicate their mathematical thinking. Now, I want you to think back to the Mars Rover Activity that we worked on in class today. Can someone tell me how many expeditions they completed during this activity using fractions? I'm going to give you a minute to think about it so you can use a fraction in your answer. (*Wait Time talk move*).

D'Angelo: I finished 5 out of 8 expeditions.

Teacher: That's great, D'Angelo! Can anyone tell me how you would say that as a fraction? (*Adding On talk move*)

Ayanna: I would say five eighths.

Teacher: That's right! If you completed 5 expeditions and there are 8 expeditions in the game, you completed 5/8 of the available expeditions in the game. If there were only 6 expeditions in the game, what fraction of the game would D'Angelo have completed?

Marco: He would have finished five sixths of the game.

Teacher: That's right! Thank you, Marco. Can you tell me more about what part of the fraction changed and why? (*Reasoning talk move)*

Marco: The bottom part of the fraction changed because there weren't eight expeditions in one game anymore. There were six expeditions in one game.

Teacher: Marco is saying that the bottom part of the fraction, which is called the denominator, had to change because it was no longer referring to the same whole. (*Revoicing talk move*)

The whole game was separated into 8 equal parts. In this case, these equal parts are called expeditions. Each expedition would represent 1/8 of the total game. If there were 6 expeditions, each expedition would represent 1/6 of the whole game. Let's go back to our experience, where we had 8 total expeditions. If I was able to complete all 8 expeditions, how much of the game would I have completed?

Paris: You would have completed 8/8 of the whole game.

Teacher: That's correct. Thank you, Paris. Can anyone say what Paris said in a different way? *(Repeat/Rephrase talk move)*

Kamari: Paris said 8/8. I would have just said that you completed 1 game.

Brennan: That's like the same thing.

Kamari: Yeah, but in a different way. Paris said she completed 8 out of 8 expeditions in the whole game. That's 1 whole game. It doesn't matter if there were 8 expeditions or 6 expeditions. If she finished all the expeditions, the fraction would be 1 because it would be all of the equal parts of the game.

Teacher: You are all doing a great job of explaining your thinking like mathematicians. I appreciate how you respectfully helped others to understand what you were thinking. Now, I want you to use your whiteboards to write down how many expeditions you were able to complete as a fraction. I want you to write "numerator" next to the top number and "denominator" next to the bottom number. Now, describe this fraction using a sentence. I will give you a minute to write that down. *(Wait Time talk move)*

Teacher: What fractions did you write on your whiteboard?

Jack: I wrote 2/8.

Gina: I wrote 7/8.

Teacher: Wonderful! Can you two use that in a sentence?

Jack: I finished 2 out of 8 expeditions. 2 is how many parts out of the total number of parts that I completed.

Gina: I finished 7 out of 8 expeditions. 7 is how many expeditions out of the total number of expeditions that I completed.

Teacher: Great work, everyone! Now we know that fractions represent equal parts of the same whole.

Differentiate Further as Needed

If students are struggling with the material and need more scaffolding, present them with an appropriate hint card and/or offer them manipulatives so that they can show their thinking in the best way possible for them. If students completed their tiered lesson and need more activities to engage them, you may present them with a challenge card to keep them thinking about the concept of whole numbers as fractions.

Hint Cards

Hint Card

This card is specifically useful for Tier 1 #2

You have 8 samples. Each sample has one of the 6 elements, but you do not have to use all of the elements in your answer. You can make this question easier by choosing a factor of 8. Remember that factors are positive integers that can divide numbers evenly.

What are factors of 8?

How can you use these factors to answer your question?

Hint Card

This card is specifically useful for Tier 1 #4

This question asks for the total number of samples. How can you use the information from the visual models to figure out the total number of samples? What clues can you find in the lines that separate the models?

You are asked to represent the amount of nitrogen in the samples with a fraction. How can you use the shaded section of the visual models to figure out the total amount of nitrogen in the samples?

Hint Card

This card is specifically useful for Tier 3 #3ab

You are asked to come up with numbers that are equal to 1/3. One way to do this is to examine the multiples of 3. Multiples are numbers created by multiplying 2 numbers. A factor is the number that is multiplied. For example, in the equation 2 x 4 = 8, 2 and 4 are factors and 8 is the multiple.

What are some examples of multiples of 3?

How can these multiples help you answer the question?

Sample Responses

Hint Cards

This card is specifically useful for Tier 1, #2

You have 8 samples. Each sample has one of the 6 elements, but you do not have to use all of the elements in your answer. You can make this question easier by choosing a factor of 8. Remember that factors are positive integers that can divide numbers evenly.

What are factors of 8?

Factors of 8 include 1, 2, 4, and 8.

How can you use these factors to answer your question?

I can choose one of these factors, like 2, as the number of elements in my sample. I might choose carbon and nitrogen, for example. 4 of my samples will be carbon and 4 will be nitrogen.

Hint Card

This card is specifically useful for Tier 1 #4

This question asks for the total number of samples. How can you use the information from the visual models to figure out the total number of samples? What clues can you find in the lines that separate the models?

The lines that separate the models show how many parts separate the whole group. Counting each section separated by the lines helps us figure out the total number of samples that the team collected.

You are asked to represent the amount of nitrogen in the samples with a fraction. How can you use the shaded section of the visual models to figure out the total amount of nitrogen in the samples?

The shaded sections of the visual models tell us how many samples contained nitrogen. Counting these sections can help us figure out the total amount of nitrogen in the samples.

Hint Card

This card is specifically useful for Tier 3 #3ab

You are asked to come up with numbers that are equal to 1/3. One way to do this is to examine the multiples of 3. Multiples are numbers created by multiplying 2 numbers. A factor is the number that is multiplied. For example, in the equation 2 x 4 = 8, 2 and 4 are factors and 8 is the multiple.

1, 3

What are some examples of multiples of 3?

6, 9, 12, 15

How can these multiples help you answer the question?

Choosing a multiple of 3 can help me figure out fractions that are equal to 1/3. Multiplying 1/3 by 2 is 2/6, which is one answer to the question.

Challenge Cards

Challenge Card

During today's lesson, you and your classmates learned that fractions represent parts of a whole. You answered questions about fractions that helped expand that knowledge. Think of an important step you need to take when dividing a whole into different parts. What could happen if you don't take that step?

Challenge Card

One of the following visual models shows a shape divided into 3 different parts. The second shape is divided into 4 different parts. Shade 2/3 of the first shape. Shade 1/4 of the second shape. Then, add these two fractions. Create another visual model to demonstrate how much of the shape has been shaded.

Challenge Card

Everyone makes mistakes sometimes. Can you see what is wrong in the following examples? How might you help your friend clear up their understanding?

1) Jose says that 2/4 is greater than 5/8.
2) Vivienne says that 1/4 plus 1/4 is 8.
3) Rohan says that 1/4 is closer to 1 than 3/4.

Sample Responses

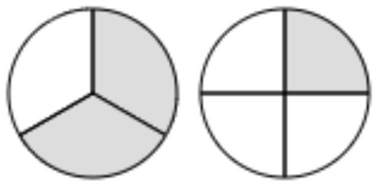
Challenge Card

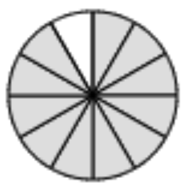
During today's lesson, you and your classmates learned that fractions represent parts of a whole. You answered questions about fractions that helped expand that knowledge. Think of an important step you need to take when dividing a whole into different parts. What could happen if you don't take that step?

*When dividing a whole into different parts, it is important to make sure that you are dividing into **equal** parts. If you don't take this step, it becomes difficult to communicate mathematically about these different parts.*

Challenge Card

One of the following visual models shows a shape divided into 3 different parts. The second shape is divided into 4 different parts. Shade 2/3 of the first shape. Shade 1/4 of the second shape. Then, add these two fractions. Create another visual model to demonstrate how much of the shape has been shaded.





1/4 + 2/3 = 11/12

Challenge Card

Everyone makes mistakes sometimes. Can you see what is wrong in the following examples? How might you help your friend clear up their understanding?

1) Jose says that 2/4 is greater than 5/8.

2/4 is 4/8, which is smaller than 5/8. I could help my friend understand that 5/8 is the greater number by showing them that 2/4 is equal to 4/8 (four parts of one whole divided into 8 even parts) and 5/8 is equal to five parts of that same whole.

2) Vivienne says that 1/4 plus 1/4 is 8.

I see that 1/4 plus 1/4 is 2/4, not 8. I think Vivienne was trying to add the denominators instead of the numerators. I could try to show her what happens when we add the numerators instead of the denominators.

3) Rohan says that 1/4 is closer to 1 than 3/4.

In this problem, one whole has been separated into 4 equal parts: 3/4 is three parts of the whole. I could show Rohan that 3/4 is closer to 1 than 1/4 by using a picture, where I color one section, then I color three sections. Finally, I will color all the sections, saying that I have colored 4/4, or 1. I could also show them with a number line divided into fourths to show that 3/4 is closer to 1 than 1/4.

Debrief and Look Ahead

4.

Debrief Content and Skills

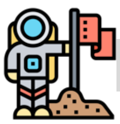
Remind students that they learned about how to partition whole numbers in this unit. Review students' understanding of fractions and remind them that we use numerators and denominators to represent specific amounts of equally divided wholes. Ask the students to describe what a fraction is, using these terms and an example.

Sample Response: *I have 5 members of my family. Each family member is 1/5 of the whole family. If I were to write that as a fraction, it would be 1/5. 1 is the numerator, that tells us the number of equal pieces we are looking at. If we were looking at 2 family members, 2 would be the numerator. 5 is the denominator, which tells us the number of equal pieces in the whole group.*

Debrief Thinking Like Mathematicians

Remind students that the mathematical practice for this lesson focused on how mathematicians solve problems and work together. Review some of the ideas students brainstormed at the beginning of class and have students offer examples of how they acted like mathematicians while they worked together during the lesson. Ask the students: How did you represent mathematics in different ways today? Why was it useful to present these numbers in different ways?

Sample Response: *Today, I represented fractions in many different ways. I drew diagrams to represent fractions, rewrote whole numbers as fractions, and used these fractions to communicate my ideas through mathematical writing.*



Assess

5. What Students Learned

Prior to assigning students into tiered groups, they will complete an entrance ticket. This will serve as a type of formative assessment and the teacher will use the results to split students into the groups. Students will submit their tiered student pages into the teacher at the end of the lesson. These student pages will serve as a type of summative assessment and will show the teacher if the students achieved the lesson objectives.

Resources

Francis, R., Estlin, T., Doran, G., Johnstone, S., Gaines, D., Verma, V., Burl, M., Frydenvang, J., Montano, S., Wiens, R., Schaffer, S., Gasnault, O., Defloers, L., Blaney, D., & Bornstein, B. (2017). AEGIS autonomous targeting for ChemCam on Mars Science Laboratory: Deployment and results of initial science team use. *Science Robotics*, *2*(7), eaan4582. https://www.science.org/doi/abs/10.1126/scirobotics.aan4582#con9

Jet Propulsion Laboratory. (2013, March 6). *Mars in a minute: Why is curiosity looking for organics*. https://www.jpl.nasa.gov/videos/mars-in-a-minute-why-is-curiosity-looking-for-organics

NASA Science. (2022, March 4). *Explore Mars: A Mars rover game*. https://spaceplace.nasa.gov/explore-mars/en/

NASA Science. (2014, February 19). *Curiosity's color view of Martian dune after crossing it*. https://mars.nasa.gov/resources/6010/curiositys-color-view-of-martian-dune-after-crossing-it/

NASA Science. (n.d.). *Curiosity landing site: Gale crater*. https://mars.nasa.gov/msl/timeline/prelaunch/gale-crater/

NASA Science. (n.d.). *Curiosity rover mission overview*. https://mars.nasa.gov/msl/mission/overview/

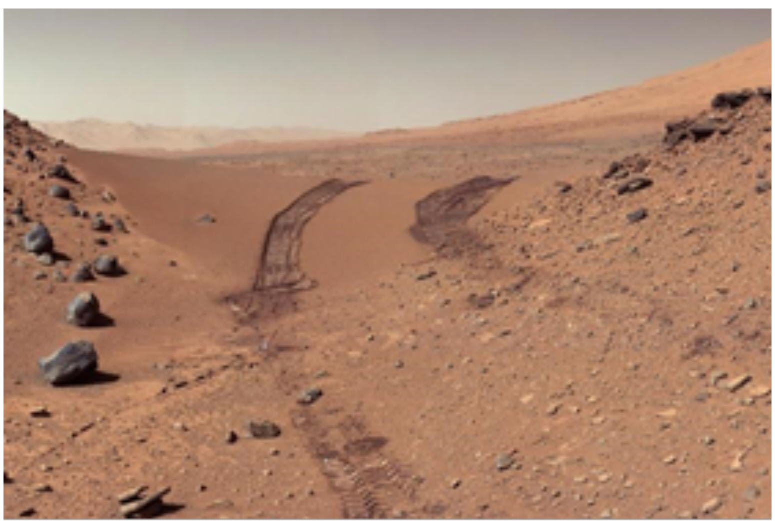
NASA Science. (n.d.). *Mission contributions to NASA's Mars exploration program science goal*. https://mars.nasa.gov/msl/mission/science/goals/

Mission to Mars Student Pages

Scientist ________________________________ Date _________________

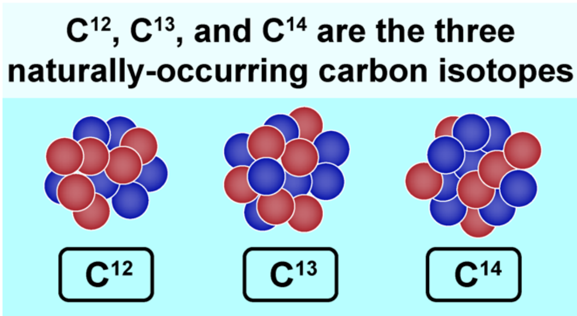
Peggy Whitson Tier 1 Lesson

Scientists from the Mars Science Laboratory want to figure out what elements make up the Martian atmosphere! They do this by measuring the *stable isotopes* of elements such as carbon. *Isotope* is the name that scientists use for elements with the same number of protons but a different number of neutrons. Unstable isotopes decay over time, so scientists are most interested in finding elements with two or more *stable isotopes*. The Mars Science Laboratory allows scientists to look for *biosignatures* (or signs of life) such as abrupt changes in isotopes. You can learn more about these biosignatures by following this link (https://mars.nasa.gov/msl/mission/science/goals/). You will learn a lot more about atoms, neutrons, and isotopes later, but don't worry! For now, we will be using fractions to communicate what scientists are finding in relation to the signs of life in Martian rock and soil samples.



(Source: https://mars.nasa.gov/resources/6010/curiositys-color-view-of-martian-dune-after-crossing-it/)

1. As you saw in the video (https://www.jpl.nasa.gov/videos/mars-in-a-minute-why-is-curiosity-looking-for-organics) earlier during class, the Curiosity Rover is looking for samples that may contain the necessary elements for life on Earth. Scientists are looking for stable isotopes in these samples such as carbon. Imagine that your sample of Martian soil and rocks contains the *three stable isotopes* of carbon that are pictured below.



a) Divide these three stable isotopes into equal groups. Show each answer using fractions.

   How many stable isotopes are in each group? _____________

b) Demonstrate this number using a fraction and a visual model.

2. You are working in a group of scientists. The scientists in your group are asked to report how many of the samples contain nitrogen. They have shown how many of their samples contain nitrogen through shapes. Each shaded part represents how much of the sample contains nitrogen. You are asked to depict these fractions in writing.

   How would you show how much nitrogen is in this sample, using a written fraction?

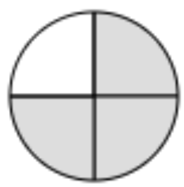
   ____

How would you report how much nitrogen is in this sample, using a fraction?

____

How would you report how much nitrogen is in this sample, using a fraction?

____



3. As you have seen before, there are 6 elements that are necessary for life on earth: **carbon**, **hydrogen**, **nitrogen**, **oxygen**, **phosphorus**, and **sulfur**. You have 8 samples, as shown below. Each box represents 1 sample. Assume each box contains **just one element** that is necessary for life on Earth. You do not have to use all of the elements in your answer. Choose a color to represent each element and list these colors below. With your colored pencil, shade the boxes to show which elements are in each box. Then, use fractions to explain what elements are in your samples.

Carbon: ___________ Hydrogen ___________ Nitrogen ___________

Oxygen ___________ Phosphorous ___________ Sulfur ___________

a) How many samples do you have altogether? ___________

b) Choose an element that is in your samples. Describe how many samples contain that element. ______________

c) Write this answer as a fraction: _____________

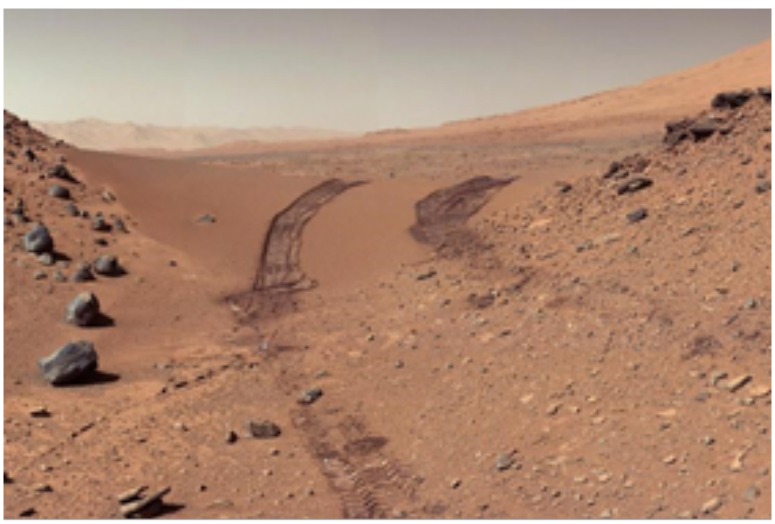
4. Use the information you gathered in the previous questions to write a report for your supervisors. You will need to describe using fractions a) the total number of samples that each group member collected; 2) the amount of nitrogen in each group member's sample; 3) the total number of samples; and 4) the amount of nitrogen in the total samples.

Mission to Mars Student Pages

Scientist ___________________________________ Date __________________

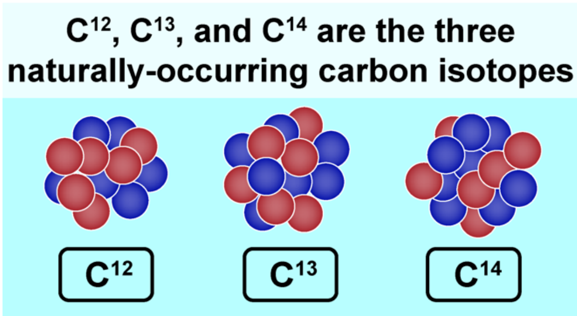
Guion Bluford Tier 2 Lesson

Scientists from the Mars Science Laboratory want to figure out what elements make up the Martian atmosphere! They do this by measuring the *stable isotopes* of elements such as carbon. *Isotope* is the name that scientists use for atoms of the same element that have different masses because they have a different number of neutrons in the nucleus. Scientists are most interested in finding elements that have two or more *stable isotopes*. The Mars Science Laboratory allows scientists to look for biosignatures (or signs of life) such as abrupt changes in isotopes. You can learn more about these biosignatures by following this link (https://mars.nasa.gov/msl/mission/science/goals/). You will learn a lot more about atoms, neutrons, and isotopes later, but don't worry! For now, we will look at using fractions to discuss the signs of life in Martian rock and soil samples.



(Source: https://mars.nasa.gov/resources/6010/curiositys-color-view-of-martian-dune-after-crossing-it/)

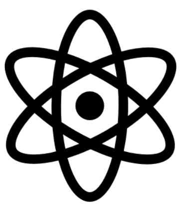
1. As you saw in the video (https://www.jpl.nasa.gov/videos/mars-in-a-minute-why-is-curiosity-looking-for-organics) earlier during class, the Curiosity Rover is looking for samples that may contain the necessary elements for life on Earth. Scientists are looking for stable isotopes in these samples such as carbon. Imagine that your sample of Martian soil and rocks contains the *three stable isotopes* of carbon that are pictured below.



The number of protons and neutrons for each isotope is provided below. Demonstrate these numbers using fractions, showing the relationship between the parts of the isotope and the whole isotope.

a) C12 Isotope- Protons: 6. Neutrons: 6.

Use a fraction to show how many protons and neutrons there are out of the total number of items in the nucleus for the first isotope.



Protons ______________

Neutrons ______________

b) C13 Isotope- Protons: 6. Neutrons: 7.

Use a fraction to show how many protons and neutrons there are out of the total number of items in the nucleus in the second isotope.

Protons ______________

Neutrons ______________

c) C14 Isotope- Protons: 6. Neutrons: 8.

Use a fraction to show how many protons and neutrons there are out of the total number of items in the nucleus in the third isotope.

Protons ______________

Neutrons ______________

2. As you have seen before, there are six elements that are necessary for life on earth: carbon, hydrogen, nitrogen, oxygen, phosphorus, and sulfur. Your supervisors are wanting to do a specific report on nitrogen. They want you to report how many Martian rocks in your sample contain nitrogen. Luckily, four of your total number of samples contain nitrogen! You must write your own report that depicts the number of samples that contain nitrogen. The number of samples are depicted below. Each box represents 1 sample.

   a) Use a line to separate the samples into two groups: One group contains nitrogen and the other group does not.

   b) How many samples are in each group? ______________

   c) Write this answer as a fraction compared to the whole. ______________

   d) Explain how you would use the information that you have found to write your report.

   ____

   ____

   ____

3. You have a friend in the Mars Science Laboratory who has also been asked to report how much of their total sample contains nitrogen. They are asking for your help submitting their report, as they have been asked to give a talk about the Autonomous Exploration for Gathering Increased Science (AEGIS) system. They know that they have more than 1 and less than 8 samples total. They also believe that at least 2 of their total samples contain nitrogen.

   a) How many total samples might you expect to find?

      Give three examples.

      i. ____________

      ii. ____________

      iii. ____________

For each example, write how many nitrogen samples you might expect to find.

i. ____________

ii. ____________

iii. ____________

b) Use fractions to write how many nitrogen samples there are in each of your examples.

Example 1: ______________

Example 2: ______________

Example 3: ______________

4. When you find your friend’s samples, you discover that there are 6 total samples. When you run analyses on each of the samples, you find that your friend’s rough estimate (at least 2 of the samples contain nitrogen) was correct.

   Using a colored pencil, **shade** the number of samples that contain nitrogen.

   How many samples contain nitrogen? Use a fraction in your answer.

   ____

   How many samples do not contain nitrogen? Use a fraction in your answer.

   ____

   How do fractions help us describe our mathematical thinking? Explain.

   ____

   ____

   ____

Mission to Mars Student Pages

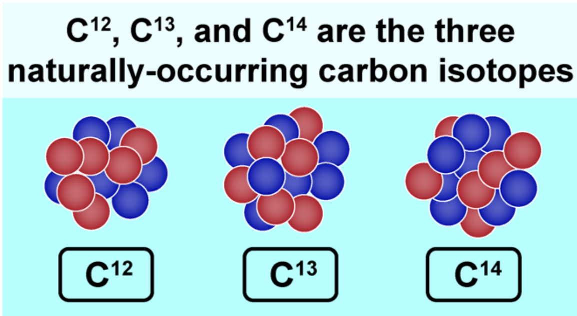
Scientist ___________________________________ Date __________________

Ellen Ochoa Tier 3 Lesson

Scientists from the Mars Science Laboratory want to precisely determine the composition of the Martian atmosphere by measuring the *stable isotopes* of elements such as carbon. *Isotopes* are atoms of the same element that have different masses because they have a different number of neutrons in the nucleus. Scientists are most interested in finding elements that have two or more *stable isotopes*. The Mars Science Laboratory allows scientists to look for biosignatures (or signs of life) such as abrupt changes in isotopic abundance. In their quest to find signs of life, scientists look at different rocks, soils, and other landforms that could help explain changes in Mars' atmosphere over time. For more information about the Martian atmosphere, you may visit this link (https://mars.nasa.gov/msl/mission/science/goals/). We can use fractions to discuss the signs of life in Martian rock and soil samples.

(Source: https://mars.nasa.gov/resources/6010/curiositys-color-view-of-martian-dune-after-crossing-it/)

1. As you saw in the video (https://www.jpl.nasa.gov/videos/mars-in-a-minute-why-is-curiosity-looking-for-organics) earlier during class, the Curiosity Rover is looking for samples that may contain the necessary elements for life on Earth. Scientists are looking for stable isotopes in these samples such as carbon. Imagine that your sample of Martian soil and rocks contains *three stable isotopes* of carbon, pictured below. Each has the **same** number of protons, but a **different** number of neutrons.



a) How many protons and neutrons are in each isotope? Show each answer using fractions. (*In this answer, you should focus on dividing the numbers 12,13, and 14 into two equal parts. You may use the above picture to help you. The red circles refer to protons.*)

C12: Protons _____ Neutrons _____

C13: Protons _____ Neutrons _____

C14: Protons _____ Neutrons _____

b) Demonstrate the C12 isotope using a visual fraction model.

2. As you have seen before, there are six elements that are necessary for life on earth: carbon, hydrogen, nitrogen, oxygen, phosphorus, and sulfur. Your supervisors are wanting to do a specific report on nitrogen. They want you to report how many Martian rocks in your sample contain nitrogen. You must create your **own** report that explains a) how many total samples you have; b) how many of your samples contain elements that are necessary for life on Earth (biosignatures), and c) how many of your samples contain nitrogen.

a) ____________ (total samples)

b) ____________ (# of samples that contain biosignatures)

c) ____________ (# of samples that contain nitrogen)

Write each answer as a fraction

a) _____________

b) _____________

c) _____________

3. You have a friend in the Mars Science Laboratory who reports that 1/3 of the total Martian rocks contain microbes that have at least one element of interest (biosignatures). You are asked to verify that your friend's report is correct.

   Write three examples of the number of samples (and the total number of Martian rocks altogether) that you might be asked to collect. *(Hint: These example fractions should be equal to the one found in your friend's report)*.

   Example 1:

   # of Samples that Contain Biosignatures _____________

   # of Total Samples _____________

   Example 2:

   # of Samples that Contain Biosignatures _____________

   # of Total Samples _____________

   Example 3:

   # of Samples that Contain Biosignatures _____________

   # of Total Samples _____________

   Rewrite the above examples using fractions.

   Example 1: _____________

   Example 2: _____________

   Example 3: _____________

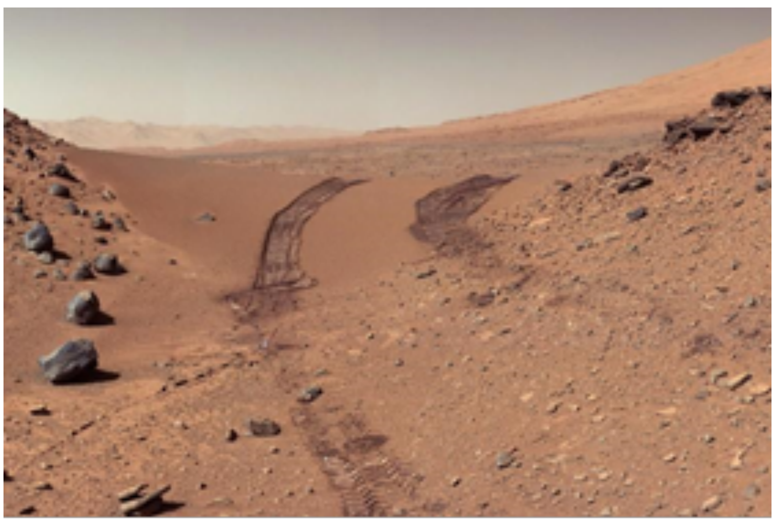
4. You found that the fraction your friend reported was not correct. As you can see below, the total number of samples is 10 (which is not divisible by 3). The samples are depicted below in rectangular boxes. You found that 2/10 of the total number of samples contained elements that are necessary for life.

   Using a colored pencil, **shade** the number of samples that contain biosignatures.

Is there another way to write 2/10 as a fraction? Explain.

Peggy Whitson Tier 1 Lesson

Scientists from the Mars Science Laboratory want to figure out what elements make up the Martian atmosphere! They do this by measuring the *stable isotopes* of elements such as carbon. *Isotope* is the name that scientists use for elements with the same number of protons but a different number of neutrons. Unstable isotopes decay over time, so scientists are most interested in finding elements with two or more *stable isotopes*. The Mars Science Laboratory allows scientists to look for *biosignatures* (or signs of life) such as abrupt changes in isotopes. You can learn more about these biosignatures by following this link (https://mars.nasa.gov/msl/mission/science/goals/). You will learn a lot more about atoms, neutrons, and isotopes later, but don't worry! For now, we will be using fractions to communicate what scientists are finding in relation to the signs of life in Martian rock and soil samples.



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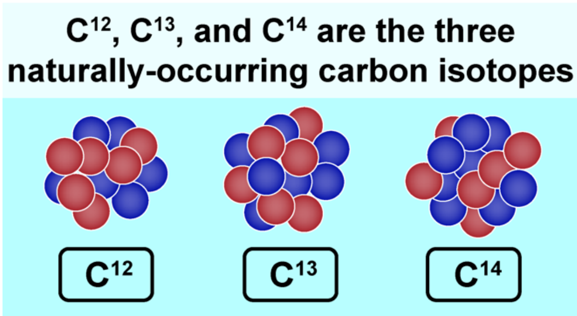
1. As you saw in the video (https://www.jpl.nasa.gov/videos/mars-in-a-minute-why-is-curiosity-looking-for-organics) earlier during class, the Curiosity Rover is looking for samples that may contain the necessary elements for life on Earth. Scientists are looking for stable isotopes in these samples such as carbon. Imagine that your sample of Martian soil and rocks contains the *three stable isotopes* of carbon that are pictured below.



a) Divide these three stable isotopes into equal groups. Show each answer using fractions.

   How many stable isotopes are in each group? ______*1*______

b) Demonstrate this number using a fraction and a visual model.

1/3

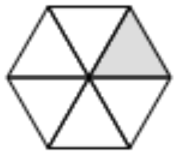
2. You are working in a group of scientists. The scientists in your group are asked to report how many of the samples contain nitrogen. They have shown how many of their samples contain nitrogen through shapes. Each shaded part represents how much of the sample contains nitrogen. You are asked to depict these fractions in writing.

   How would you show how much nitrogen is in this sample, using a written fraction?

   ______*2/5*______

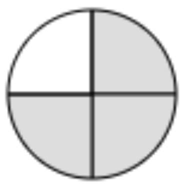
How would you report how much nitrogen is in this sample, using a fraction?

_______*1/6*______



How would you report how much nitrogen is in this sample, using a fraction?

_______*3/4*______



3. As you have seen before, there are 6 elements that are necessary for life on earth: **carbon**, **hydrogen**, **nitrogen**, **oxygen**, **phosphorus**, and **sulfur**. You have 8 samples, as shown below. Each box represents 1 sample. Assume each box contains **just one element** that is necessary for life on Earth. You do not have to use all of the elements in your answer. Choose a color to represent each element and list these colors below. With your colored pencil, shade the boxes to show which elements are in each box. Then, use fractions to explain what elements are in your samples.

Carbon: ___*Yellow*___ Hydrogen ____*Pink*____ Nitrogen ___*Blue*____

Oxygen ___*Green*___ Phosphorous ___*Orange*___ Sulfur ____*Red*____

a) How many samples do you have altogether? _____*8*_____

b) Choose an element that is in your samples. Describe how many samples contain that element.

Element: ___*Nitrogen*___ Number of Samples: _______*3*_______

c) Write this answer as a fraction: _____*3/8*_____

4. Use the information you gathered in the previous questions to write a report for your supervisors. You will need to describe using fractions a) the total number of samples that each group member collected; 2) the amount of nitrogen in each group member's sample; 3) the total number of samples; and 4) the amount of nitrogen in the total samples.

The 3 scientists in our team have collected 15 samples in total. The first scientist collected 5 samples. The second scientist collected 6 samples. The third scientist collected 4 samples. Nitrogen was in 2/5 of the first sample, 1/6 of the second sample, and 3/4 of the third sample. Nitrogen is in 6/15 of the total samples.

Guion Bluford Tier 2 Lesson

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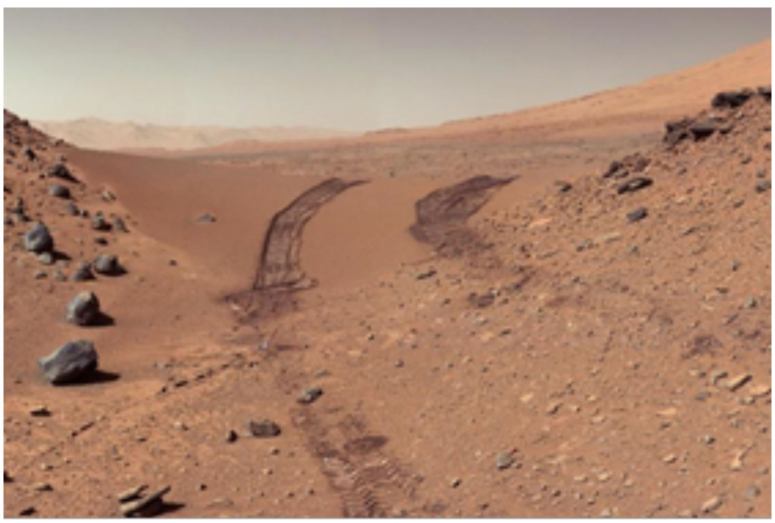

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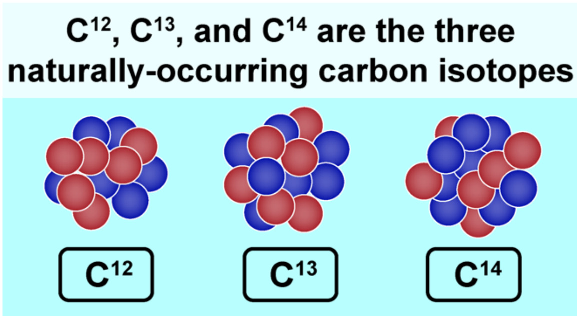
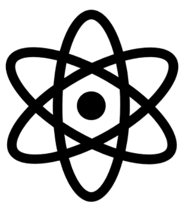


The number of protons and neutrons for each isotope is provided below. Demonstrate these numbers using fractions, showing the relationship between the parts of the isotope and the whole isotope.

a) C12 Isotope- Protons: 6. Neutrons: 6.

Use a fraction to show how many protons and neutrons there are out of the total number of items in the nucleus for the first isotope.



Protons _____*6/12*_____

Neutrons _____*6/12*_____

b) C13 Isotope- Protons: 6. Neutrons: 7.

Use a fraction to show how many protons and neutrons there are out of the total number of items in the nucleus in the second isotope.

Protons _____*6/13*_____

Neutrons _____*7/13*_____

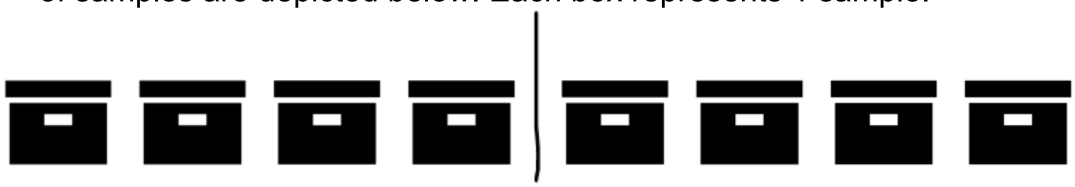
c) C14 Isotope- Protons: 6. Neutrons: 8.

Use a fraction to show how many protons and neutrons there are out of the total number of items in the nucleus in the third isotope.

Protons _____*6/14*_____

Neutrons _____*8/14*_____

2. As you have seen before, there are six elements that are necessary for life on earth: carbon, hydrogen, nitrogen, oxygen, phosphorus, and sulfur. Your supervisors are wanting to do a specific report on nitrogen. They want you to report how many Martian rocks in your sample contain nitrogen. Luckily, four of your total number of samples contain nitrogen! You must write your own report that depicts the number of samples that contain nitrogen. The number of samples are depicted below. Each box represents 1 sample.



   a) Use a line to separate the samples into two groups: One group contains nitrogen and the other group does not.

   b) How many samples are in each group? *4*

   c) Write this answer as a fraction compared to the whole. *4/8*

   d) Explain how you would use the information that you have found to write your report.

   Now that I know there are four samples in each of the two groups, I know that the number of nitrogen samples I have (4/8), represents one of the two groups (1/2). I would tell the scientists that 4/8 of my samples are nitrogen samples.

3. You have a friend in the Mars Science Laboratory who has also been asked to report how much of their total sample contains nitrogen. They are asking for your help submitting their report, as they have been asked to give a talk about the Autonomous Exploration for Gathering Increased Science (AEGIS) system. They know that they have more than 1 and less than 8 samples total. They also believe that at least 2 of their total samples contain nitrogen.

   a) How many total samples might you expect to find?

   Give three examples.

   i. *4*

   ii. *6*

   iii. *7*

For each example, write how many nitrogen samples you might expect to find.

i. *2*

ii. *4*

iii. *6*

b) Use fractions to write how many nitrogen samples there are in each of your examples.

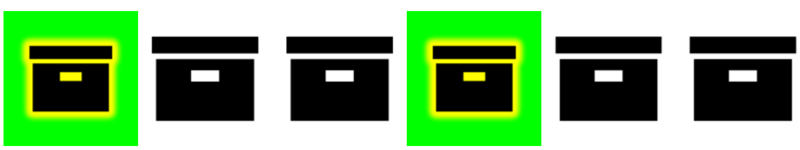
Example 1: *2/4*

Example 2: *4/6*

Example 3: *6/7*

4. When you find your friend's samples, you discover that there are 6 total samples. When you run analyses on each of the samples, you find that your friend's rough estimate (at least 2 of the samples contain nitrogen) was correct.

Using a colored pencil, **shade** the number of samples that contain nitrogen.



How many samples contain nitrogen? Use a fraction in your answer.

2/6

How many samples do not contain nitrogen? Use a fraction in your answer.

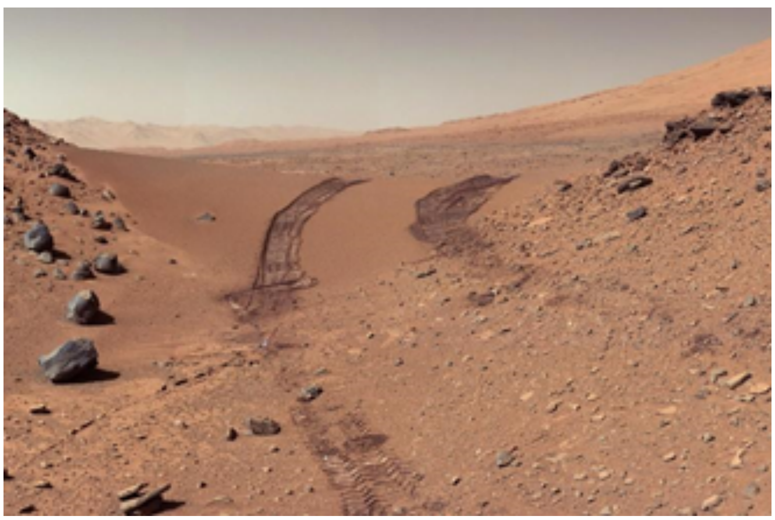
4/6

How do fractions help us describe our mathematical thinking? Explain.

Fractions help us describe our mathematical thinking because they help us to easily show one part of a whole group. I can use fractions to talk about a whole group and parts of that group.

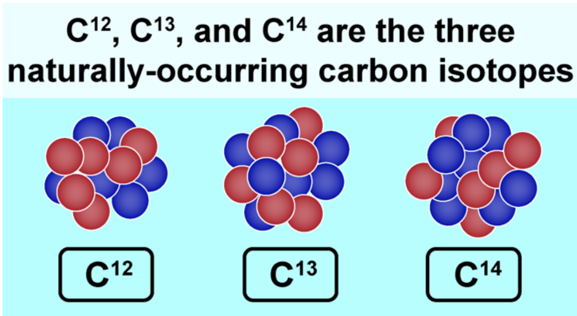
Ellen Ochoa Tier 3 Lesson

Scientists from the Mars Science Laboratory want to precisely determine the composition of the Martian atmosphere by measuring the stable isotopes of elements such as carbon. Isotopes are atoms of the same element that have different masses because they have a different number of neutrons in the nucleus. Scientists are most interested in finding elements that have two or more stable isotopes. The Mars Science Laboratory allows scientists to look for biosignatures (or signs of life) such as abrupt changes in isotopic abundance. In their quest to find signs of life, scientists look at different rocks, soils, and other landforms that could help explain changes in Mars' atmosphere over time. For more information about the Martian atmosphere, you may visit this link (https://mars.nasa.gov/msl/mission/science/goals/). We can use fractions to discuss the signs of life in Martian rock and soil samples.



(Source: https://mars.nasa.gov/resources/6010/curiositys-color-view-of-martian-dune-after-crossing-it/)

1. As you saw in the video (https://www.jpl.nasa.gov/videos/mars-in-a-minute-why-is-curiosity-looking-for-organics) earlier during class, the Curiosity Rover is looking for samples that may contain the necessary elements for life on Earth. Scientists are looking for stable isotopes in these samples such as carbon. Imagine that your sample of Martian soil and rocks contains *three stable isotopes* of carbon, pictured below. Each has the **same** number of protons, but a **different** number of neutrons.



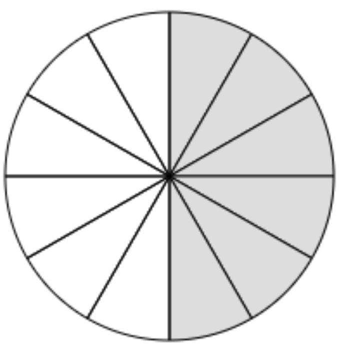
a) How many protons and neutrons are in each isotope? Show each answer using fractions. (*In this answer, you should focus on dividing the numbers 12,13, and 14 into two equal parts. You may use the above picture to help you. The red circles refer to protons.*)

C12: Protons ___6___ Neutrons ___6___

C13: Protons ___6___ Neutrons ___7___

C14: Protons ___6___ Neutrons ___8___

b) Demonstrate the C12 isotope using a visual fraction model.



2. As you have seen before, there are six elements that are necessary for life on earth: carbon, hydrogen, nitrogen, oxygen, phosphorus, and sulfur. Your supervisors are wanting to do a specific report on nitrogen. They want you to report how many Martian rocks in your sample contain nitrogen. You must create your **own** report that explains a) how many total samples you have; b) how many of your samples contain elements that are necessary for life on Earth (biosignatures), and c) how many of your samples contain nitrogen.

a) ______10______ (total samples)

b) ______4_______ (# of samples that contain biosignatures)

c) ______2_______ (# of samples that contain nitrogen)

Write each answer as a fraction

a) _____*10/10*____

b) _____*4/10*_____

c) _____*2/10*_____

3. You have a friend in the Mars Science Laboratory who reports that 1/3 of the total Martian rocks contain microbes that have at least one element of interest (biosignatures). You are asked to verify that your friend's report is correct.

   Write three examples of the number of samples (and the total number of Martian rocks altogether) that you might be asked to collect. *(Hint: These example fractions should be equal to the one found in your friend's report)*.

   Sample answers may vary. Each example should be equivalent to 1/3.

   Example 1:

   # of Samples that Contain Biosignatures ______*2*______

   # of Total Samples ______*6*______

   Example 2:

   # of Samples that Contain Biosignatures ______*3*______

   # of Total Samples ______*9*______

   Example 3:

   # of Samples that Contain Biosignatures ______*4*______

   # of Total Samples ______*12*_____

   Rewrite the above examples using fractions.

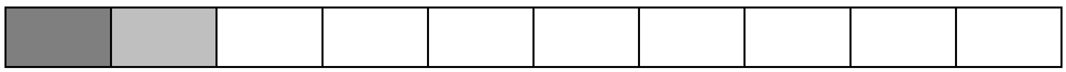
   Example 1: _____*2/6*_____

   Example 2: _____*3/9*_____

   Example 3: _____*4/12*____

4. You found that the fraction your friend reported was not correct. As you can see below, the total number of samples is 10 (which is not divisible by 3). The samples are depicted below in rectangular boxes. You found that 2/10 of the total number of samples contained elements that are necessary for life.

   Using a colored pencil, **shade** the number of samples that contain biosignatures.

   Is there another way to write 2/10 as a fraction? Explain.

Sample answers may vary. This question is included to prepare students to think about equivalent fractions. Some students may be able to recognize that the fraction 1/5 represents 2 out 10 samples.

Sample Response

The whole sample collection has 10 parts. Scientists want to look at 2 of these parts. This would be 1 group of 2 parts. If each sample was separated like this, there would be 5 groups altogether and the scientists would want to look at the group that had the right elements in it.

Thinking Like Mathematicians:
Challenging All Grade 3 Students

Common Core State Standard

Defining and Labeling Number Lines

CCSS.MATH.CONTENT.3.NF.A.2
Understand a fraction as a number on the number line; represent fractions on a number line diagram.

> **CCSS.MATH.CONTENT.3.NF.A.2.A**
> Represent a fraction 1/*b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size 1/*b* and that the endpoint of the part based at 0 locates the number 1/*b* on the number line.
>
> **CCSS.MATH.CONTENT.3.NF.A.2.B**
> Represent a fraction *a*/*b* on a number line diagram by marking off a lengths 1/*b* from 0. Recognize that the resulting interval has size *a*/*b* and that its endpoint locates the number *a*/*b* on the number line.

Standards in Plain Language:
Students will use number lines to understand fractions as a whole that has been divided into equal parts. They will recognize that these number lines go from 0 to 1 and each interval represents a unit fraction (e.g., 1/2 or 1/4). Students will use these number lines to represent fractions visually.

Lesson Designer: Lisa DaVia Rubenstein

LESSON 6

Lesson 6: Finding and Labeling Fractions on Number Lines—

Practice Saving the Shuttle: Finding and Repairing Loose Screws

Big Ideas

Numbers provide a consistent method to communicate a precise quantity being considered. Specifically, fractions are numbers that precisely describe a situation where a whole has been broken up into *equal* parts. The “denominator” communicates how many equal parts there are in the whole, and it is written on the bottom of the fraction. The “numerator” communicates how many of those equal parts are present, and it is recorded at the top of the fraction.

In this lesson series, we are examining number lines that span from 0 to 1. In this case, [0 to 1] is the whole that will be split into equal parts. Fractions can be used to communicate the precise distance between 0 and 1. Precision is important to establish a common understanding of distance that most closely communicates the true or desired value. For example, let’s suppose you want to tell your friend how far you are to their house. A fraction can be used to communicate the relative distance from your starting point to their house, like “I am halfway there.” The more precise you are; your friend will be better able to estimate when to expect you.

Generally, understanding fractions on number lines will support the measurement of distance, reading gauges, and generally comparing relative lengths. The big idea of this lesson is to develop fractional reasoning to determine where the shuttle needs to be repaired.

Lesson Objectives	• Students will describe that a fraction communicates when a whole unit is divided into equal parts. In this lesson, students will conceptualize the whole unit as the distance between 0 and 1 on a number line.

	- Students will define "denominator" as the bottom number in the fraction and recognize a denominator communicates how many total parts make up the whole. In this lesson, students will describe the denominator as the total parts of the whole, when the whole is 0 to 1 on a number line. - Students will define "numerator" as the top number in the fraction and recognize the numerator communicates how many parts of the whole are present. In this lesson, students will describe the how the numerator communicates the precise distance from 0, when the whole is equally divided into the denominator's equal parts. - Students will build/label their own number lines. - Students will be able to compare fractions using a number line to determine relative size. - Students will use fractions to communicate precisely with others. Students provide precise explanations and definitions in their communication. **Common Core State Standard** Develop understanding of fractions as numbers. <u>CCSS.MATH.CONTENT.3.NF.A.2</u> Understand a fraction as a number on the number line; represent fractions on a number line diagram. <u>CCSS.MATH.CONTENT.3.NF.A.2.A</u> Represent a fraction 1/*b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size 1/*b* and that the endpoint of the part based at 0 locates the number 1/*b* on the number line. <u>CCSS.MATH.CONTENT.3.NF.A.3.A</u> Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line. <u>CCSS.MATH.CONTENT.3.NF.A.3.D</u> Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the

	results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.
Materials 	All students need the appropriate dice and a stopwatch. Students should also have access to a variety of other materials, such as graph paper, tiles, fraction rods, paper clips, scissors, scrap paper, and other materials teachers may have available. Students may not choose to build their responses using these tools, but in general, mathematicians have various tools at their disposal that they can use to test their hypotheses.
Mathematical Terms 	• **Denominator:** bottom number in a fraction that identifies the number of pieces the whole has been divided into equal parts • **Equal:** the same portion, piece, or segment • **Fraction:** a number that represents part of a whole • **Number Line:** a line with numbers placed in their correct position • **Numerator:** top number in a fraction that identifies the number of equal pieces considered as part of the whole • **Precise:** describes responses that are exact, accurate, careful about details
Selected Mathematical Practices 	• MP1: Make sense of problems and persevere in solving them. *I never give up on a problem and I do my best to get it right.* • MP3: Construct viable arguments and critique the reasoning of others. *I can explain my math thinking and talk about it with others.* • MP5: Use appropriate tools strategically. *I know how to choose and use the right tools to solve a math problem.* • MP6: Attend to precision. *I can work carefully and check my work.*

Differentiation

Content

Guiding Questions

- prior knowledge or learner readiness
  What evidence do you have about students' current knowledge and skills?
- tiered activities
  How will you design tiered activities on the same mathematical concept with varied levels of difficulty?
- formative assessment
  What techniques will you use to assess students' prior knowledge and skills?
- varied levels of challenge
  How will you vary the level of difficulty for each tiered activity?
- "teaching up" (aim high, provide scaffolding)
  How will you increase the depth, breadth, complexity, and abstractness of lessons to challenge and support student learning?

Process

Guiding Questions

- questioning strategies
  How will you pose and how will you encourage students to pose open-ended, closed-ended, lower-level, and higher-level questions to promote mathematical discourse?
- 4Cs (21st Century Skills)
  - creative thinking

  How will you encourage students to "think outside the box" and synthesize information in new, different, and useful ways?
- 4Cs (21st Century Skills)
  - collaboration

  How will you encourage students to work with other students and appreciate their contributions to solving problems or making connections to prior work?
- 4Cs (21st Century Skills)
  - communication

  How will you promote students' opportunities to communicate face-to-face, in large and small groups, in online environments, and with print and non-print resources using their oral, written, and non-verbal skills?

	• hands-on activities/manipulatives *How will you incorporate activities promoting the use of manipulatives to clarify or illustrate mathematical concepts?* **Product** ***Guiding Questions*** • oral, visual, and written opportunities *How will you encourage students to represent their thinking and problem solving using different communication strategies?* • multiple ways to demonstrate knowledge, understanding, and skills *How will you encourage students to share their understanding of mathematical concepts and skills?* • multiple models and representations *What techniques of lesson design will you include to support students' deep understanding and the ability to apply mathematical concepts and skills?* • summative assessment *How will you assess student learning upon completion of the lesson?* **Learning Environment** ***Guiding Questions*** • flexible grouping *How will you use your tiered lesson to support flexible grouping?* • whole group/small group/individual instruction *How will you incorporate different grouping plans to address students' learning needs?*

Lesson Preview

This lesson builds from the concept that fractions require **equal** parts. Within this lesson, students will use **number lines** to communicate a precise location. It builds on the previous lesson by requiring students to create and label the equal parts on a number line. Further, students will gain practice in developing number lines with a variety of denominators. They will be building mathematical reasoning skills regarding the size of fractions with different denominators and equivalent fractions, which will be supported further in the next lesson.

In the current lesson, students will explore these concepts within a Mission to Mars scenario. The mission premise is the astronauts must practice finding and fixing loose screws on the space shuttle. Mission Control will provide a location, but they must be able to find it on a diagram. Mission Control is providing a fraction that is located on the center line of the shuttle.

Launch

1. **Thinking like Mathematicians: Centering the Mathematical Practice**

Reminder of Key CCSS.Math.Practice.MP6: Attend to Precision
Within this lesson, students will be developing the following mathematical practices:

- Precise communication by specifying how many equal parts are present.
- Precise use of tools and visual representations of equal parts.
- Determining the degree of precision appropriate for specific contexts.
- Use of clear definitions in discussion with others and in their own reasoning.

Explain the Current Task: Throughout this lesson series, we have learned that mathematicians and astronauts need to be precise in their communication. For today's mission, the astronauts must locate and tighten certain screws that Mission Control has identified as problematic.

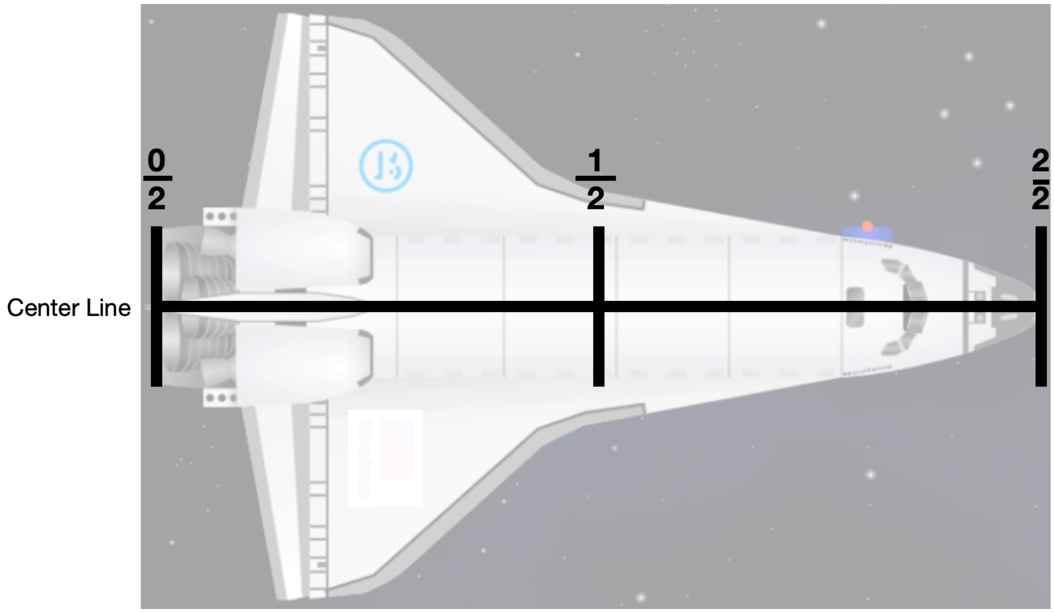
In the previous lesson, we prepared for a spacewalk by checking our equipment, and now we are going to practice finding specific locations on the shuttle.

Remember how the space station had specific labels along the outside to help the spacewalkers find their work sites? [If they don't remember, check out minute 3:23-4:23 in the spacewalk recording: https://www.youtube.com/watch?v=qStW1FysHLY] This is an example of precisely labeling parts of the space station. Why does precision matter?

Explore

2. **Activate Simulation: Saving the Shuttle**
Use this introductory task to determine which differentiated group may be most appropriate for each student.

Introduction Task
The space shuttle has been labeled using a number line. When a part of the space shuttle needs repaired, Mission Control tells the astronauts where the problem is located on the number line.

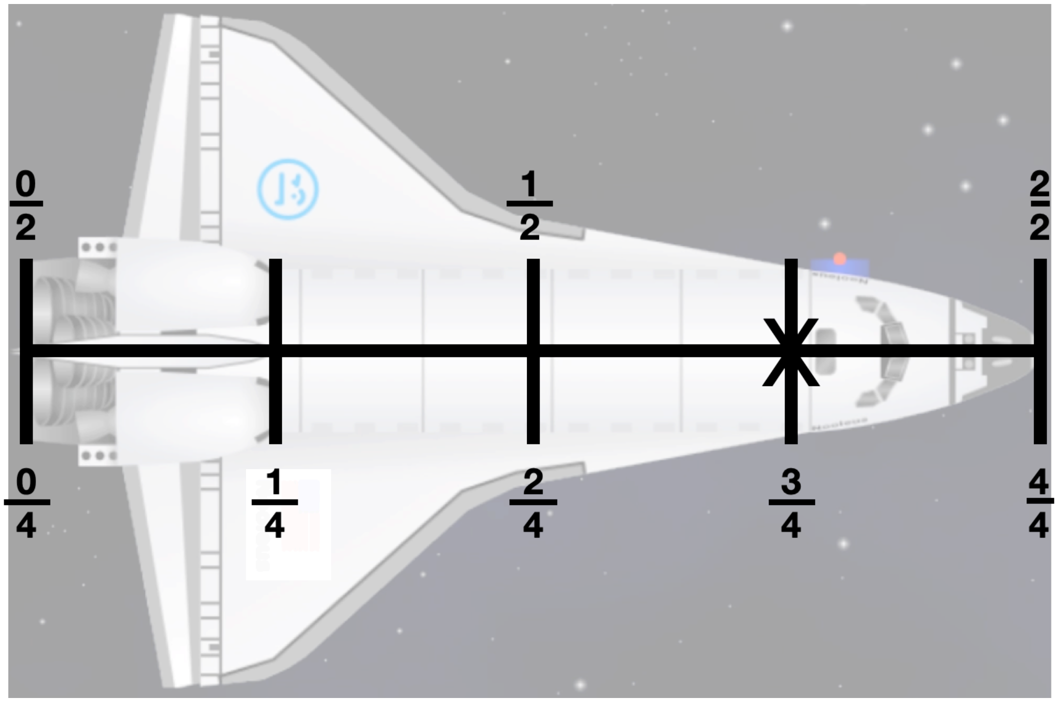


Mission Control radioed: “Screw lose along Center Line, Position 3/4.”

- Where should the astronaut go to tighten the screw? Mark it on the ship and describe your process below.
- Why is precision important when labeling the space shuttle?

Using the Introduction Task to Determine Differentiated Groups
Teachers should evaluate students’ responses using both students’ knowledge of the content and their mathematical processes.

- **Fraction Concept:** It is most important that students display the concept of equal parts. They need to demonstrate that the number line needs to be broken into equal parts as dictated by the denominator. Then the X should be placed in the numerator’s position. In this case, there should be 4 equal parts (denominator), and the “X” on the third line (numerator) as follows:



- **Mathematical Practice 6:** Students should demonstrate the precise use of tools and visual representations of equal parts. They may use a variety of tools, anything from a ruler, paper clips, or folding the paper, yet anything used must be used as a method for communicating equal parts. If the number line is not precisely broken into components, students may not arrive at the correct answer. The process should be precisely communicated in the first question. In the second part, precision is important in locating the precise issue on the space shuttle. Astronauts need to know where the problem is and how far it is from other points of reference.

In this investigation, students will be working on one of the Student Pages in their differentiated groups, based on readiness levels. The groups are based on teacher's observations of students' conceptual understanding and mathematical practice acumen as demonstrated in the Introductory Task.

- **Tier 1:** Students who do not demonstrate a conceptual understanding of fractions on number lines should be placed in Tier 1. They will be provided with additional manipulatives to emphasize equality of parts.
- **Tier 2:** If students demonstrate a vague sense of fractions (i.e., they try to establish equal parts) but do not demonstrate a specific strategy or fail to arrive at the correct answer, they should be in Tier 2. These students may also struggle with using precise language to explain their responses.

- **Tier 3:** If students communicate the appropriate location of the loose screw, and they used a specific strategy to precisely establish equal parts, they should be placed in Tier 3. They should demonstrate they can precisely communicate what a denominator is because in their tier, they will be making comparisons of denominators and starting to wrestle with the idea of common denominators.

Groups Formed by Student Readiness		
Tier 1: Peggy Whitson	**Tier 2: Guion Bluford**	**Tier 3: Ellen Ochoa**
Student Names	**Student Names**	**Student Names**

Collaborate and Communicate

Have students record their ideas for on their individual worksheets or one for the small group. Help them clarify their ideas by asking questions like, "What do you mean here?" and "How might you share that idea with the rest of the class?" Remind students that mathematicians use precise definitions, examples/non-examples, and various representations/tools to help support their conclusions. Mathematician can demonstrate their precision by using appropriate tools and precise language, including units and fractions.

Below are some possible student responses, and you can record additional ones you observed in your own class in the blank boxes.

A. [Possible response] *This group . . .*	**B.** [Possible response] *This group . . .*	**C.** [Possible response] *This group . . .*

Examine and Elaborate

3. Highlight Students' Mathematical Thinking

Mathematicians think about possible solutions in a variety of ways. Therefore, it is important for students to realize that they, too, can approach problems using different strategies. Ultimately, students need to understand that a possible solution should be judged by the correctness of the mathematics, and there might even be some valid ideas within a solution when a student has an incorrect answer.

Share and Discuss

After the groups have an opportunity to explore their assigned tasks, bring the class back together for a full group discussion. In this discussion, it is important to stress that mathematicians explain their thoughts using mathematical definitions, examples/non-examples, and concrete evidence using tools. After the groups share their approaches…

Guiding MP questions for all groups should include:

- What tools could you use to make your conclusion? (MP5: Encourages appropriate use of tools)
- How precise are your tools? (MP6: Attend to precision)
- What is another way you could determine the location of the screw that would be more precise? Less precise? (MP6: Attend to precision and promotes fluency of thought, which is a component of creativity)
- What precise definition are using for denominators? Numerators? (MP6: Attend to precision)

In this discussion, it is important to stress that mathematicians decide to be either more or less precise in their responses based on the situation and materials. In this case, astronauts need to be very precise regarding the location of a loose screw.

During this discussion, continue to discuss how a number line supports precise communication, how 0 to 1 is the whole that can be broken into equal parts, and then, fractions and fraction notation can be used to describe a location. Fractions only describe when a whole is divided into EQUAL parts. As they share, connect back to fraction notation and vocabulary.

Teacher: I want to explore your strategies for helping you find the loose screw as quickly as possible. Both the Whitson and Bluford groups used dice that had 1, 2, 3, and 6. And Whitson group had the opportunity to use paper clips. Were the paper clips helpful?

Monroe: Yes, kind of.

Teacher: Can you elaborate on your thinking? *(Creative Thinking Question Strategy: Elaboration.)*

Monroe: I found that six paper clips perfectly fit across the space shuttle, so whenever I rolled a 6 and placed it in the denominator, I could use the paper clips to help find the loose screw.

Teacher: Could anyone in any group think of another way the paper clips could be used to help find other denominators? (*Adding On talk move)*

Avalyn: Well, if 2 paperclips are used as the equal parts, then there are 3 total parts. So, you could use them if "3" is the denominator.

Teacher: I think I understand what you are saying. Could any rephrase what Avalyn just explained? *(Rephrase talk move)*

Differentiate Further as Needed

Please see the Hint and Challenge cards at the end of the lesson. The hint cards remind students to incorporate precise language and how fractions require equal parts. The challenge cards begin to probe students thinking on equivalent fractions on number lines.

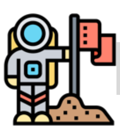
Debrief and Look Ahead

4.

Debrief Content and Skills

Remind students that the mathematical practice for this lesson focused on how mathematicians use precision to communicate locations. Fractions are one way to increase precision because they can communicate a relative amount. A fraction communicates how far you are from the back of the shuttle as well as where the loose screw is.

Finally, have students offer examples of how they acted like mathematicians while they worked together during the lesson.



Assess

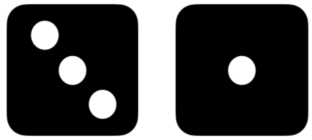
5.

What Students Learned

Use the following exit card to assess what students learned from this lesson. Students may also want to play more rounds of the Mission Control Simulator. This could be turned into a game to see which team or student can find the loose screw the fastest. It could be extended by using different numbered/sided dice. Google provides simple simulated dice rolls of various sided dice: https://www.google.com/search?client=firefox-b-1-d&q=ROLL+DICE

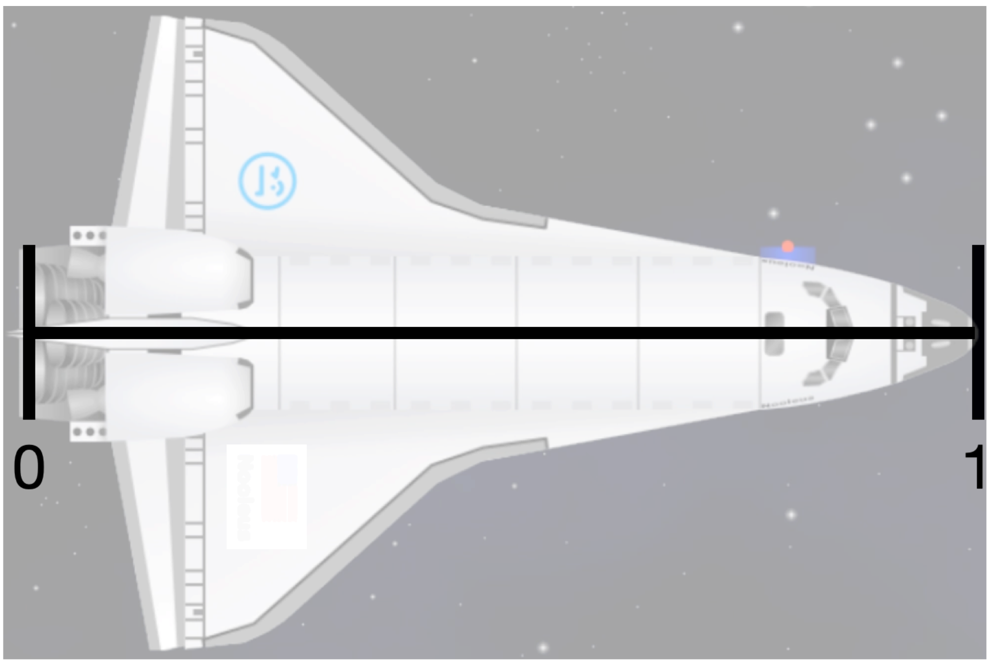
Exit Card

Mission Control simulation provides you the following location on the center line:



Where is the loose screw? Mark the location with a large “X” on the shuttle below. (Remember in our Mission Control Simulation, the bigger number serves as the denominator.)

Center Line



How did you **precisely** locate the loose screw? Include the words “denominator” and “numerator.”

____

____

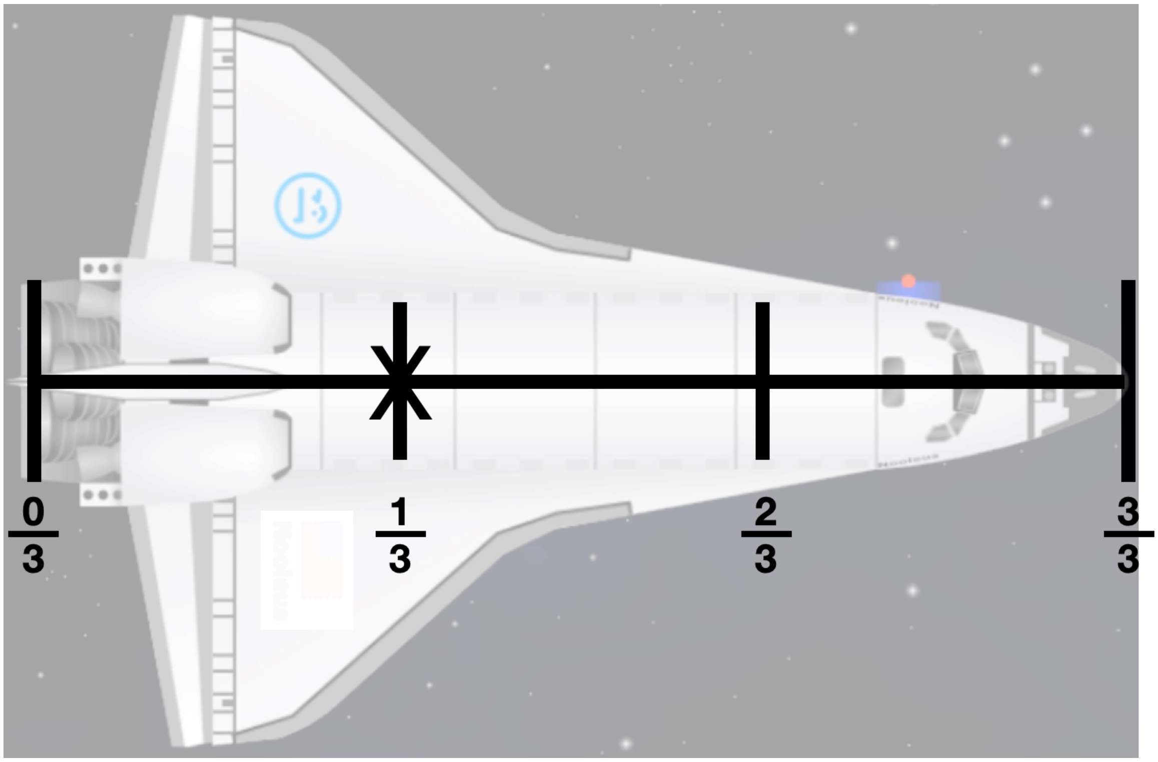
____

____

____

Exit Card Answer Key

1. *Students should locate the loose screw as follows:*

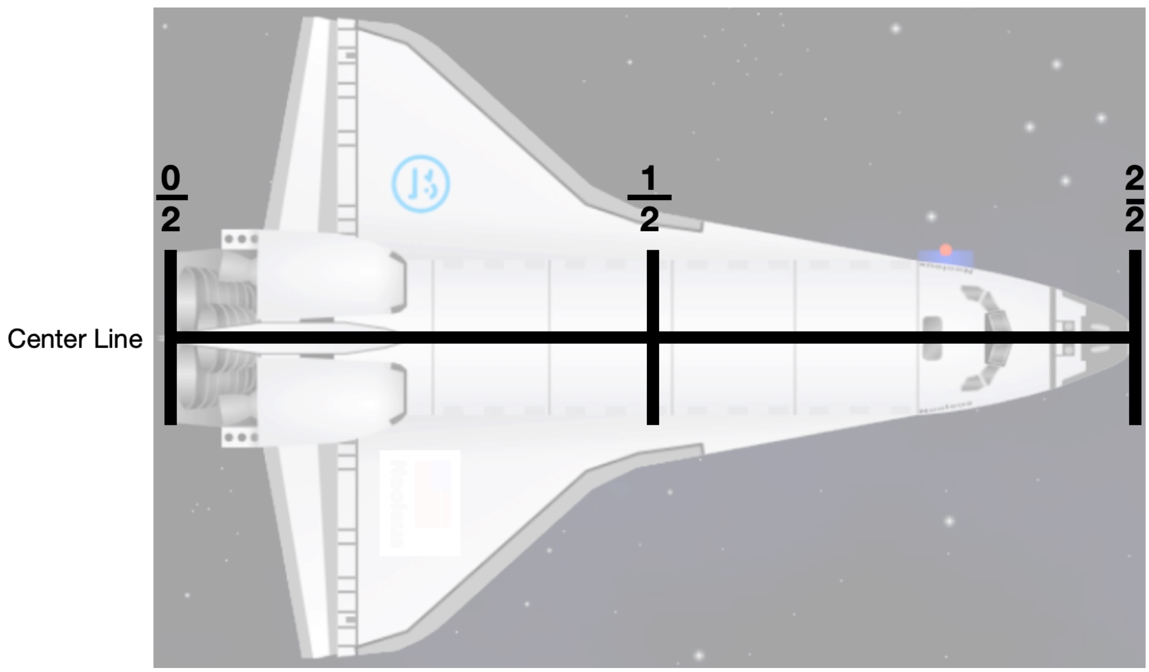


2. *Students' responses may vary. They should describe how they located 1/3. First, they needed to divide the spaceship into 3 equal pieces, as that is the denominator. To divide the space shuttle into equal pieces, students could use strips of paper, folding, or rulers; they just needed to use a strategy that yielded three equal pieces. Then, the numerator suggests that screw is located one line after 0, so that is where the X should be placed.*

Shuttle Mechanic ______________________________ Date __________________

Opening Task

The space shuttle has been labeled using a number line. When a part of the space shuttle needs repaired, Mission Control tells the astronauts where the problem is located on the number line.



Mission Control radioed: "Screw lose along Center Line, Position 3/4."

Where should the astronaut go to tighten the screw? Mark it on the ship and describe your process below.

____

____

____

____

____

Why is precision important when labeling the space shuttle?

Shuttle Mechanic ______________________________ Date _________________

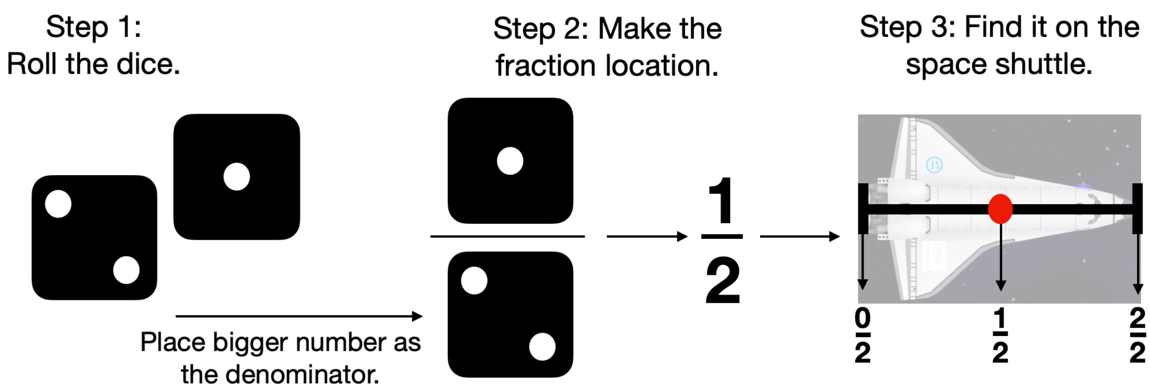
Practice Saving the Shuttle!

Peggy Whitson Tier 1 Lesson

Your goal is to find the shuttle's loose screw as fast as possible. The Mission Control Simulator will provide you the location, and you need to find it on the space shuttle.

For this practice session, the Mission Control Simulator uses two, six-sided dice with the following numbers 1, 1, 2, 2, 3, 6. The larger number is the denominator, and the smaller number is the numerator.

Here is an example:



Let's simulate shuttle repair mission. Find a partner.

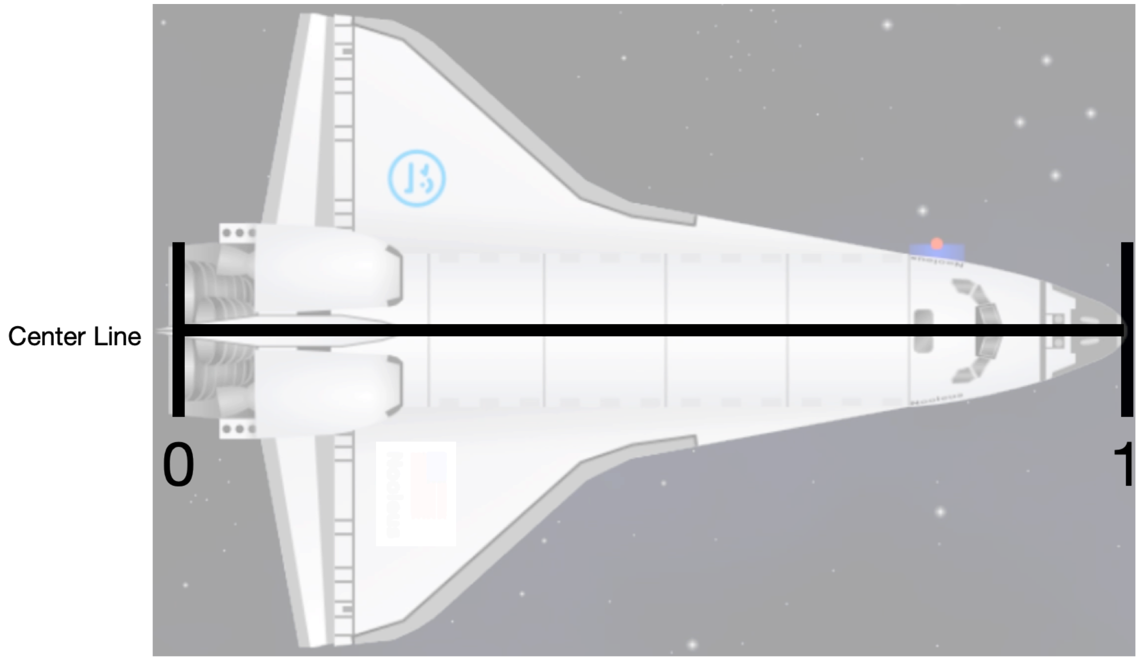
> Partner 1: Roll the dice. Record the fraction location. Share with Partner 2. Start the timer.
> Partner 2: Find the loose screw as precisely and accurately as possible.
> Partner 1: Record the time.
> Work together to answer the questions.
> Then switch roles.

Mission Control Simulation 1

1. Record the two dice: ______ and ______

2. Where is the screw's location? ______

3. Find the loose screw. Mark with an X below.



4. How long did it take to locate it? __________

5. How did you **precisely** locate the loose screw? Include the words “denominator” and “numerator.”

____

____

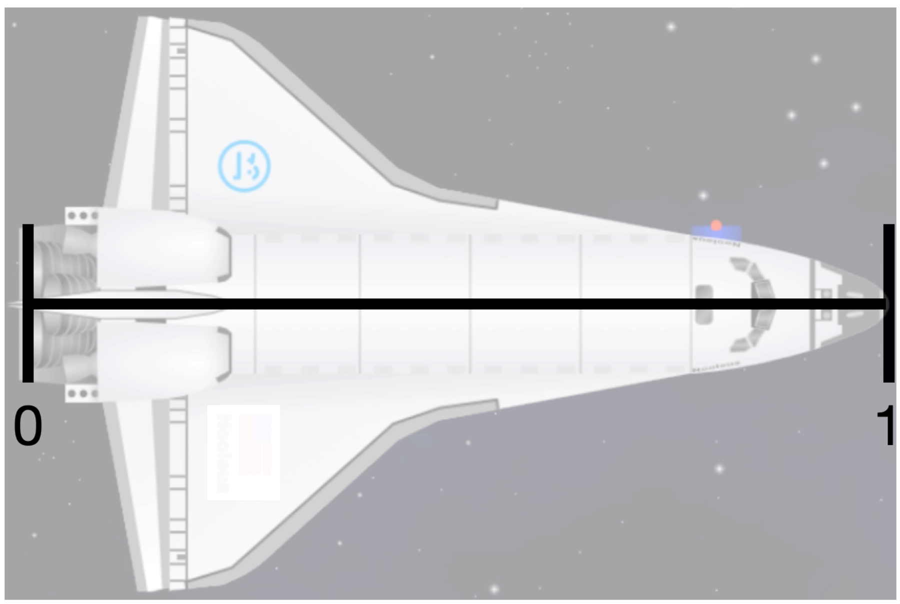
____

____

____

6. How might you label the shuttle ahead of time to help you more quickly locate the screw? Could you label the shuttle to help you locate ANY denominator?

7. How might you use paper clips to help you? You may want to cut out the paper clips below.

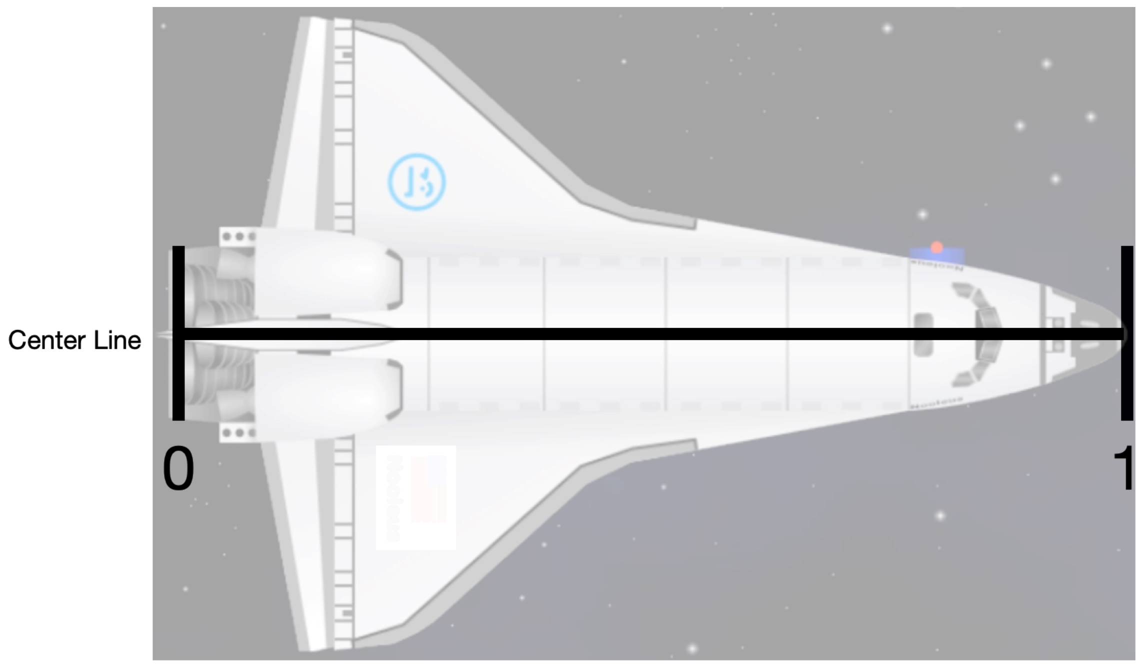


Mission Control Simulation 2—SWITCH ROLES!

8. Record the two dice: ______ and ______

9. What is the fraction location? ______

10. Find the loose screw. Mark with an X below.



11. How long did it take to locate it? __________

12. How did you **precisely** locate the loose screw? Include the words "denominator" and "numerator."

____

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____

____

13. Conclusion: What would be the easiest dice roll for you to locate? Explain your response. (You may use the diagram below if it would help.)

____

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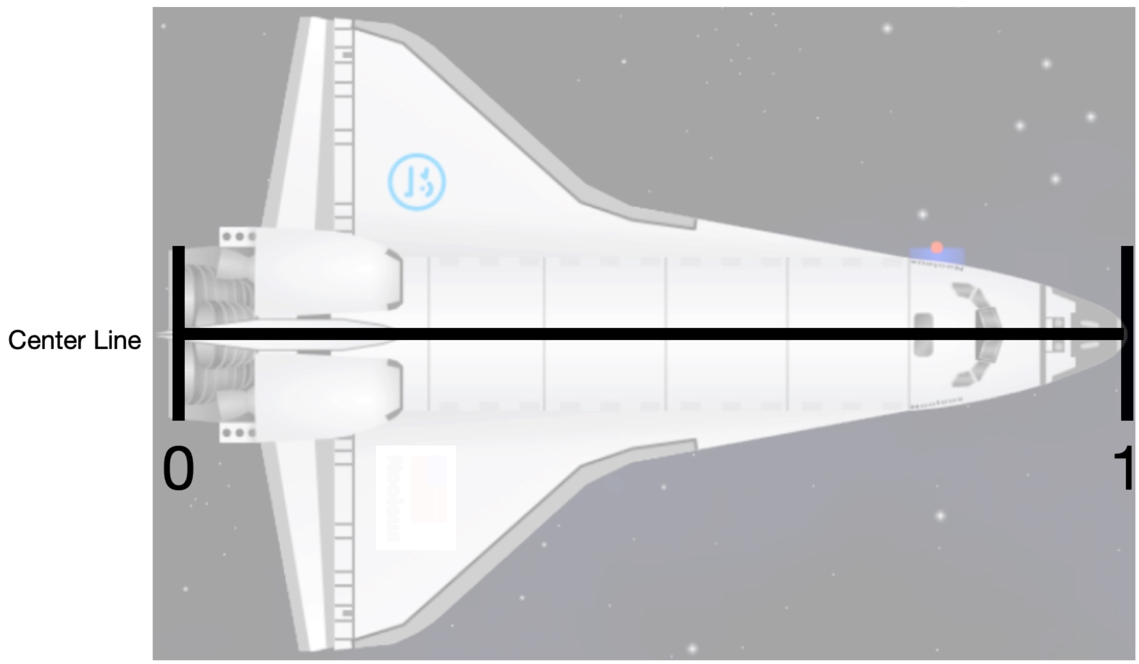


Shuttle Mechanic ______________________________ Date __________________

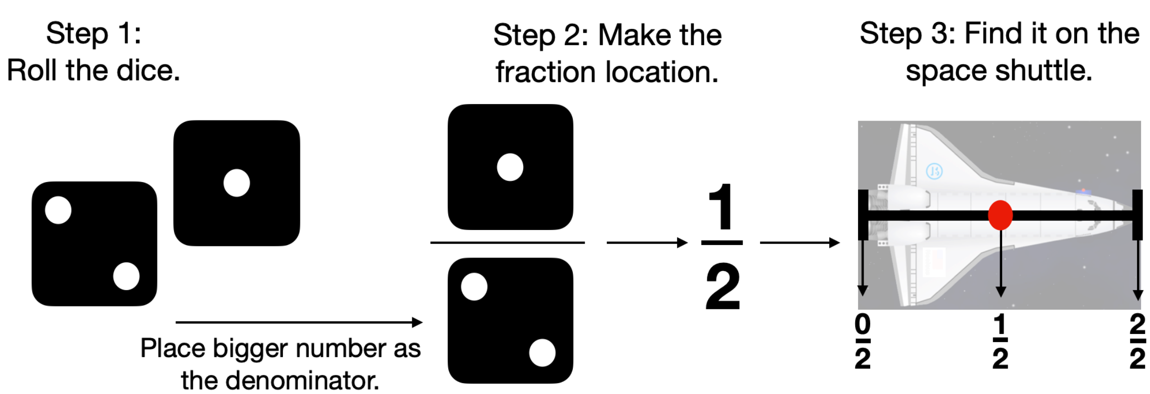
Practice Saving the Shuttle!

Guion Bluford Tier 2 Mission

Your goal is to find the shuttle's loose screw as fast as possible. The Mission Control Simulator will provide you the location, and you need to find it on the space shuttle.

For this practice session, the Mission Control Simulator uses two, six-sided dice with the following numbers 1, 1, 2, 2, 3, 6. The larger number is the denominator, and the smaller number is the numerator.

Here is an example:



Let's simulate a shuttle repair mission. Find a partner.

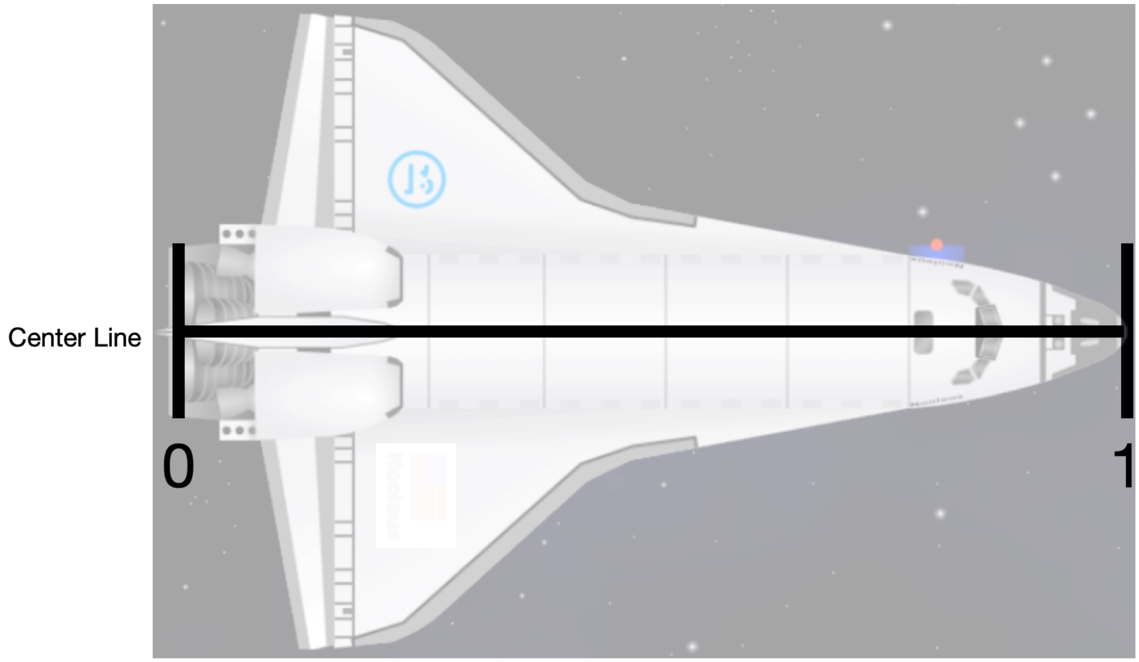
> Partner 1: Roll the dice. Record the fraction location. Share with Partner 2. Start the timer.
> Partner 2: Find the loose screw as precisely and accurately as possible.
> Partner 1: Record the time.
> Work together to answer the questions.
> Then switch roles.

Simulation 1

1. Record the two dice: ______ and ______

2. What is the fraction location? ______

3. Find the loose screw. Mark with an X below.



4. How long did it take to locate it? __________

5. How did you **precisely** locate the loose screw? Include the words “denominator” and “numerator.”

____

____

____

____

____

6. How might you **label** the shuttle to help you more quickly locate the screw?

____

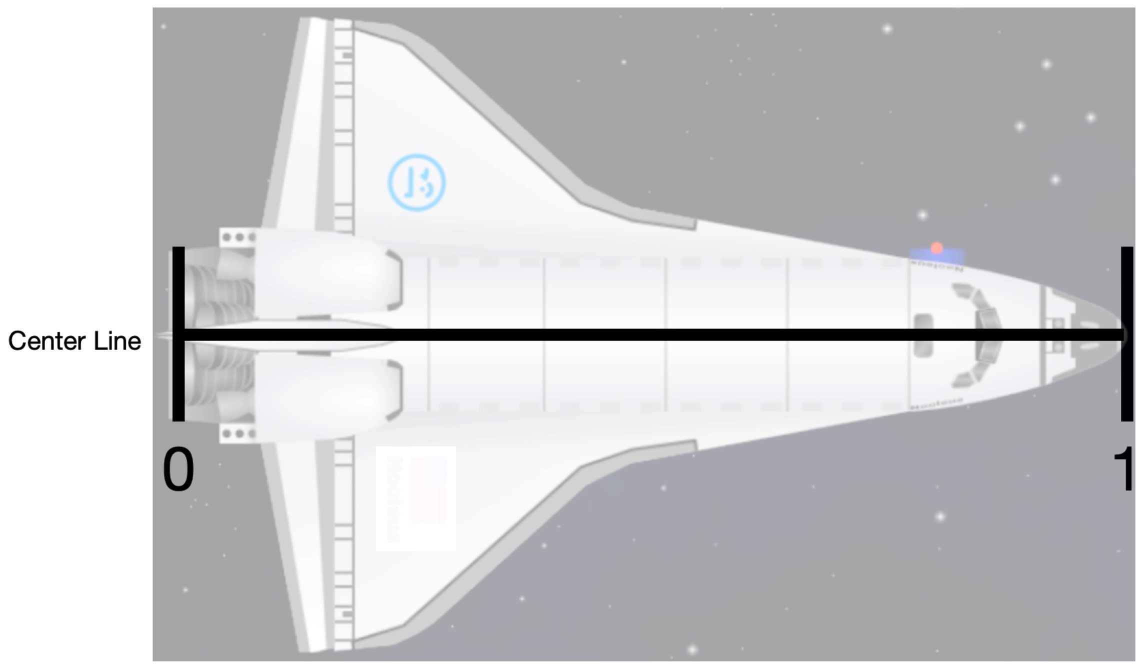
____

____

____

____

7. What tool(s) might you use to help you label your shuttle?

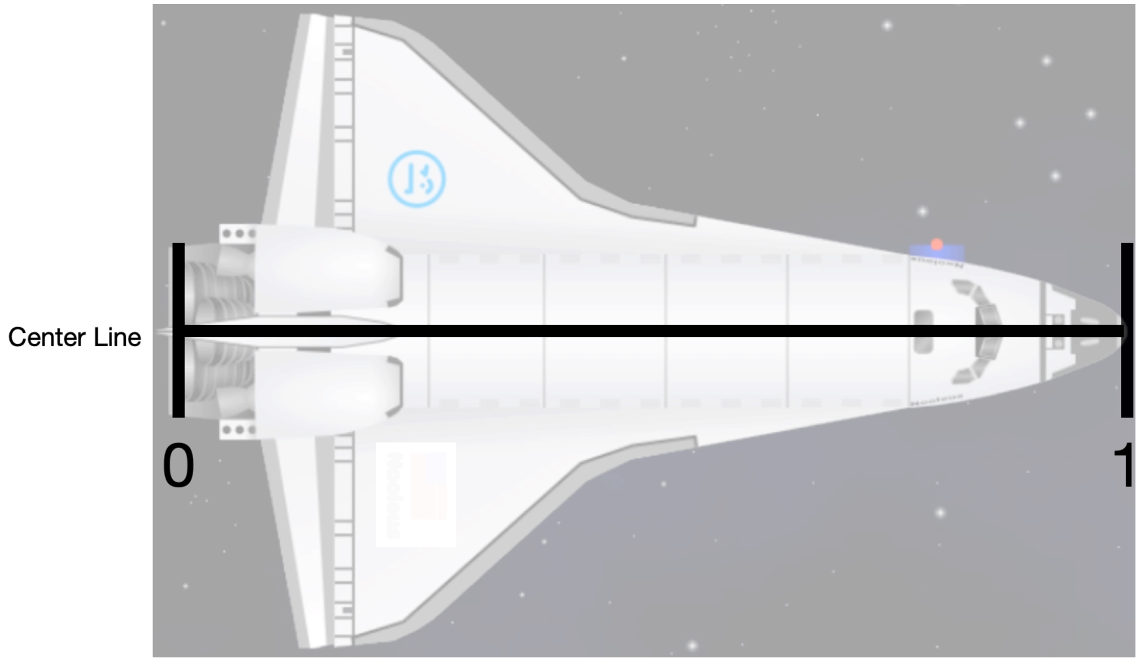


Simulation 2—SWITCH ROLES!

8. Record the two dice: ______ and ______

9. What is the fraction location? ______

10. Find the loose screw. Mark with an X below.



11. How long did it take to locate it? __________

12. How did you **precisely** locate the loose screw? Include the words “denominator” and “numerator.”

____

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13. Conclusion: What would be the easiest dice roll for you to locate? Explain your response.

____

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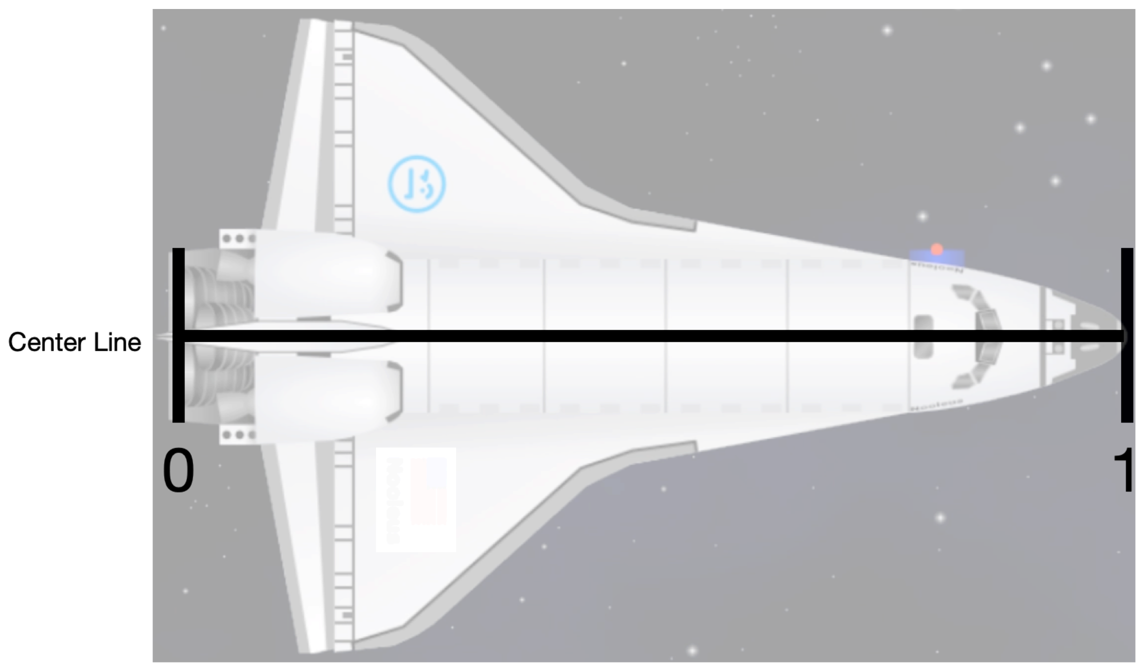
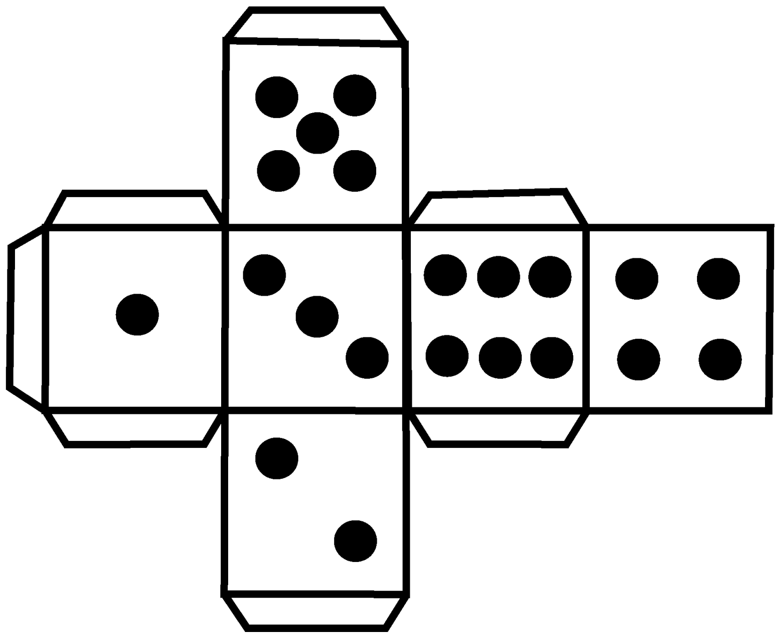
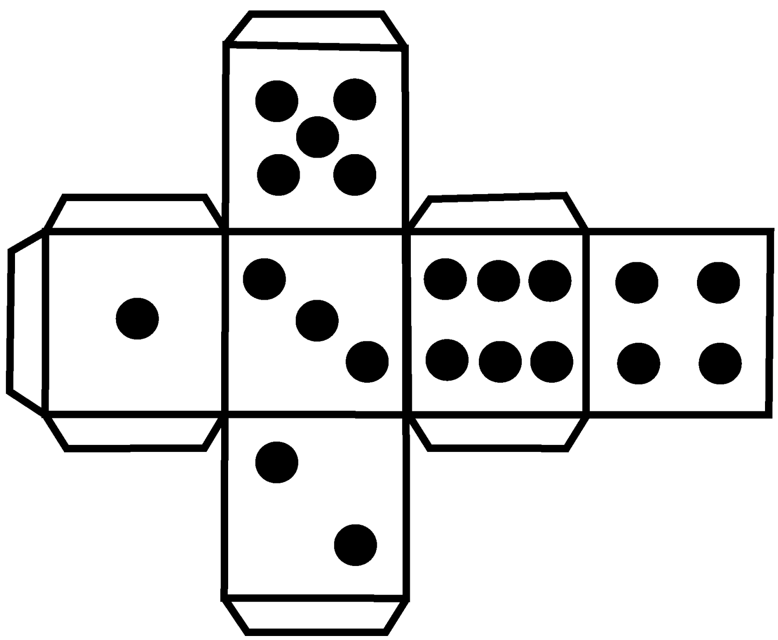


Peggy Whitson and Guion Bluford
Tier 1 and 2 Dice





(Source: https://commons.wikimedia.org/wiki/File:Dice_template.svg)

Shuttle Mechanic ______________________________ Date __________________

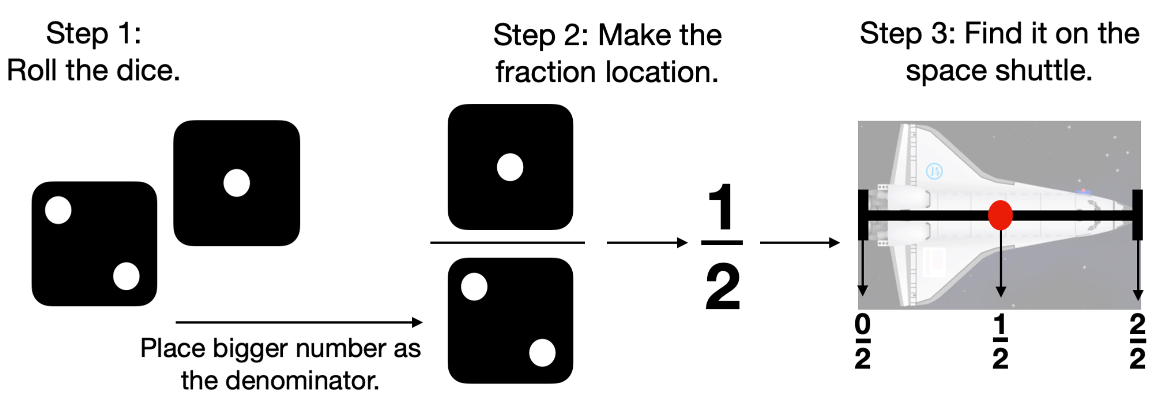
Practice Saving the Shuttle!

Ellen Ochoa Tier 3 Lesson

Your goal is to find the shuttle's loose screw as fast as possible. The Mission Control Simulator will provide you the location, and you need to find it on the space shuttle.

For this practice session, the Mission Control Simulator uses two, six-sided dice with the following numbers 1, 2, 2, 3, 5, 6. The larger number is the denominator, and the smaller number is the numerator.

Here is an example:



Let's simulate another shuttle repair mission. Find a partner.

> Partner 1: Roll the dice. Record the fraction location. Share with Partner 2. Start the timer.
> Partner 2: Find the loose screw as precisely and accurately as possible.
> Partner 1: Record the time.
> Work together to answer the questions.
> Then switch roles.

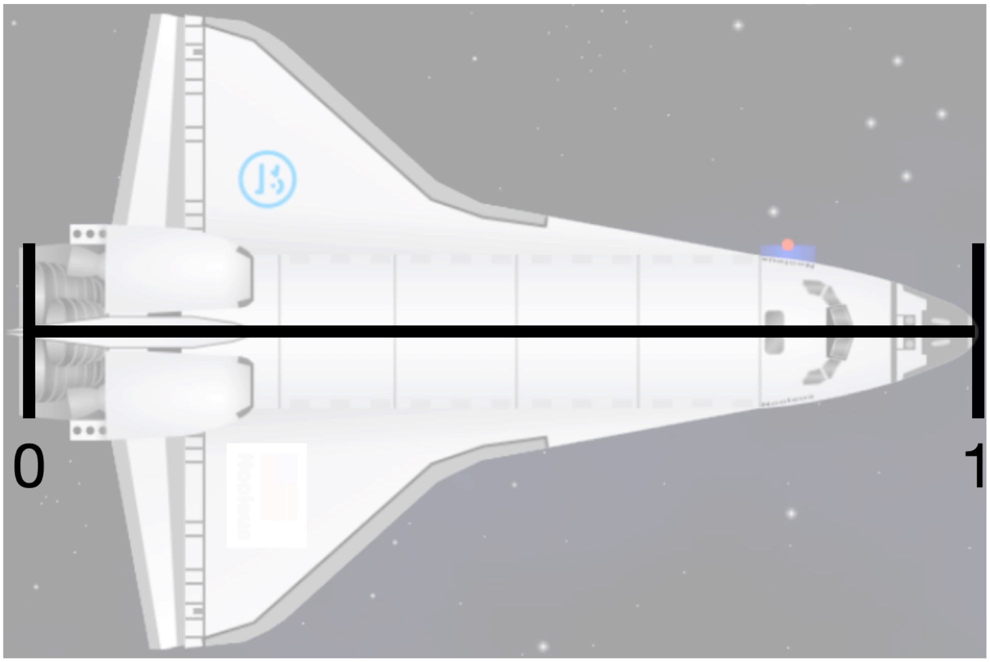
Simulation 1

1. Record the two dice: ______ and ______

2. What is the fraction location? ______

3. Find the loose screw. Mark with an X below.

Center Line



4. How long did it take to locate it? __________

5. How did you **precisely** locate the loose screw? Include the words "denominator" and "numerator."

____

____

____

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____

6. How might you improve your speed? How could you locate the loose screw faster?

____

____

____

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____

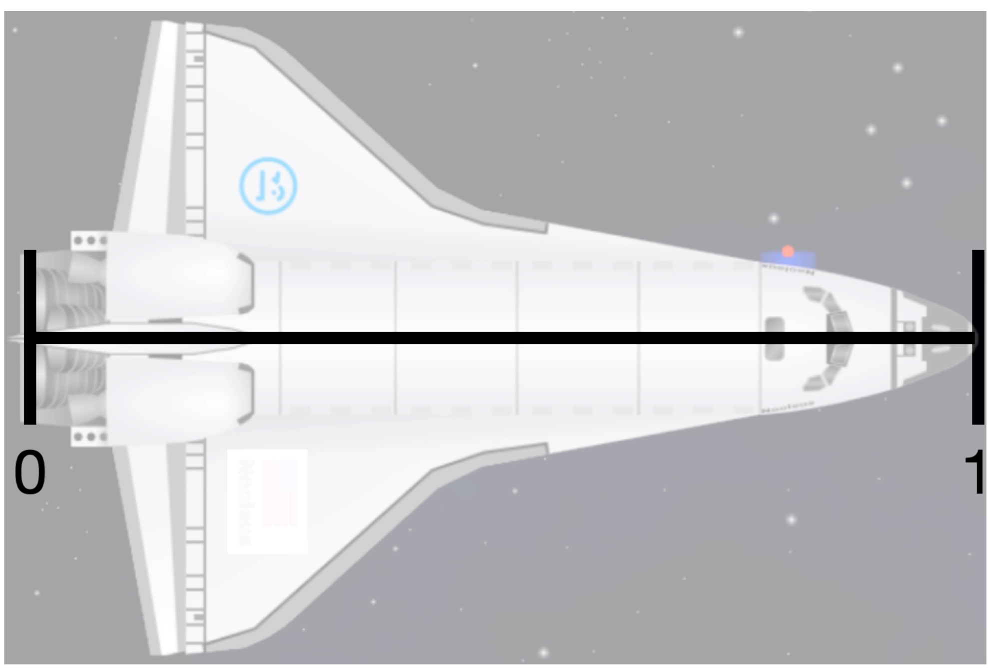
Simulation 2—SWITCH ROLES!

7. Record the two dice: ______ and ______

8. What is the fraction location? ______

9. Find the loose screw. Mark with an X below.

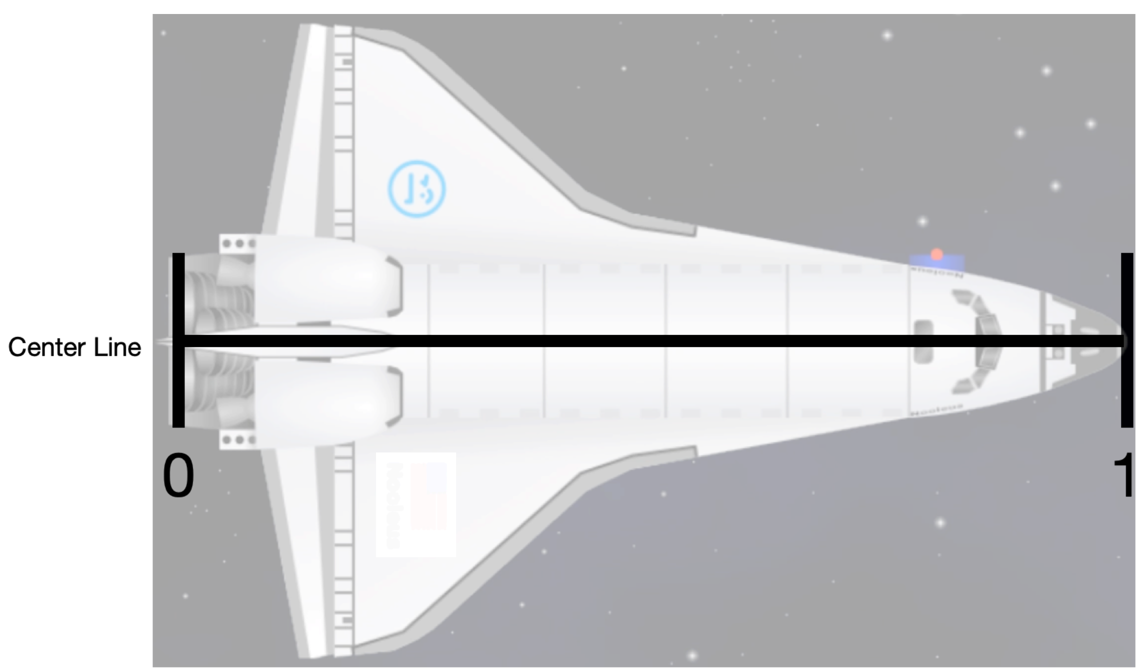
Center Line



10. How long did it take to locate it? __________

11. How did you **precisely** locate the loose screw? Include the words “denominator” and “numerator.”

12. How might you label locations along the shuttle such that you could find any loose screw quickly?



13. How many parts would you need to label? How did you determine that?

____

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____

14. Conclusion: What would be the easiest dice roll for you to locate? Explain your response.

____

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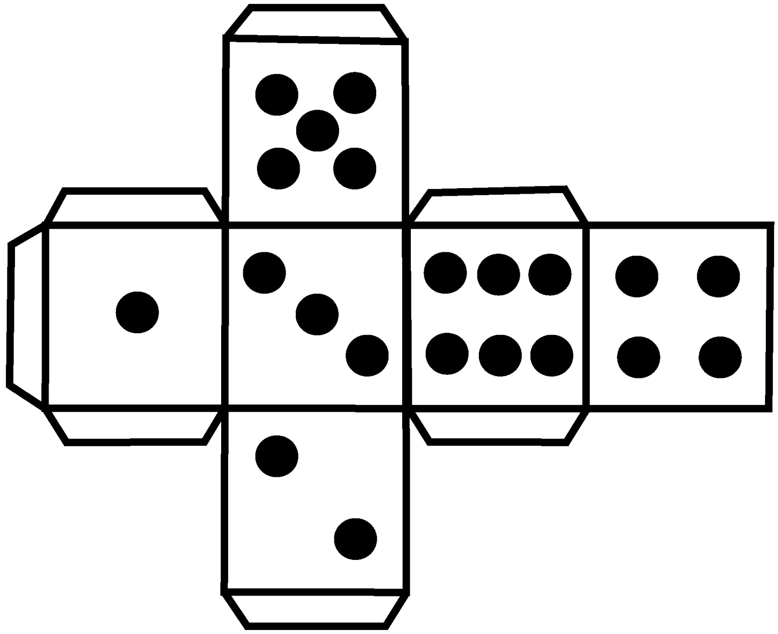
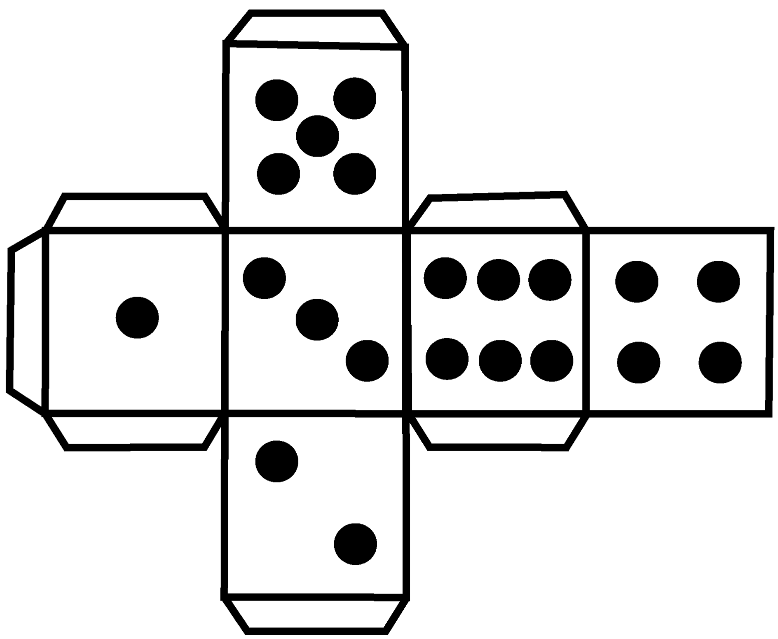
(Source: All space shuttle images: https://spaceagency.fandom.com/wiki/Space_Shuttle?file=Sketch-1537542336514.png)

Ellen Ochoa
Tier 3 Dice





(Source: https://commons.wikimedia.org/wiki/File:Dice_template.svg)

Mission to Mars Student Pages with Answer Keys

Peggy Whitson Tier 1 Lesson

1. *Answers will vary based on dice roll.*

2. *Answers will vary, but the larger number should be the denominator, and the dice numbers should be written in fraction form.*

3. *Answers will vary.*

4. *Answers will vary.*

5. *Students' responses may vary. They should describe how they located the specific fraction they rolled. First, they needed to divide the spaceship into (the larger number) equal pieces, as that is the denominator. To divide the space shuttle into equal pieces, students could use strips of paper, folding, rulers or any number of other tools; they just needed to use a strategy that yielded the appropriate number of equal pieces. Then, students should use the numerator to locate how may parts away from 0 the screw is located.*

6. *Labeling the number line will prevent students taking time to do it after the roll. The tricky thing is to consider all the possible denominators: 1, 2, 3, and 6. Is there an efficient way to label it so that they could find any fraction? This is a challenge question that should prompt a conversation about common denominators, even if that term is not introduced. If split into 6 pieces, any fraction could be found relatively easily. In other words, if every 6 piece was labeled, any fraction could be found.*

7. *The paper clip manipulatives should provide additional scaffolding to help students arrive at the conclusion presented in question 6. The paper clips are already sized at 1/6, so if they label the line with sixths, they should be able to find any fraction.*

8. *Answers will vary based on dice roll.*

9. *Answers will vary, but the larger number should be the denominator, and the dice numbers should be written in fraction form.*

10. *Answers will vary.*

11. *Answers will vary. But, hopefully, if they used the labels, their time should improve.*

12. *Students' responses may vary. They should describe how they located the specific fraction they rolled. First, they needed to divide the spaceship into (the larger number) equal pieces, as that is the denominator. To divide the space shuttle into equal pieces, students could use strips of paper, folding, rulers or any number of other tools; they just needed to use a strategy that yielded the appropriate number of equal pieces. Then, students should use the numerator to locate how may parts away from 0 the screw is located.*

13. *Answers will vary. One possible answer is that it is easiest if both the numerator and denominator are the same. This will result in simply going to the front of the ship because when the numerator and denominators are the same, the fraction is equal to one. Students may not arrive at that answer, but as long as they provide some reasoning, it could be counted correct. However, if they don't see the ease of 1, this should be highlighted in the debriefing conversation.*

Guion Bluford Tier 2 Lesson

1. *Answers will vary based on dice roll.*

2. *Answers will vary, but the larger number should be the denominator, and the dice numbers should be written in fraction form.*

3. *Answers will vary.*

4. *Answers will vary.*

5. *Students' responses may vary. They should describe how they located the specific fraction they rolled. First, they needed to divide the spaceship into (the larger number) equal pieces, as that is the denominator. To divide the space shuttle into equal pieces, students could use strips of paper, folding, rulers or any number of other tools; they just needed to use a strategy that yielded the appropriate number of equal pieces. Then, students should use the numerator to locate how may parts away from 0 the screw is located.*

6. *Labeling the number line will prevent students taking time to do it after the roll. The tricky thing is to consider all the possible denominators: 1, 2, 3, and 6. Is there an efficient way to label it so that they could find any fraction? This is a challenge question that should prompt a conversation about common denominators, even if that term is not introduced. If split into 6 pieces, any fraction could be found relatively easily. In other words, if every 6 piece was labeled, any fraction could be found.*

7. *In this second tier, students are not provided the paper clip manipulatives; although that is one approach they may use on their own. The paper clips are already sized at 1/6, so if they label the line with sixths, they should be able to find any fraction. Students in this tier, however, should be encouraged to explore multiple tools to label the number line.*

8. *Answers will vary based on dice roll.*

9. *Answers will vary, but the larger number should be the denominator, and the dice numbers should be written in fraction form.*

10. Answers will vary.

11. Answers will vary. Hopefully, if they used the labels, their time should improve.

12. Students' responses may vary. They should describe how they located the specific fraction they rolled. First, they needed to divide the spaceship into (the larger number) equal pieces, as that is the denominator. To divide the space shuttle into equal pieces, students could use strips of paper, folding, rulers or any number of other tools; they just needed to use a strategy that yielded the appropriate number of equal pieces. Then, students should use the numerator to locate how may parts away from 0 the screw is located.

13. Answers will vary. One possible answer is that it is easiest if both the numerator and denominator are the same. This will result in simply going to the front of the ship because when the numerator and denominators are the same, the fraction is equal to one. Students may not arrive at that answer, but as long as they provide some reasoning, it could be counted correct. However, if they don't see the ease of 1, this should be highlighted in the debriefing conversation.

Ellen Ochoa Tier 3 Lesson

1. *Answers will vary based on dice roll.*

2. *Answers will vary, but the larger number should be the denominator, and the dice numbers should be written in fraction form.*

3. *Answers will vary.*

4. *Answers will vary.*

5. *Students' responses may vary. They should describe how they located the specific fraction they rolled. First, they needed to divide the spaceship into (the larger number) equal pieces, as that is the denominator. To divide the space shuttle into equal pieces, students could use strips of paper, folding, rulers or any number of other tools; they just needed to use a strategy that yielded the appropriate number of equal pieces. Then, students should use the numerator to locate how may parts away from 0 the screw is located.*

6. *This is a general question for students to experiment with different approaches. At the end of this tier's work, there is a more targeted series of questions to explore common denominators and labeling. Students may start the discussion here, but if not see the answer to Questions 14 and 15.*

7. *Answers will vary based on dice roll.*

8. *Answers will vary, but the larger number should be the denominator, and the dice numbers should be written in fraction form.*

9. *Answers will vary.*

10. *Answers will vary. Hopefully, if they developed an adaptive strategy in #6, their time should improve.*

11. *Students' responses may vary. They should describe how they located the specific fraction they rolled. First, they needed to divide the spaceship into (the larger number) equal pieces, as that is the denominator. To divide the space shuttle into equal pieces, students could use strips of paper, folding, rulers or any number of other tools; they just needed to use a strategy that yielded the appropriate number of equal pieces. Then, students should use the numerator to locate how may parts away from 0 the screw is located.*

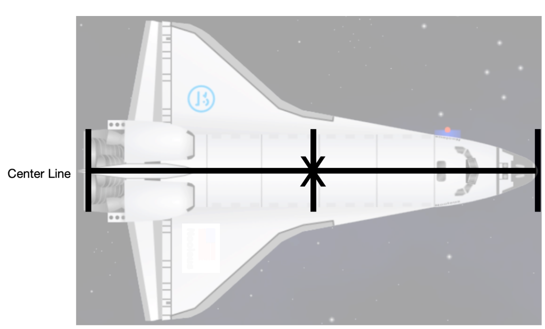
12 ***AND*** *13. Labeling the number line will prevent students taking time to do it after the roll. The tricky thing is to consider all the possible denominators: 1, 2, 3, 5, and 6. Is there an efficient way to label it so that they could find any fraction? This is a challenge question that should prompt a conversation about common denominators, even if that term is not introduced. This tier is using different dice, so unlike the other tiers, splitting it into 6 pieces will not account for fifths. So, the most efficient label would be 30. Or students may think of additional ways of labeling. If you notice multiple lines drawn on top of each other, ask if there is a way they could split it into equal pieces, yet find every fraction using the same equal pieces. Students should also explore how to increase speed with different tools such as paper clips or paper strips.*

14. *Answers will vary. One possible answer is that it is easiest if both the numerator and denominator are the same. This will result in simply going to the front of the ship because when the numerator and denominators are the same, the fraction is equal to one. Students may not arrive at that answer, but as long as they provide some reasoning, it could be counted correct. However, if they don't see the ease of 1, this should be highlighted in the debriefing conversation.*

Hint Cards for Communicating Precise Locations on a Number Line

Hint 1 What is the definition of a fraction?	**Hint 2** How can you divide the line into EQUAL parts?
Hint 3 What **tools** might you use to divide the line into equal parts? What is your process?	**Hint 4** How might you use folding paper strips to help divide the line into equal parts?

Challenge Cards for Understanding Fractions on Number Lines

Challenge 1 Imagine you have traditional dice that have sides labeled 1, 2, 3, 4, 5, and 6. How many different combinations would lead you to the following location? Do you have all the possible combinations? How do you know? 	**Challenge 2** Imagine you have traditional dice that have sides labeled 1, 2, 3, 4, 5, and 6. Could you label a number line that would allow you to locate any possible fraction? How many equal pieces would be on this number line? How do you know?

Lesson Designer: Lisa DaVia Rubenstein

Lesson 7: Defining/Labeling Number Lines—

Preparing for a Space Walk: Gauging Oxygen, Water, and Fuel

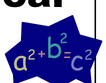
LESSON 7

Big Ideas

Numbers provide a consistent method to communicate a precise quantity, and specifically, fractions are numbers that precisely describe a situation where a whole has been broken up into **equal** parts. The “denominator” communicates how many equal parts there are in the whole, and it is written on the bottom of the fraction. The “numerator” communicates how many of those equal parts are present, and it is recorded at the top of the fraction. In this lesson series, we will be examining number lines that span from 0 to 1. In this case, [0 to 1] is the whole that will be split into equal parts. Fractions can be used to communicate the precise distance between 0 and 1. Precision is important to establish a common understanding of distance that most closely communicates the true or desired value.

For example, let’s suppose you want to tell your friend how far you are to their house. A fraction could communicate the **relative distance** from your starting point to their house, like “I am halfway there.” The more precise you are; your friend will be better able to estimate when to expect you. When completing a fund raiser, a number line could be used to show how close a group is to meeting their goal. Generally, understanding fractions on number lines will support the measurement of distance, reading gauges, and comparing relative lengths. Number lines are helpful to visualize and compare distances and amounts.

Lesson Objectives	• Students will recognize and describe that a fraction communicates when a whole unit is divided into **equal parts**. In this lesson, students will conceptualize the whole unit as the distance between 0 and 1 on a number line. • Students will break down the distance between 0 and 1 into equal parts. • Students will use mathematical language to communicate precisely with others. Students

	provide precise explanations and definitions in their communication. **Common Core State Standard** Develop understanding of fractions as numbers. <u>CCSS.MATH.CONTENT.3.NF.A.2.A</u> Represent a fraction 1/*b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size 1/*b* and that the endpoint of the part based at 0 locates the number 1/*b* on the number line.
Materials 	Students should have access to graph paper, tiles, fraction rods, paper clips, paper strips, scissors, scrap paper, and any other available materials. Students may not choose to build their responses using these tools, but in general, mathematicians have various tools at their disposal that they can use to test their hypotheses.
Mathematical Terms 	• **Denominator:** bottom number in a fraction that identifies the number of pieces the whole has been divided into equal parts • **Equal:** the same portion, piece, or segment • **Fraction:** a number that represents part of a whole • **Number Line:** a line with numbers placed in their correct position • **Numerator:** top number in a fraction that identifies the number of equal pieces considered as part of the whole • **Precise:** describes responses that are exact, accurate, careful about details
Selected Mathematical Practices 	• MP1: Make sense of problems and persevere in solving them. *I never give up on a problem and I do my best to get it right.* • MP2: Reason abstractly and quantitatively. *I can solve problems in more than one way.* • MP3: Construct viable arguments and critique the reasoning of others. *I can explain my math thinking and talk about it with others.*

	• MP5: Use appropriate tools strategically. *I know how to choose and use the right tools to solve a math problem.* • MP6: Attend to precision. *I can work carefully and check my work.*
Differentiation 	**Content** ***Guiding Questions*** • prior knowledge or learner readiness *What evidence do you have about students' current knowledge and skills?* • tiered activities *How will you design tiered activities on the same mathematical concept with varied levels of difficulty?* • formative assessment *What techniques will you use to assess students' prior knowledge and skills?* • varied levels of challenge *How will you vary the level of difficulty for each tiered activity?* • "teaching up" (aim high, provide scaffolding) *How will you increase the depth, breadth, complexity, and abstractness of lessons to challenge and support student learning?* **Process** ***Guiding Questions*** • questioning strategies *How will you pose and how will you encourage students to pose open-ended, closed-ended, lower-level, and higher-level questions to promote mathematical discourse?* • 4Cs (21st Century Skills) ○ creative thinking *How will you encourage students to "think outside the box" and synthesize information in new, different, and useful ways?* • 4Cs (21st Century Skills) ○ collaboration *How will you encourage students to work with other students and appreciate their contributions to solving problems or making connections to prior work?*

	4Cs (21st Century Skills) - communication *How will you promote students' opportunities to communicate face-to-face, in large and small groups, in online environments, and with print and non-print resources using their oral, written, and non-verbal skills?* - hands-on activities/manipulatives *How will you incorporate activities promoting the use of manipulatives to clarify or illustrate mathematical concepts?* **Product** ***Guiding Questions*** - oral, visual, and written opportunities *How will you encourage students to represent their thinking and problem solving using different communication strategies?* - multiple ways to demonstrate knowledge, understanding, and skills *How will you encourage students to share their understanding of mathematical concepts and skills?* - multiple models and representations *What techniques of lesson design will you include to support students' deep understanding and the ability to apply mathematical concepts and skills?* - summative assessment *How will you assess student learning upon completion of the lesson?* **Learning Environment** ***Guiding Questions*** - flexible grouping *How will you use your tiered lesson to support flexible grouping?* - whole group/small group/individual instruction *How will you incorporate different grouping plans to address students' learning needs?*

Lesson Preview

The content goal of this lesson is to establish fractions require **equal** parts, and specifically, a number line must be broken into **equal** parts to precisely communicate distance between 0 and 1. As students are preparing for a spacewalk, they must be able to solve unanticipated problems, and in this lesson, they are provided a scenario in which their gauge readers fail, and they must be able to communicate how much oxygen, water, and fuel are present in their space suit to Mission Control. The mathematical practice emphasis is communicating precisely to others.

Launch

1. Thinking like Mathematicians: Centering the Mathematical Practice

CCSS.Math.Practice.MP6: Attend to Precision

Within this lesson, students will be developing the following mathematical practices:

- Precise communication by specifying how many equal parts are present.
- Precise use of tools and visual representations (or other strategy) of equal parts.
- Determining the degree of precision appropriate for specific contexts.
- Use of clear definitions of fractions, denominators, and numerators in discussion with others and in their own reasoning.

Explain: Mathematicians often need to be precise in their answers. Imagine you received a phone call, informing you that you won a major contest! The person said, “You won our third-grade student of the year award that comes with a cash prize that is somewhere between $1 and $1 million!

Would you be excited? Why or why not? [Gather student responses.]

Explain: Honestly, I am not sure how I would feel. There is a big difference between $1 and 1 million dollars, so I know I would need the person to be a little more precise in their description. Similarly, mathematicians also need to be precise to communicate clearly.

Let’s imagine now the person states you won between $1 and $7.

While you may be disappointed, is that a more precise description of your winnings? [Yes.]

Is it precise enough? [It depends.] Knowing you won between $1 and $7 helps you decide not to buy a private jet. So, it is precise enough to make that decision.

However, if you were trying to decide on a fast-food meal, you might need ever more precision. With $7 you could get a full meal with a drink, fries, and sandwich, and with $1 you would not be able to purchase anything with tax.

Let's brainstorm- in general, **when** would you need to be more precise? When could you be less precise? (If students struggle, you might want to be very precise when building a house so all the walls are straight, but you may want to be less precise in building with blocks.

Ask: How might mathematicians be more precise in their work?

If students struggle to generate ideas, consider mathematicians can be more precise by communicating the context, using clear definitions and units to explain their reasoning, and expressing answers with a degree of precision appropriate for the problem context. They may also use appropriate tools and language to provide more precise responses.

Being precise can be very important for mathematicians as well as for astronauts, which we will see today.

Situating the Lesson Context: Strolling in Space

Explain: In this 3-Lesson Series, we will be practicing and planning for a spacewalk. In the past, several astronauts have had to troubleshoot equipment and spacesuits that were malfunctioning. They must be extremely precise to ensure the success of their missions.

Watch: Let's watch a clip of actual astronauts completing a spacewalk. This is a condensed 15-minute clip of an actual spacewalk outside of the International Space Station that took close to 8 hours!

https://youtu.be/qStW1FysHLY

While you are watching, look for examples of **precise communication**. (EVA is an acronym that stands for **E**xtra **V**ehicular **A**ctivity, so EV1 is the first person out of the vehicle/station.

Ask: How were the astronauts precise in their work?
Potential examples:

- As they were leaving the station, they reported information on all their gauges, for example their suit gauges (e.g., 4.4 for EV2).
- There were specific numbers along the side of the station. As they moved down the station, there were markers spaced equidistant from each other. These numbers helped to give them information on where they were on the station. They stated they were going to the "P6 Truss site." They label all the sites.

- While they were working, they were given specific instructions on which bolt needed tightened and how many rotations were necessary (e.g., 17.9 turns on the Nader bolt).
- In general, both Mission Control and the astronauts were precise on all their instructions.

Explain: These astronauts demonstrated extreme precision with their communication. When might astronauts not be so precise? (Sample responses may include the amount of toothpaste they use or the number of minutes they sleep. Astronauts may round to the nearest whole number in those cases. Often, however, astronauts are extraordinarily precise, even in their daily routines.)

Why is precision important for astronauts? While everything went well for these astronauts, that is not always the case. For example, when Italian astronaut Luca Parmitano was on a spacewalk, water started to fill his spacesuit helmet. He had to navigate his way back to the hatch while water was flooding in. After that event, NASA started to run considerable tests on the helmets to fix the issue. They needed to replicate the problem. Below you can see the empty spacesuit helmet in an Aug. 27, 2013 test of the faulty spacewalking gear. This water leak confirmation helped NASA engineers devise repair methods for the spacesuit. If one component is slightly off, it may be a matter of survival.

(Source: APPEL News Staff listed below in References)

References

APPEL News Staff. (2014, February 27). *Mishap investigation board briefing on spacesuit water intrusion*. https://appel.nasa.gov/2014/02/27/mishap-investigation-board-briefing-on-spacesuit-water-intrusion/

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Current Task
Explain: For today's Mission to Mars task, we are going to practice **precisely** communicating and understanding data from our space suit gauges. oxygen and water gauges. This will ensure our spacewalks will be successful.

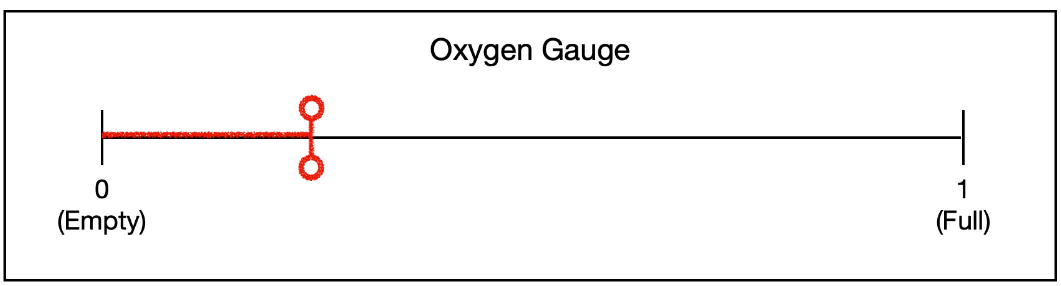
Explore

2. Communicating Information From Number Lines

Use this introductory task to place students in appropriate differentiated groups.

Say: When astronauts prepare for their spacewalk, they check their equipment. They examine their oxygen tank gauge. This gauge tells them how much oxygen they have in their suit.

Let's imagine you are preparing for a spacewalk. Strangely, the numbers had worn off your gauge. How would you communicate to Mission Control how much oxygen is in your tank?



Encourage students to write down ideas in their journals and share with their neighbors. After students have this entry discussion, start to probe their thinking by asking for evidence.

Differentiated Examination of Additional Options
During this introductory task discussion, teachers should look for developmental levels of two key concepts to demonstrate students' readiness levels:

- **Fraction Concept:** The correct answer is 1/4 (or an equivalent fraction, like 2/8), but it is more important that students display the concept of equal parts. They need to demonstrate that the number line needs to be broken into equal parts to determine and communicate the fraction. They also need to use the total parts as the denominator, and the level of oxygen present as the numerator.

- **Mathematical Practice 6:** Students should demonstrate the precise use of tools and visual representations of equal parts. They may use a variety of tools, anything from a ruler, paper clips, or folding the paper, yet anything used must be used as a method for communicating equal parts. They should be able to use to precise mathematical terms: fraction, denominator, and numerator appropriately by the end of this lesson.

In this investigation, students will be working on one of the Student Pages in their differentiated groups based on readiness levels. The groups are based on teacher's observations of students' conceptual understanding and mathematical practice acumen as described above.

- **Tier 1:** Students who do not demonstrate a conceptual understanding of fractions on number lines should be placed in Tier 1.
- **Tier 2:** If students demonstrate a vague sense of fractions (i.e., they try to establish equal parts) but do not demonstrate a specific strategy or fail to arrive at the correct answer, they should be in Tier 2.
- **Tier 3:** If students can communicate the fraction is 1/4 (or an equivalent fraction) and they used a specific strategy, they should be placed in Tier 3.

Explain to students that you are excited to see so many interesting approaches to determine and communicate the level of oxygen. Now, they are going to consider the level of water the astronauts have in their space suits, which is also key for astronaut survival on a spacewalk.

Groups Formed by Student Readiness		
Tier 1: Peggy Whitson	**Tier 2: Guion Bluford**	**Tier 3: Ellen Ochoa**
Student Names	**Student Names**	**Student Names**

Collaborate and Communicate
Have students record their ideas for on their individual worksheets or one for the small group. Help them clarify their ideas by asking questions like, “What do you mean here?” and “How might you share that idea with the rest of the class?” Point out that mathematicians use definitions, examples/non-examples, and various representations to help support their conclusions. Below are some possible student responses, and you can record additional ones you observed in your own class in the blank boxes.

A. [Possible response]	**B.** [Possible response]	**C.** [Possible response]
This group . . .	*This group . . .*	*This group . . .*

Examine and Elaborate

3. **Highlight Students’ Mathematical Thinking**
Mathematicians think about possible solutions in a variety of ways. Therefore, it is important for students to realize that they, too, can approach problems using different strategies. Ultimately, students need to understand that a possible solution should be judged by the correctness of the mathematics, and there might even be some valid ideas within a solution when a student has an incorrect answer.

Share and Discuss
After the differentiated groups have an opportunity to explore communicating fractions on number lines, bring the class back together for a full group discussion.

Guiding MP questions for all groups should include:
- What tools could you use to make your conclusion? (MP5: Encourages appropriate use of tools)
- How precise are your tools? (MP6: Attend to precision)
- What is another way you could determine the level of oxygen that would be more precise? Less precise? (MP6: Attend to precision and promotes fluency of thought, which is a component of creativity)

In this discussion, it is important to stress that mathematicians decide to be either more or less precise in their responses based on the situation and materials. In this case, astronauts need to be very precise regarding how much oxygen or water they have, but they may not need to be as precise with how much shampoo they have left.

During this discussion, continue to discuss how these gauges are number lines, how 0 to 1 is the whole that can be broken into equal parts, and then, fractions and fraction notation can be used to describe an amount. Fractions only describe when a whole is divided into EQUAL parts. As they share, connect back to fraction notation, demonstrating how the number line can be divided into equal parts.

Here is a sample conversation on how to bring all the Tiers together to develop mathematical understanding.

Teacher: Let's start with the Whitson group. You had a water gauge that had 2 paper clips. How did you use those paper clips to determine how full the water tank was?
Monroe: The water line covered one out of two paper clips, so we said it was 1/2 full.
Teacher: Interesting. I'm curious how this related to the Bluford Group's first gauge. Can you describe what is different between your gauge and the Whitson gauge?
Avalyn: Yes! We had more paper clips.
Teacher: That is true! Can someone add on to Avalyn's observation? *(Adding On talk move)*
Gerardo: I noticed that the Whitson group's paper clips are the same size, but our paper clips were different sizes.
Teacher: Woah. So why is that important? What about someone from the Ochoa group?
Katie: If the parts are not equal, we cannot use fractions.
Teacher: And why is using fractions important?
Monroe: It helps us to communicate to Mission Control! When we just said 2 paper clips, Mission Control may not understand the size of the paper clips, but if we said 1/2, Mission Control understands how full our tank is.
Teacher: I hear you saying that using fractions helps us to communicate more precisely than paper clips. Is that correct? *(Revoicing talk move)*
Monroe: Yes!

Differentiate Further as Needed

Please see the Hint and Challenge cards at the end of the lesson. The hint cards remind students to incorporate precise language and how fractions require equal parts. The challenge cards begin to probe students thinking on equivalent fractions on number lines.

Debrief and Look Ahead

4. **Debrief Content and Skills**

Remind students that the mathematical practice for this lesson focused on how mathematicians use precision to communicate amounts. Astronauts reading gauges is one example of when precision is important. In this case, students communicate the denominator is how many total EQUAL pieces are in the whole unit, and the numerator indicates how many of those pieces are present. They precisely label the number line with equal pieces using a strategy or tool. Students should be able to communicate their strategy for establishing equal parts.

Assess

5. **What Students Learned**

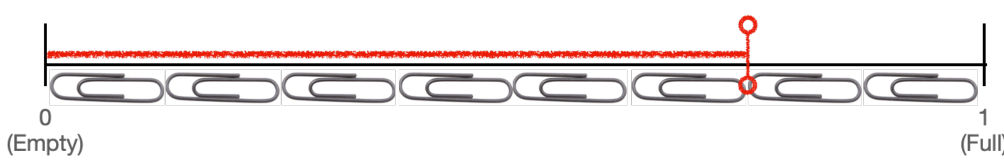
Use the following exit card to assess what students learned from this lesson.

Exit Card

The astronauts need to communicate how much fuel they have.

1. Use the paper clips to **label** the fuel gauge using fractions.

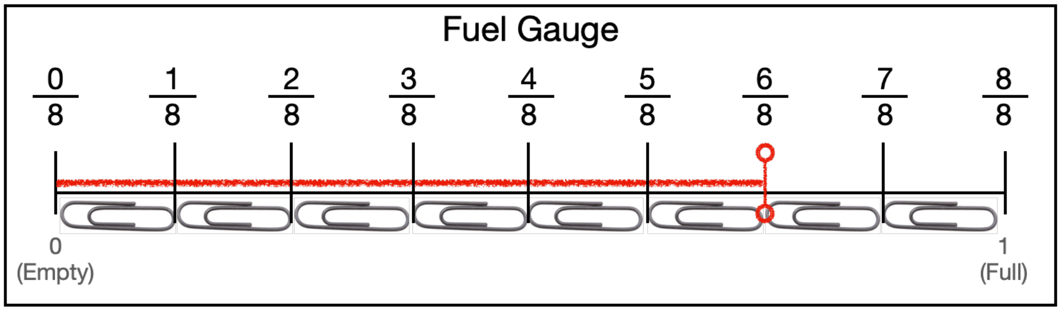
Fuel Gauge



2. How could you **precisely communicate** how much fuel is in the tank? (Use the definition of fractions in your answer.)

Exit Card Answer Key

1. *Students will label the number line like the picture below. (They may use equivalent fractions in some cases.)*



2. *Students will communicate the amount of fuel by explaining the line has been broken down into 8 equal parts, and 6 of them are filled. Therefore, the tank is 6/8 full. Students may also describe there are 6 paperclips worth of fuel, but they need to be able to communicate the fraction form. The fraction is more precise because Mission Control would not need to know the size of the paper or the gauge to determine how much fuel is available if they understand fractions. Using fractions to communicate is more efficient and precise than paper clips. (Although paper clips can help break the whole into equal pieces IF they are the same size.)*

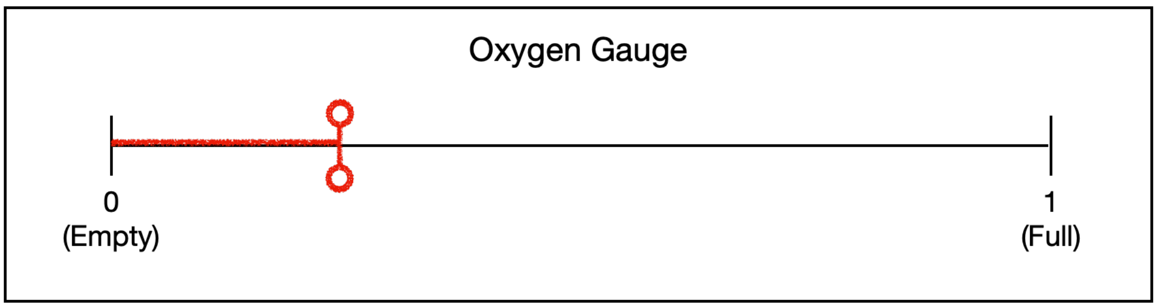
Space Walker ______________________________ Date _________________

Opening Task

You are preparing for your spacewalk and checking your equipment.

Mission Control: Good Morning Team! Please give us an update on your oxygen tank levels.

You examine your oxygen tank gauge to report how much oxygen you have. Strangely, all the markings had worn off! How would you communicate to Mission Control how much oxygen is in your tank?



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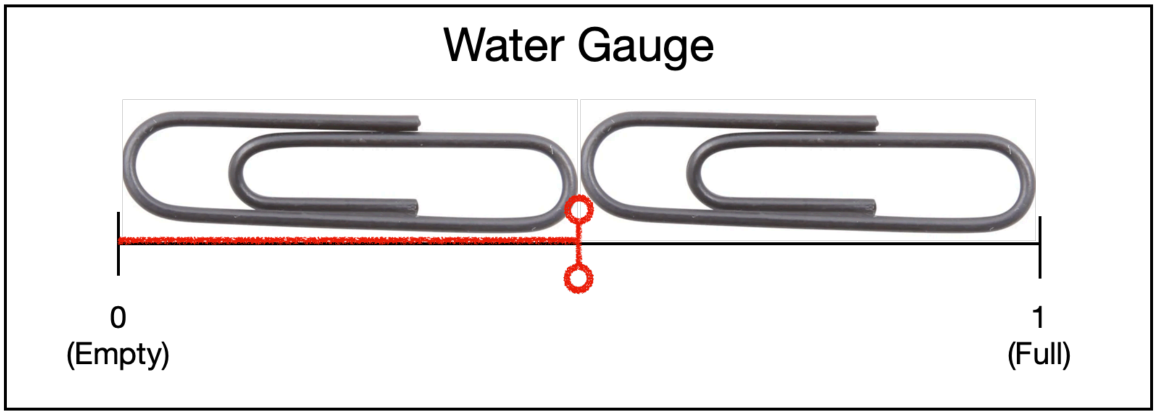
Space Walker ________________________________ Date __________________

Time for a Space Walk!

Peggy Whitson Tier 1 Lesson

Before you can go on your spacewalk, you need to make sure you have enough water in your suit to remain cool and hydrated. Again, Mission Control wants you to report on all your life support systems, but the markings have worn off. Li and Joe discuss using some items you have in the shuttle to describe how much water is in the tank. Joe placed paper clips on the gauge, like this:



Mission Control Conversation

Mission Control: We cannot see your water gauge. Report on level of water remaining.

Joe: Water tank is 1 paper clip past empty.

Mission Control: Copy 1 paper clip past empty.

1. After this conversation, does Mission Control have a **precise** understanding of how much water is in the tank? Explain why or why not. What questions might they ask?

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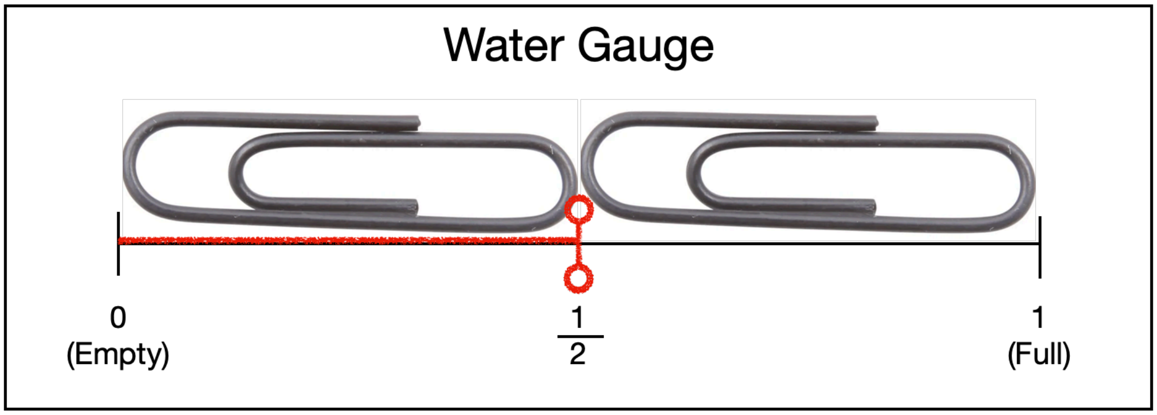
Li examined Joe’s paper clips, and then, she labeled a fraction:



Mission Control Conversation

Mission Control: Could you report a precise fraction of your water levels?

Li: Yes, water tank is 1/2 full.

Mission Control: Copy 1/2 full.

2. Li reported they had 1/2 of a tank of water. Is she correct? Explain why or why not using the prompts below.

Guiding Questions	Your Response
Define: What does 1/2 communicate?	
Use of Tools: How did Li use the paper clips to determine the water tank is half full?	
Evaluate: How could you use other tools to check Li’s conclusion?	

3. Joe communicated the tank was 1 paper clip full. Li reported the tank was 1/2 full. Which astronaut was more precise? Explain. Why is being precise important in this situation?

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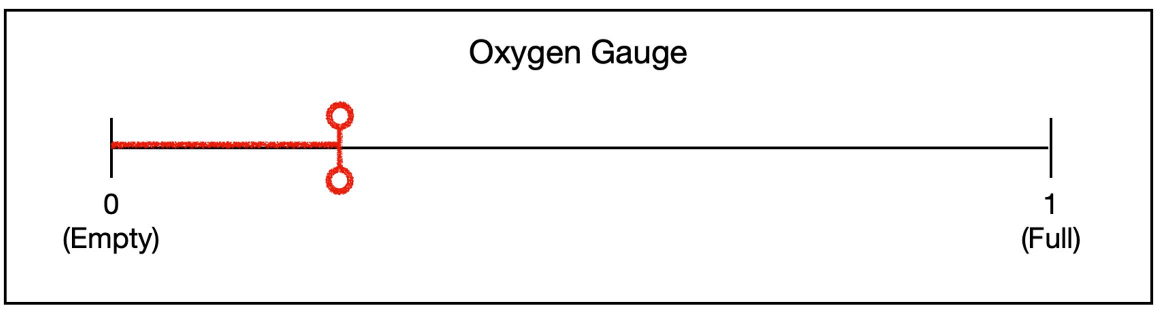
4. Now, back to our opening task. Using the materials you have in the classroom, what is another way you might **precisely** communicate what fraction of the tank is full? How could you check?



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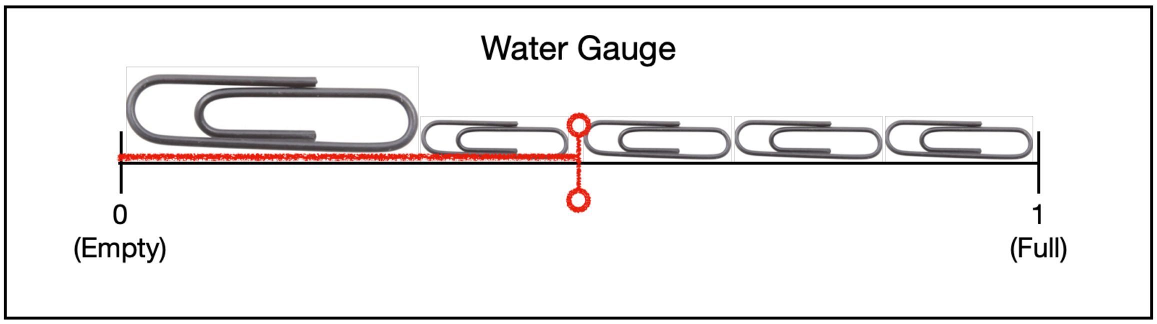
Space Walker ______________________________ Date __________________

Strolling in Space!

Guion Bluford Tier 2 Lesson

Before you can go on your spacewalk, you need to make sure you have enough water in your suit to remain cool and hydrated. Mission Control wants you to report on your water level, but the markings have worn off. Li and Joe discuss using some items you have in the shuttle to describe how much water is in the tank. Joe placed paper clips on the gauge, like this:



Mission Control Conversation

Mission Control: We cannot see your water gauge. Report on level of water remaining.

Joe: Water tank is 2 paper clips past empty.

Mission Control: Copy 2 paper clips past empty.

1. After this conversation, does Mission Control have a **precise** understanding of how much water is in the tank? Explain why or why not. What question should they ask?

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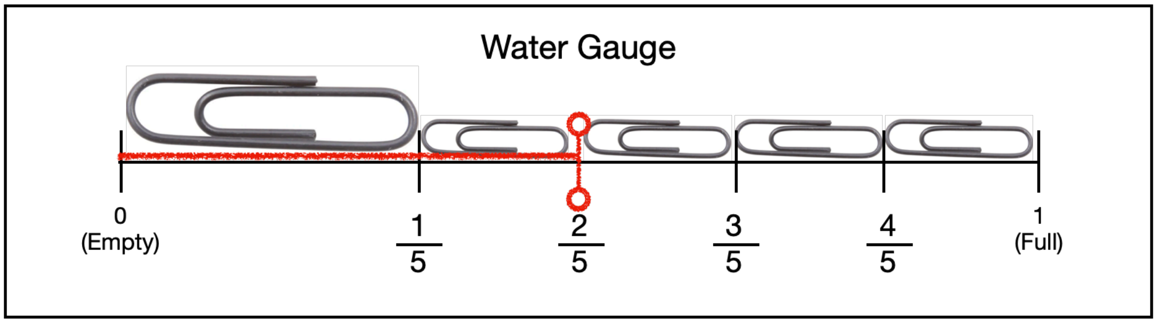
Li looked at Joe's paper clips and added her own marks using fractions:



Mission Control Conversation

Mission Control: Could you report a precise fraction?

Li: Yes, water tank is 2/5 full.

Mission Control: Copy 2/5 full.

2. She reported they had 2/5 of a tank of water. Is she correct? Explain why or why not. (Include a definition of a fraction.)

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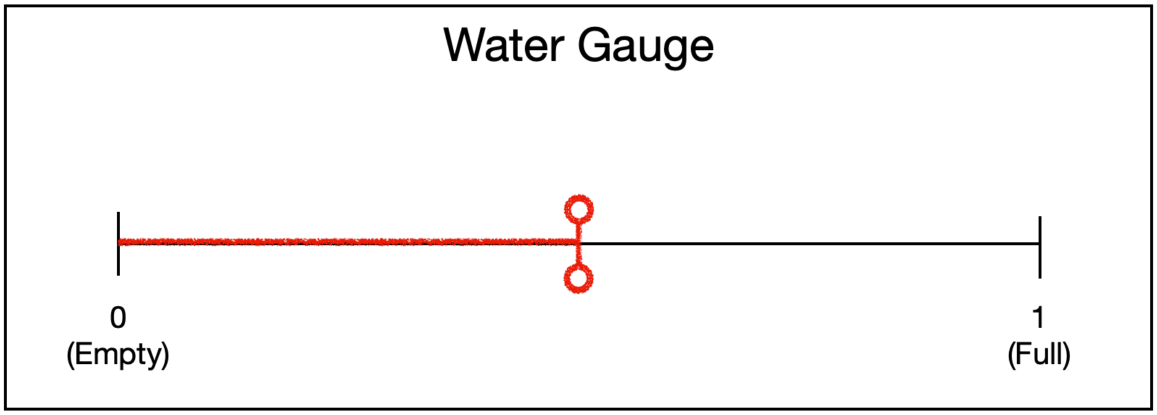
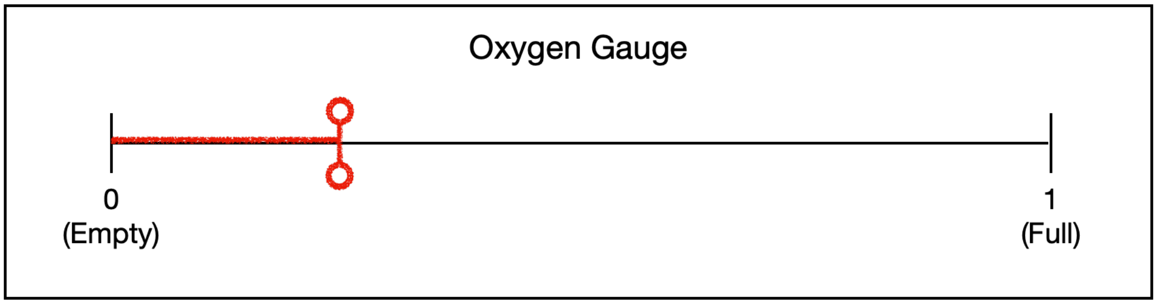
3. What is an additional **more precise** and **accurate** way you might communicate what fraction of the tank is full? What are 2 ways you can demonstrate your conclusion?



4. Now, back to our opening task. Using the materials you have in the classroom, what is another way you might **precisely** communicate what fraction of the tank is full? How could you check?



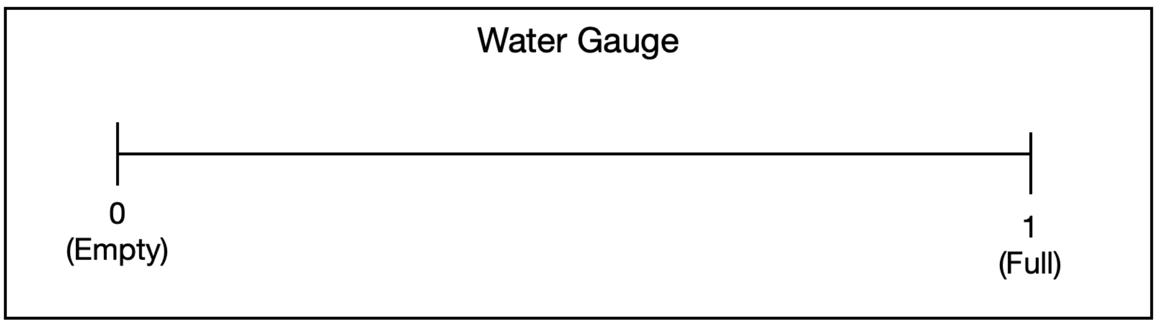
Mission Control Team Member _________________ Date _________________

Strolling in Space

Ellen Ochoa Tier 3 Lesson

Before the astronauts can go for a spacewalk, Mission Control and the astronauts need to know how much water is in the suit's water tank to help the spacewalkers stay cool and hydrated, but the markings have worn off.

Imagine you are in Mission Control. You know they have three sizes of paper clips available to them that they could use to determine how much water they have in their tank. You also know they have 20 paper clips in each size.



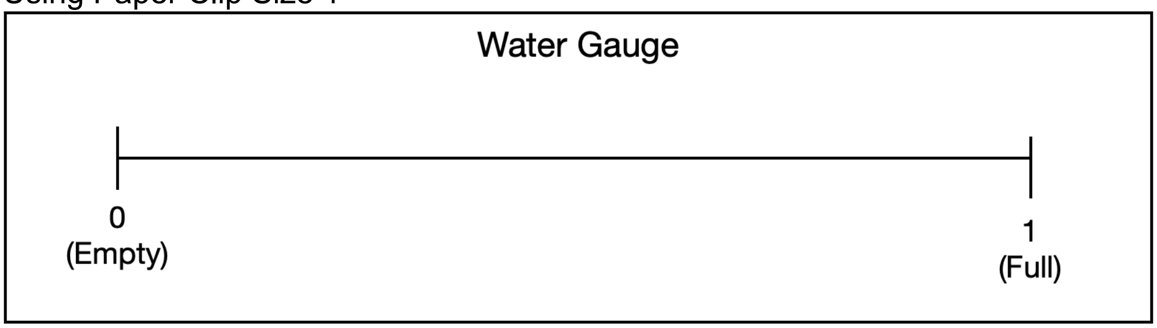
Paper Clip Size 1

Paper Clip Size 2

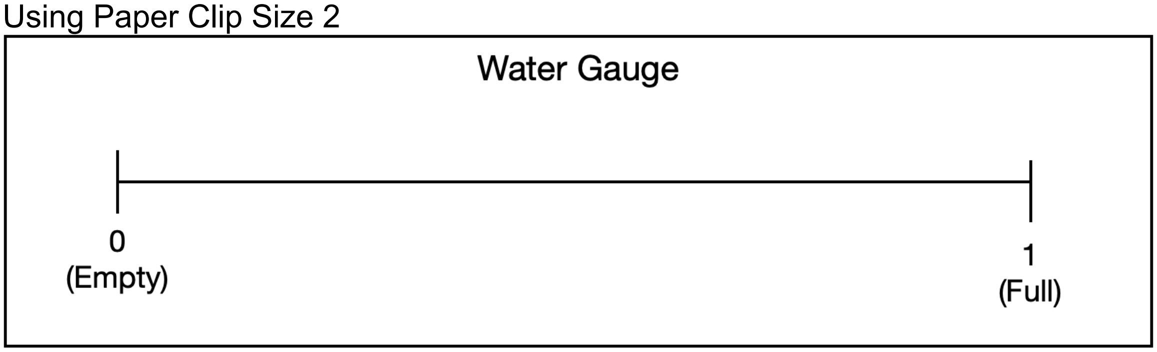
Paper Clip Size 3

1. The astronauts reported that they have 1/2 tank of oxygen. How could they have used any of the paper clip sizes to determine this amount? Explain. Draw each of their options and label the fractions.

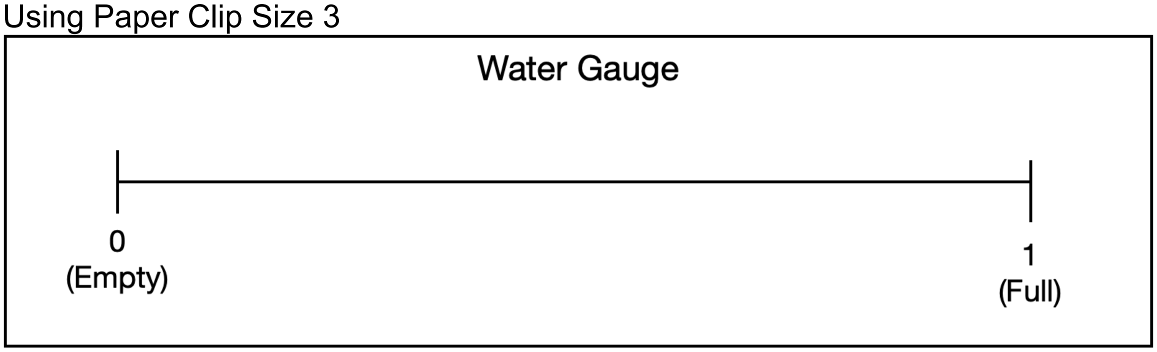
Using Paper Clip Size 1



Using Paper Clip Size 2



Using Paper Clip Size 3



The astronauts successfully completed their first spacewalk, but now they need to report back how much water they have left.

Mission Control Conversation

You: We need the astronauts to report on how much water they now have.

Mission Control, Tina: Tell the astronauts to use Paper Clip Size 3 to report how much water they have.

Mission Control, Devi: No! Wait! They should use Paper Clip Size 2. It is right in the middle.

Mission Control, Wallace: Why would that matter? Tell them Paper Clip Size 1 will be best.

2. As leader of Mission Control, you need to give the astronauts **precise instructions** on how to use their paper clips to report back on their water levels. Explain to the astronauts which paper clip size they should use and why.

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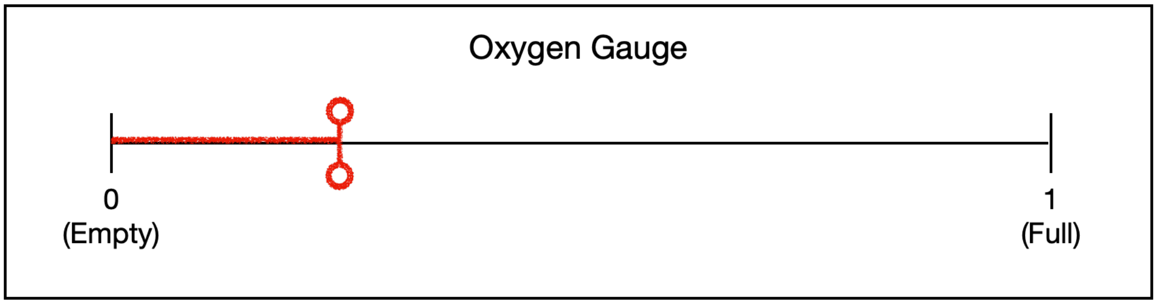
3. Now, back to our opening task. Using your classroom materials:
   a. What are 2 different ways you might **precisely** communicate what fraction of the tank is full?
   b. What makes one of your approaches better or worse than your other approach?



Mission to Mars Student Pages with Answer Keys

General Conceptual Framework
Throughout their responses, students need to incorporate the concept of equality. Fractions are only fractions when the portions are divided equally. The final full class discussion will synthesize across their experiences to demonstrate that fractions describe when a whole, in this case the space on a number line between 0 and 1, is divided into EQUAL parts.

Students should communicate precisely using definitions, examples, different representations, and various tools. They should use precise mathematical language of denominator and numerator to communicate, including the denominator is how many total EQUAL pieces are in the whole unit, and the numerator indicates how many of those pieces are present. They should precisely label the number line with equal pieces using a strategy or tool. They should recognize that fractions are helpful for their ability to communicate precise locations and amounts of parts of a whole.

Opening Task
Fraction Answer: 1/4

- **Fraction Concept:** The correct answer is 1/4 (or an equivalent fraction, like 2/8), but it is more important that students display the concept of equal parts. They need to demonstrate that the number line needs to be broken into equal parts to determine and communicate the fraction.
- **Mathematical Practice 6:** Students should demonstrate the precise use of tools and visual representations of equal parts. They may use a variety of tools, anything from a ruler, paper clips, or folding the paper, yet anything used must be used as a method for communicating equal parts.

Peggy Whitson Tier 1 Lesson

Tier 1 is differentiated as students only need to recognize the paper clips are already equal sizes. They still need to leverage those paper clips to communicate a fraction on a number line. They are also given additional scaffolding to craft their response.

1. *Answers may vary. Students should communicate that Mission Control only has a precise understanding IF they know how long the paper clip is AND how long the gauge is. If Mission Control has that information, then, the method could be precise. As it is, they do not know what "1 paper clip past empty" really means.*

2. *Answers may vary. Define-1/2 is a fraction that communicates the number line was broken into 2 equal pieces and that the tank is filling 1 of those 2 pieces. Use of Tools-Li used the paper clips to break the gauge into those 2 equal pieces and saw that 1 of the 2 were full, so she was able to communicate the tank was 1/2 full. Evaluate-students could check Li's reasoning using a ruler, folding, or any other consistent size tool, like tiles.*

3. *Li is more precise because it communicates the relationship of the part to the whole. Her response does not require knowing the length of the paper clip or of the gauge. It is consistently understood as a fraction. Being precise in this situation is important so Mission Control will be aware of how much water is available for the remaining parts of the spacewalk. Astronauts need water to cool their suits. If they are on their spacewalk without enough water, it may be a matter of survival.*

4. *Fraction Answer: 1/4*
    - ***Fraction Concept:** The correct answer is 1/4 (or an equivalent fraction, like 2/8), but it is more important that students display the concept of equal parts. They need to demonstrate that the number line needs to be broken into equal parts to determine and communicate the fraction.*
    - ***Mathematical Practice 6:** Students should demonstrate the precise use of tools and visual representations of equal parts. They may use a variety of tools, anything from a ruler, paper clips, or folding the paper, yet anything used must be used as a method for communicating equal parts.*

Guion Bluford Tier 2 Lesson

Tier 2 is differentiated as students need to recognize how when the paper clips are unequal sizes, the number of paper clips communicates nothing. Tier 2 does not have scaffolding in writing their responses, and they have to wrestle with fifths.

1. *Answers may vary. Students should communicate that Mission Control only has a precise understanding IF they know how long each paper clip is AND how long the gauge is. If Mission Control has that information, then, the method could be precise. As it is, they do not know what "2 paper clips past empty" really means. Further, they do not realize the paper clips are different sizes. If Joe insists on this method, Mission Control must ask for the lengths of the paper clips and the gauge.*

2. *Answers may vary. Define-2/5 is a fraction that communicates the number line was broken into 5 equal pieces and that the tank is filling 2 of those 5 pieces. Use of Tools-Li used the paper clips to break the gauge into those 5 pieces and saw that 2 of the 5 were full, so she was able to communicate the tank was 2/5 full. HOWEVER, this is overlooking the key idea that fractions communicate equal parts of a whole. The first paper clip is much larger than the others, so the whole is not equally divided and therefore, 2/5 is not accurate. Evaluate-students could check Li's reasoning using a ruler, folding, or any other consistent size tool, like tiles.*

3. *Answers may vary. The key idea is to break this line into equal parts and then communicate the filled portion using those equal parts. Students may use tiles, graph paper, rulers, or any number of other tools to demonstrate their response.*

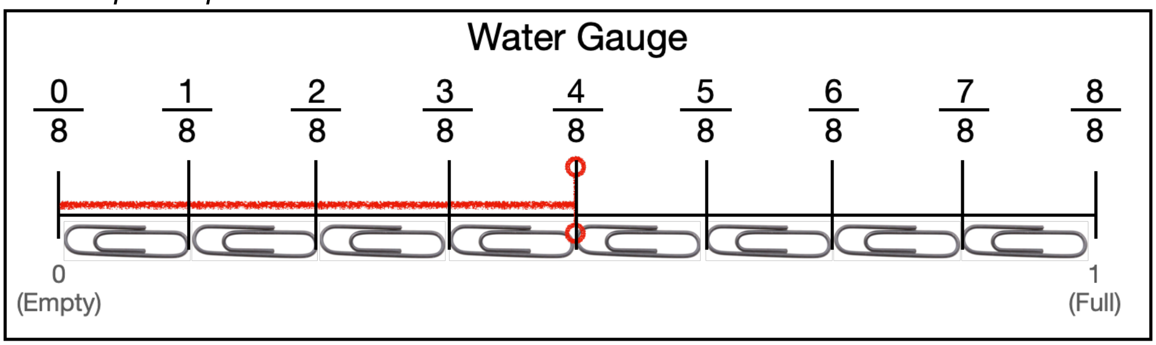
4. *Fraction Answer: 1/4*
   - ***Fraction Concept:** The correct answer is 1/4 (or an equivalent fraction, like 2/8), but it is more important that students display the concept of equal parts. They need to demonstrate that the number line needs to be broken into equal parts to determine and communicate the fraction.*
   - ***Mathematical Practice 6:** Students should demonstrate the precise use of tools and visual representations of equal parts. They may use a variety of tools, anything from a ruler, paper clips, or folding the paper, yet anything used must be used as a method for communicating equal parts.*

Ellen Ochoa Tier 3 Lesson

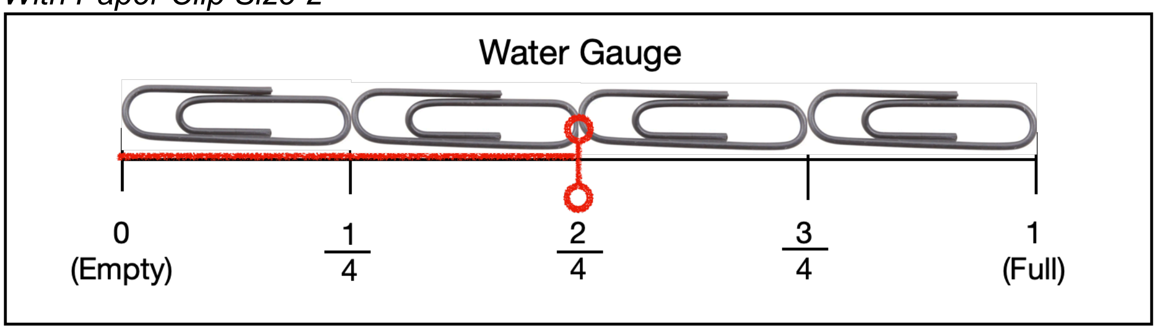
Tier 3 is differentiated as students wrestle with the idea that the smaller the parts, the more options for precision exists. They still need to construct number lines and understand fractions, but they start to uncover the benefit of breaking down the whole into smaller pieces for precision.

1. *Answers may vary. One example is as follows.*

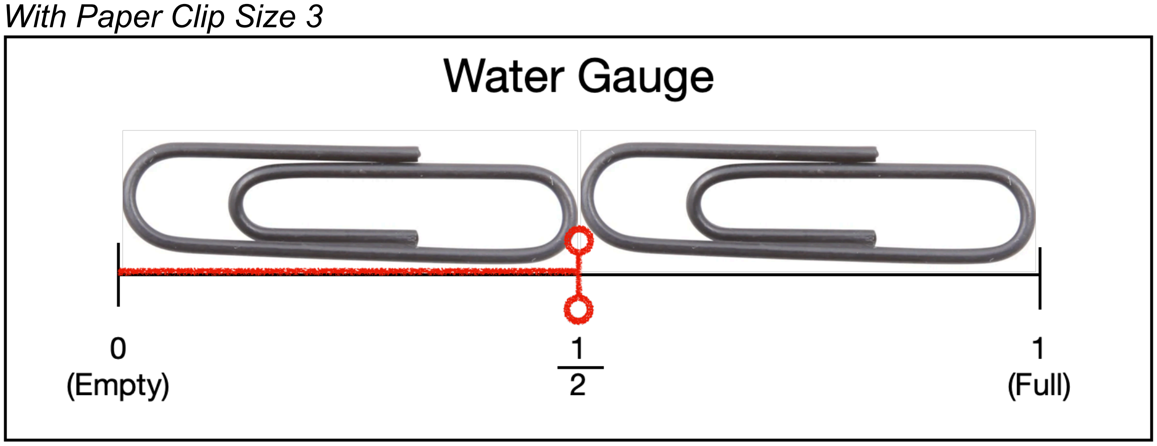
With Paper Clip Size 1



With Paper Clip Size 2



With Paper Clip Size 3



In general, students should be evaluated on breaking the parts into equal pieces. The labeled fractions could be equivalent fractions (e.g., 1/2 could be 2/4 or 4/8)

2. *Answers may vary. The smaller paper clip will be most helpful, as it can accurately determine distance down to the 1/8 of a tank, so it is able to determine 1/2, 1/4, and 1/8; whereas, the other options are not precisely able to determine 1/8.*

3. *Fraction Answer: 1/4*
   - ***Fraction Concept:*** *The correct answer is 1/4 (or an equivalent fraction, like 2/8), but it is more important that students display the concept of equal parts. They need to demonstrate that the number line needs to be broken into equal parts to determine and communicate the fraction.*
   - ***Mathematical Practice 6:*** *Students should demonstrate the precise use of tools and visual representations of equal parts. They may use a variety of tools, anything from a ruler, paper clips, or folding the paper, yet anything used must be used as a method for communicating equal parts.*
   - *Students must demonstrate two methods, but the key is to explore why one may be more precise than another. Size is one option. In this activity, students saw the smaller size of the paper clip would help communicate more precise results. However, other factors may also yield different degrees of precision, such as rigidity of the tool-if students use rubber bands, they may move between measurements, giving a less precise reading.*

Hint Cards for Communicating Precise Locations on a Number Line

Hint 1 What is the definition of a fraction?	**Hint 2** How can you divide the line into EQUAL parts?
Hint 3 What tools might you use to divide the line into equal parts? What is your process?	**Hint 4** How might you use folding to help divide the line into equal parts? What about tiles? Graph paper?

Challenge Cards for Fraction Understandings on Number Lines

Challenge 1 Are larger or smaller denominators more precise? How do you know?	**Challenge 2** Are there places on a number line when the size of the paper clip does not matter? Where? How do you know?

Thinking Like Mathematicians:
Challenging All Grade 3 Students

Common Core State Standard

Equivalent Fractions

CCSS.MATH.CONTENT.3.NF.A.3
Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

> **CCSS.MATH.CONTENT.3.NF.A.3.A**
> Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.
> **CCSS.MATH.CONTENT.3.NF.A.3.B**
> Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3. Explain why the fractions are equivalent, e.g., by using a visual fraction model.
> **CCSS.MATH.CONTENT.3.NF.A.3.C**
> Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.*

Standards in Plain Language:
Students will need to understand that equivalent fractions are fractions that are the same size but are represented by different numbers (e.g., 1/2 and 2/4). They will learn that the same value can be represented in more than one way, which can also include whole numbers (e.g., 4 = 4/1 or 3/3 = 1). Students will use tools such as number lines and fraction models to show that different fractions can be the same size.

Lesson Designer: Lisa DaVia Rubenstein

LESSON 8

Lesson 8: Equivalent Fractions—

Preparing for Take-Off: Designing the Perfect Flag One Half by Any Name

Big Ideas

Numbers provide consistent methods of communication so everyone can understand the precise quantity being considered. In all communication modes, there are multiples ways to communicate the same ideas. For example, in language, we use different words that communicate similar ideas (e.g., soda and pop). In the fraction language, 1/2 communicates the same value as 2/4. Both communicate half of the whole. In this lesson, equivalent fractions are explored using an area model, such that equivalent fractions are occupying the same amount of space, which can be demonstrated using a visual fraction model. Understanding equivalent fractions is essential as students advance into using fractions as a part of mathematical operations (e.g., solving addition/subtraction problems or even complex equations). A real-world example occurs in cooking when you only have a 1/4 measuring cup, but the recipe calls for 1/2 cup of an ingredient. Thus, two 1/4 measuring cups is equivalent to one 1/2 measuring cup.

Lesson Objectives	• Students will recognize and communicate that a fraction is a number representing a quantity reflecting when a whole has been divided equally. • Students will define equivalent fractions as fractions that communicate the same value despite having different numbers in the numerator and denominator. • Students will construct a mathematical argument that includes definitional clarity, examples/non-examples, and concrete evidence. **Common Core State Standards** Develop understanding of fractions as numbers.

	CCSS.MATH.CONTENT.3.NF.A.1 Understand a fraction 1/*b* as the quantity formed by 1 part when a whole is partitioned into *b* equal parts; understand a fraction *a*/*b* as the quantity formed by *a* parts of size 1/*b*. CCSS.MATH.CONTENT.3.NF.A.3.A Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line. CCSS.MATH.CONTENT.3.NF.A.3.B Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3. Explain why the fractions are equivalent, e.g., by using a visual fraction model.
Materials 	Students should have access to graph paper, rulers, scissors, scrap paper to support their own reasoning. Students may not choose to build their argument using these tools, but in general, mathematicians have various tools at their disposal to solve problems.
Mathematical Terms 	• **Denominator:** bottom number in a fraction that identifies the number of pieces the whole has been divided into equal parts • **Equal:** shows the same amount • **Equivalent Fractions:** fractions with different numerators and denominators that represent the same value • **Fraction:** a number that represents part of a whole • **Half:** one of two *equal* parts of a whole • **Numerator:** top number in a fraction that identifies the number of equal pieces considered as part of the whole
Selected Mathematical Practices 	• MP1: Make sense of problems and persevere in solving them. *I never give up on a problem and I do my best to get it right.* • MP2: Reason abstractly and quantitatively. *I can solve problems in more than one way.* • MP3: Construct viable arguments and critique the reasoning of others. *I can explain my math thinking and talk about it with others.*

	• MP5: Use appropriate tools strategically. *I know how to choose and use the right tools to solve a math problem.* • MP6: Attend to precision. *I can work carefully and check my work.*

Differentiation

	Content
	Guiding Questions
	• prior knowledge or learner readiness
	What evidence do you have about students' current knowledge and skills?
	• tiered activities
	How will you design tiered activities on the same mathematical concept with varied levels of difficulty?
	• formative assessment
	What techniques will you use to assess students' prior knowledge and skills?
	• varied levels of challenge
	How will you vary the level of difficulty for each tiered activity?
	• "teaching up" (aim high, provide scaffolding)
	How will you increase the depth, breadth, complexity, and abstractness of lessons to challenge and support student learning?
	
	Process
	Guiding Questions
	• questioning strategies
	How will you pose and how will you encourage students to pose open-ended, closed-ended, lower-level, and higher-level questions to promote mathematical discourse?
	• 4Cs (21st Century Skills)
	  ○ critical thinking
	How will you promote a learning environment in which students question data and view issues or problems from multiple perspectives?
	• 4Cs (21st Century Skills)
	  ○ creative thinking
	How will you encourage students to "think outside the box" and synthesize information in new, different, and useful ways?

- 4Cs (21st Century Skills)
    - collaboration

  How will you encourage students to work with other students and appreciate their contributions to solving problems or making connections to prior work?
- 4Cs (21st Century Skills)
    - communication

  How will you promote students' opportunities to communicate face-to-face, in large and small groups, in online environments, and with print and non-print resources using their oral, written, and non-verbal skills?
- hands-on activities/manipulatives

  How will you incorporate activities promoting the use of manipulatives to clarify or illustrate mathematical concepts?

Product

Guiding Questions

- oral, visual, and written opportunities

  How will you encourage students to represent their thinking and problem solving using different communication strategies?
- multiple ways to demonstrate knowledge, understanding, and skills

  How will you encourage students to share their understanding of mathematical concepts and skills?
- multiple models and representations

  What techniques of lesson design will you include to support students' deep understanding and the ability to apply mathematical concepts and skills?
- summative assessment

  How will you assess student learning upon completion of the lesson?

Learning Environment

Guiding Questions

- flexible grouping

  How will you use your tiered lesson to support flexible grouping?
- whole group/small group/individual instruction

  How will you incorporate different grouping plans to address students' learning needs?

	• learning community *How will you support a positive learning community as students are encouraged to think, work, and communicate like mathematicians?*

Lesson Preview

To recap, before the astronauts take off, they want to commission a flag to place in the Mars soil to commemorate their safe landing. Their team is made up of both Chinese and American astronauts, so they wanted to develop a flag that included one color from each country, equally. The American astronauts chose blue (the color of the stripes on their flag), and the Chinese astronauts chose yellow (the color of the stars on their flag). This lesson builds on the previous lesson in which students learned one-half communicated when a whole was split equally into two pieces. The focus of this lesson will be using equivalent fractions as additional approaches to communicating one-half. Students will learn that the concept of one-half can be represented in various ways and symbolically communicated using fractions like 2/4, 3/6, and 4/8. (Although there are an infinite number of fractions equivalent to 1/2, we focus on equivalent fractions with denominators of 2, 4, 6, and 8 as CCSS-M third-grade content standards emphasize these denominators.)

Launch

1. Thinking Like Mathematicians: Situating the Lesson Processes

MP3. Developing Viable Arguments

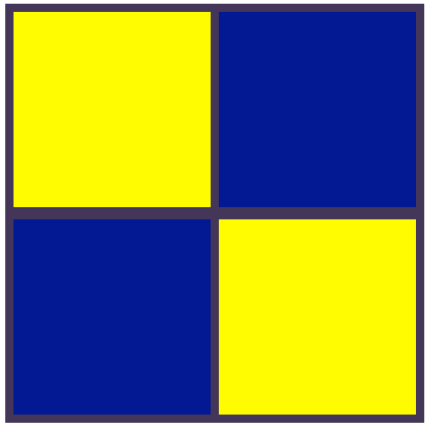
The mathematical practice emphasized in this lesson is the development of mathematically sound arguments that include definitions, examples/non-examples, and specific pieces of evidence to justify their conclusions. Further, this evidence may be gathered using a variety of tools and approaches. As a whole group, discuss how mathematicians support their answers.

Explain: Remember in our previous lesson we discussed how mathematicians often need to provide support for their answers.

Ask: What are some of the methods mathematicians use to provide that support?

If students struggle to generate ideas, suggest mathematicians share the steps they used. They use precise mathematical language/definitions. They provide examples and non-examples, and they provide evidence using manipulatives or tools.

Situating the Lesson Content
Display the following picture:



Explain: The astronauts were so grateful for your help! They really appreciated your clear explanations on determining whether a flag is fairly divided in half. They decided to experiment with a few more complex options. What do you think? Is this a fair flag?

Encourage students to write down ideas in their journals and share with their neighbors. After students have this entry discussion, start to probe their thinking by asking for evidence.

Possible process questions include:

- How do you know if it is fair? (Promotes mathematical reasoning and proof-MP3.)
- What tools could you use to make your conclusion? (Encourages appropriate use of tools-MP5.)
- Is there another way you could determine whether the flag is fair? (Promotes fluency of thought, which is a component of creativity.)
- How could you communicate how much of this flag is blue? (This will start to highlight equivalent fractions.)

Emphasize a fraction must be composed of equal parts. This flag has four equal parts and two are blue and two are yellow, so yes, this flag is fair. Explain how to write a fraction representing the different-colored spaces on this flag. The flag was split into 4 equal parts and 2 of those parts are blue, so the **numerator** would be 2 and the **denominator** would be 4. Then ask is one half of this flag blue. This will prompt students to start thinking about equivalent fractions. Explain that this is what they are going to be exploring throughout this lesson.

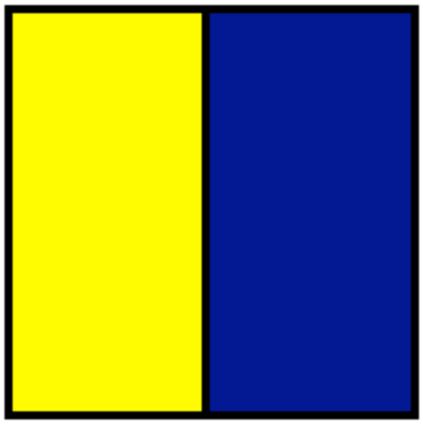
Explore

2.

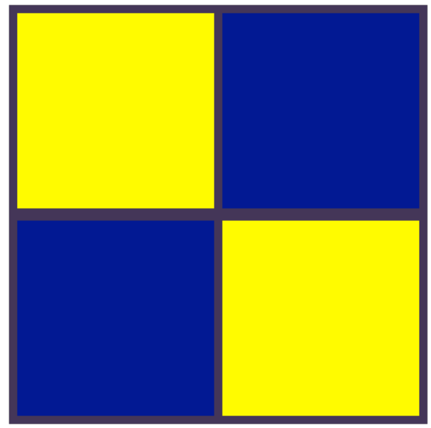
Designing a Fair Flag

Before splitting students into their differentiated groups, explore this introductory task together.

Encourage students to compare one of the previous lesson's opening-task flag with flag discussed in the current Launch section. Show students the following two flags:



and



Ask students to compare these two flags:

- Compare these two flags. What do you notice?
- What fraction of the first flag is blue? What about the second flag?
- Are they both fair flags? (Add explanation if needed: While these flags may look different, they are both fair. Both countries have equal space/area on the flag.)
- How might you provide evidence that these flags are both fair?
- So, what does that tell you about 2/4 and 1/2?

This last question begins the discussion of equivalent fractions, and that language can be introduced here.

Say: We can see and provide mathematical evidence that both of these flags are fair, that both have equal amounts of blue and equal amounts of yellow, yet we use different numbers in the numerator and denominator. This is an example of "**equivalent fractions**." Equivalent fractions are fractions that despite using different numbers they communicate the same amount. This is kind of like when some people say they are drinking "soda" and others say they drink "pop." People are using different words to communicate the same thing.

Ask: Can you think of other examples of when people may use different words to say the same thing? (This is the creative thinking strategy: metaphorical thinking. It helps students to make both proximal and distal connections.) This is just like equivalent fractions, except instead of using letters and words, in math we show this relationship using numbers.

Differentiated Examination of Additional Options
Break students into groups based on readiness levels. They will be receiving different flags to discuss and different levels of available scaffolds. Explain that they will be considering additional options for the astronauts' fair flag, options that may add a little pizazz.

In this investigation, students will be working on one of the Student Pages based on their differentiated groups. The groups are based on teacher's observations of students' understanding.

Groups Formed by Student Readiness		
Tier 1: Peggy Whitson	**Tier 2: Guion Bluford**	**Tier 3: Ellen Ochoa**
Student Names	**Student Names**	**Student Names**

Collaborate and Communicate

Have students record their ideas on their individual worksheets or one for the small group. Help them clarify their ideas by asking questions like, "What do you mean here?" and "How might you share that idea with the rest of the class?" Point out that mathematicians use various representations to help explain their thoughts and use precise language to do so.

A. [Possible response] *This group . . .*	**B.** [Possible response] *This group . . .*	**C.** [Possible response] *This group . . .*

Examine and Elaborate

3.

Highlight Students' Mathematical Thinking

Mathematicians think about possible solutions in a variety of ways. Therefore, it is important for students to realize that they, too, can approach problems using different strategies. Ultimately, students need to understand that a possible solution should be judged by the correctness of the mathematics, and there might valid ideas within a solution when a student has an incorrect answer.

Share and Discuss

After the groups have an opportunity to explore their assigned flags and complete their task worksheets, bring the class back together for a full group discussion. In this discussion, it is important to stress that mathematicians explain their thoughts using mathematical definitions, examples/non-examples, and concrete evidence using tools. In this case, students may have used graph paper, folding, and/or cutting to demonstrate how the colors are taking up equal space on the flag, although some halves might not be congruent. After the groups share their approaches to their assigned flag, invite students to share how they created their own fair flag. As they share, connect back to fraction notation, demonstrating the blue sections are 1/2 of the flag.

Teacher: Let's gather back together to discuss your different tasks!

Teacher: First, both the Whitson and Bluford groups needed to create a fair flag with 8 pieces. Do you think they came up with the same answers? Thumbs up or thumbs down?

Teacher: Interesting, let's see. A member of the Bluford group, let's see how you approached this. Come to the board and show us.

Gwyneth: I colored every other row blue and then the left-over rows I colored yellow.

Teacher: Whitson group, is this what yours looked like?

Nick: No, we colored every other column blue and the left-over columns yellow.

Teacher: How did you check to make sure this was a fair flag?

Keisha: We counted the squares.

Teacher: Would someone like to add on to Keisha's response to give us some additional details? *(Adding On talk move)*

Marie: Yes, then we compared the number of squares that were blue to those that were yellow to make sure they were the same. There were 32 blue squares and 32 yellow squares.

Teacher: Great thinking, now, we are going to take a look at the Ochoa group's problem. They were given a circle that had three parts and asked to divide them in half. Let's everyone look at this circle and see if you can figure out how to make

this a fair flag. If you were in the Ochoa group, see if you can do it a different way.
[Give everyone 2 minutes to think. Bring everyone back to share all the different ways they thought about the problem. *(Wait Time talk move)*]

Debrief and Look Ahead

4. **Debrief Content and Skills**

Remind students that the mathematical practice for this lesson focused on how mathematicians explain their responses using definitions and examples/non-examples. Further, mathematicians use a variety of tools and approaches to explain their thinking. In the next lesson, students will be expanding their conceptual understanding of halves when the flag is split into more than 2 pieces. They will start to write fractions with various denominators, including 4, 6, and 8. They should begin to connect 1/2 with 2/4, 3/6, and 4/8.

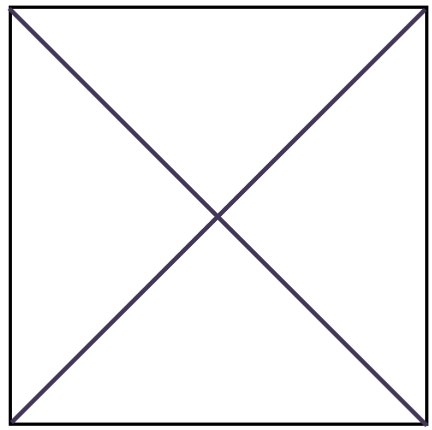
Assess

5. **What Students Learned**

Use the following exit card to assess what students learned from this lesson.

Exit Card

The astronauts are still considering their crew's flag.
They drafted the following option:



- Color this flag such that it is fair for both countries.
- What fraction would describe the yellow part?
- What fraction would describe the blue part?
- What would be a different way to write a fraction that communicates how much of the flag is blue?
- Are there any other ways?

Exit Card Answers

Two parts should be colored in yellow (2/4) and two parts should be colored in blue (2/4). Students may use any equivalent fraction to describe the flag's colors. They should be able to write both 1/2 and 2/4. For the additional ways, they may cut each section in half again, such that the fractions would be 4/8. There are other correct approaches as well, as long as the fraction is equivalent to 1/2. This last question is meant to challenge students but should not be used as a grade.

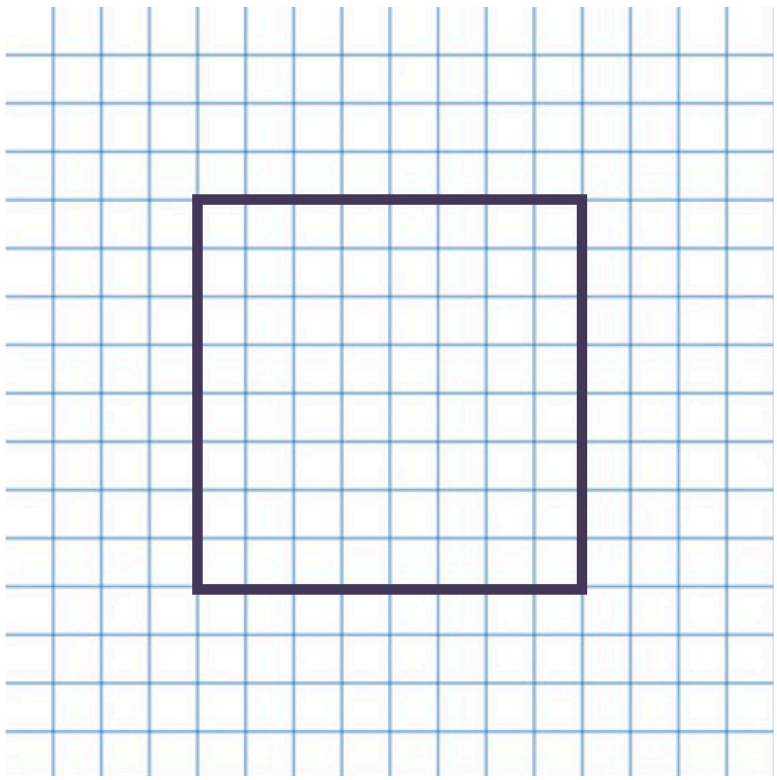
Mission to Mars Student Pages

Flag Designer ________________________________ Date __________________

Peggy Whitson Tier 1 Lesson

One of the crew members, Li, wanted to draw a flag with 8 **equal** pieces.

1. What might this flag look like?



2. Another crew member, Joe, looked at your drawing and said, “Those are not equal pieces!” Can you explain how using the graph paper helped you ensure that all 8 pieces are equal in size?

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3. Color your flag such that half is blue and half is yellow. How might you have colored in a different way?

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4. Li LOVED your flag! She said 1/2 is yellow and 1/2 is blue, but Joe argued, 4/8 are blue and 4/8 are yellow. Who is correct? Explain your thinking.

Use these prompts to guide your response.	Your Thoughts
Define: What does 1/2 mean? What does 4/8 mean?	
Evidence: How could you use the graph paper to illustrate your thoughts?	
Evidence: How could you use scissors or folding to defend your thoughts?	

Mission to Mars Student Pages

Flag Designer ______________________________ Date _________________

Guion Bluford Tier 2 Lesson

One of the crew members, Li, wanted to draw a flag with 8 **equal** pieces.

1. What might this flag look like?

2. Another crew member, Joe, looked at your drawing and said, “Those are not equal pieces!” Describe **TWO** different ways you could demonstrate you have created a flag with 8 equal pieces.

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3. Color in your flag using yellow and blue so that each color is equally represented.

What fraction of your flag is blue? _____

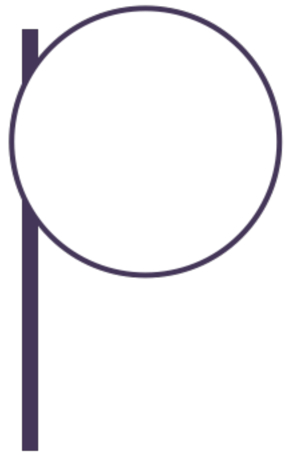
What fraction of your flag is yellow? _____

4. Li <u>LOVED</u> your flag! She said 1/2 is yellow and 1/2 is blue, but Joe argued, 4/8 are blue and 4/8 are yellow. Who is correct? Explain your thinking.

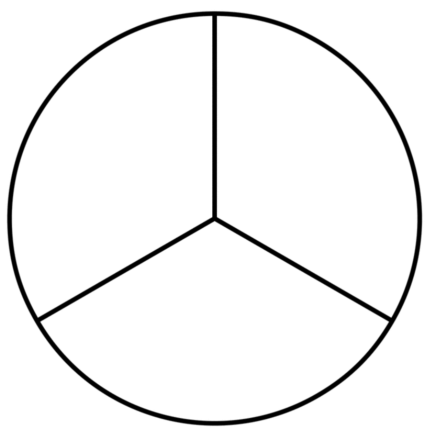
Flag Designer ______________________________ Date _________________

Ellen Ochoa Tier 3 Lesson

One of the astronauts, Li, had another idea: what if we created a circle flag? She sketched out what a circle flag would look like:



She admitted it looked a bit different, but she was excited to think about how she may be able to design a fair circle flag! She experimented and drew this flag:



1. Use her sketch to create a fair flag that represents both countries equally using both blue and yellow. You must leave her existing lines. Explain how you developed your new fair flag. How do you know it is fair?

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2. What fraction of your flag is blue? ______

   What fraction of your flag is yellow? ______

3. Li <u>LOVED</u> your flag! She said 1/2 is yellow and 1/2 is blue, but Joe argued, 3/6 are blue and 3/6 are yellow. Who is correct? Explain your thinking.

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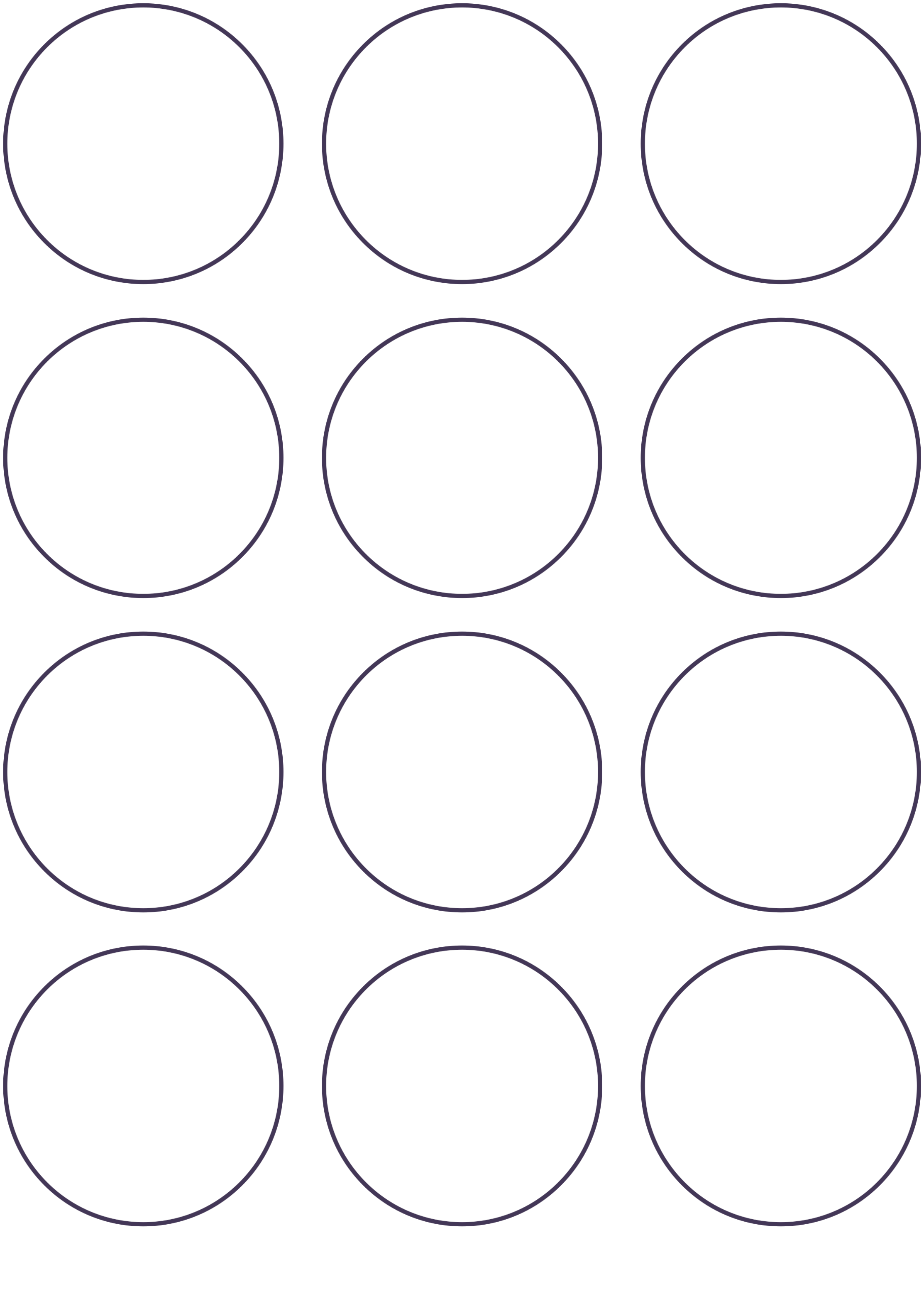
Challenge Question

4. Could you create a **fair** circle flag, equally representing both countries, with 9 total equal pieces? With 16 total equal pieces? How is 9 different from 16? Explain how you know.

Extra Circles to Test Ideas

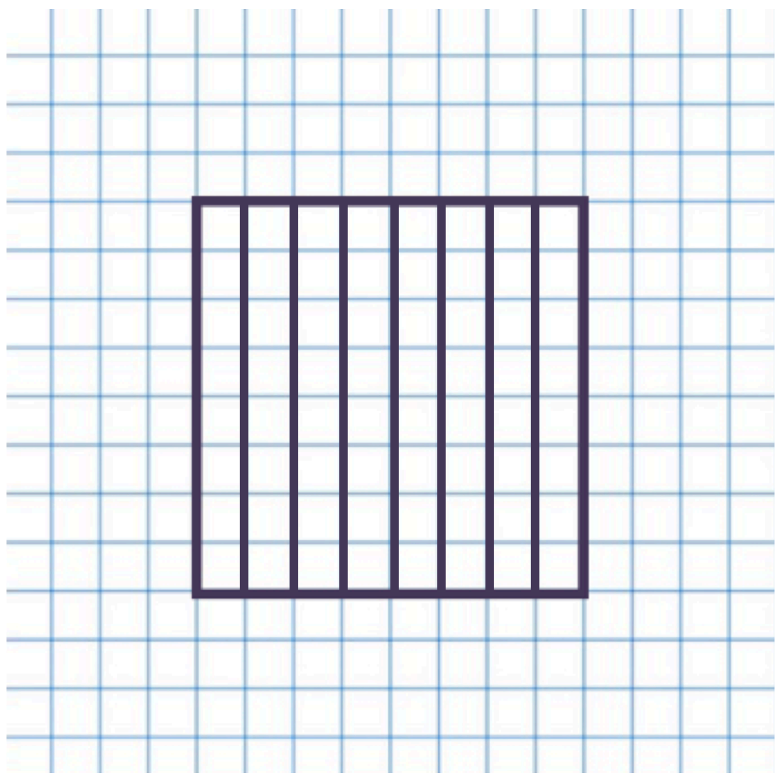


Extra Squares to Test Ideas

Throughout their responses, students need to incorporate the concept of equality and equivalence. Fractions are only fractions when the portions are divided equally. If the total number of parts are divided perfectly in half, equivalent fractions can be used to describe the flag. Students should also demonstrate their ability to construct arguments using definitions, examples, and different representations. The final full class discussion will bring their experiences together to describe fractions only describe when a whole is divided into EQUAL parts.

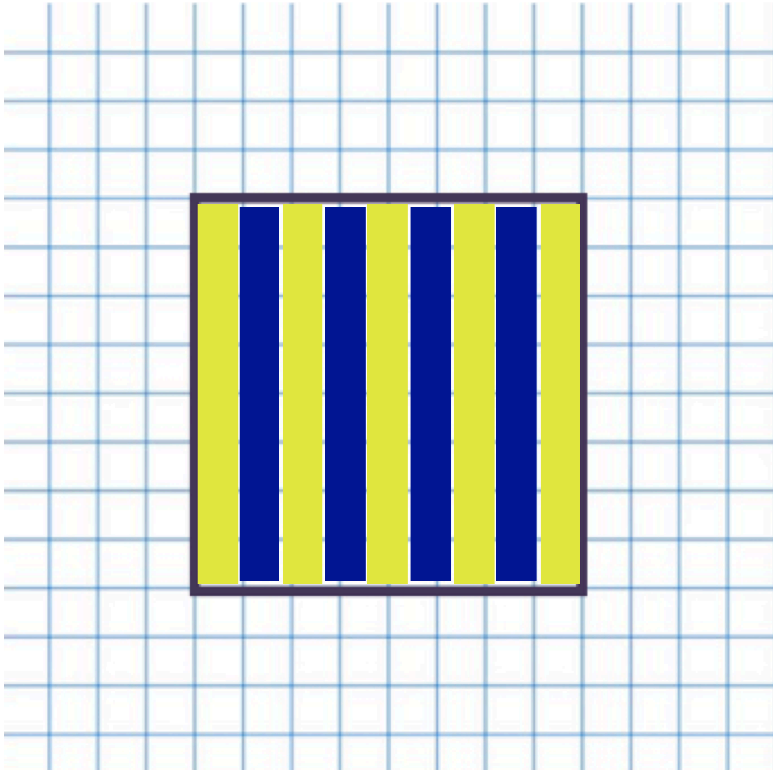
Peggy Whitson Tier 1 Lesson

1. *Students' responses will vary, but it should contain 8 equal parts. One example could be:*



2. *Students' responses will vary, but they should include that all 8 parts cover the same number of squares or the same area. The graph paper supports their efforts because each square on the graph paper is exactly the same size.*

3. *Students' responses will vary, but the blue pieces should cover the same area as the yellow pieces. For example:*

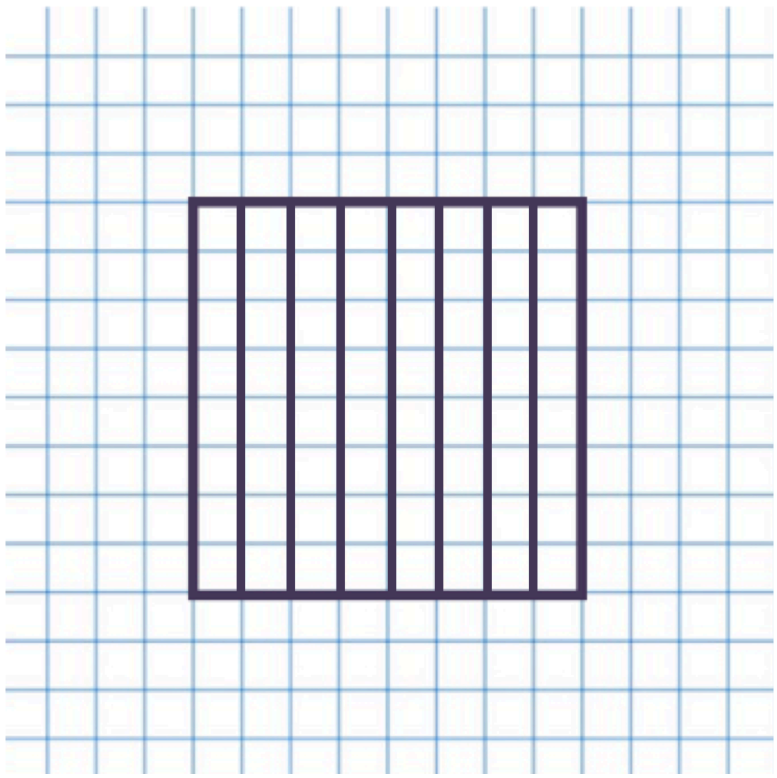


4. *The key idea is that 1/2 is equivalent to 4/8, so both Li and Joe are correct. (a) Define: 1/2 means that there are 2 equal pieces and 1 of them is colored blue; 4/8 means that there are 8 equal pieces and 4 are colored blue. Both describe a fair flag divided in half. (b) Evidence: You could compare the two fractions using graph paper by counting the number of total squares that blue occupies. In both the 1/2 and 4/8 flags, blue occupies 32 squares exactly. This demonstrates 1/2 and 4/8 represent equal areas. (c) Evidence: You could also demonstrate using folding or cutting that the blue pieces, whether 1 out of 2 or 4 out of 8 occupies the same area.*

Guion Bluford Tier 2 Lesson

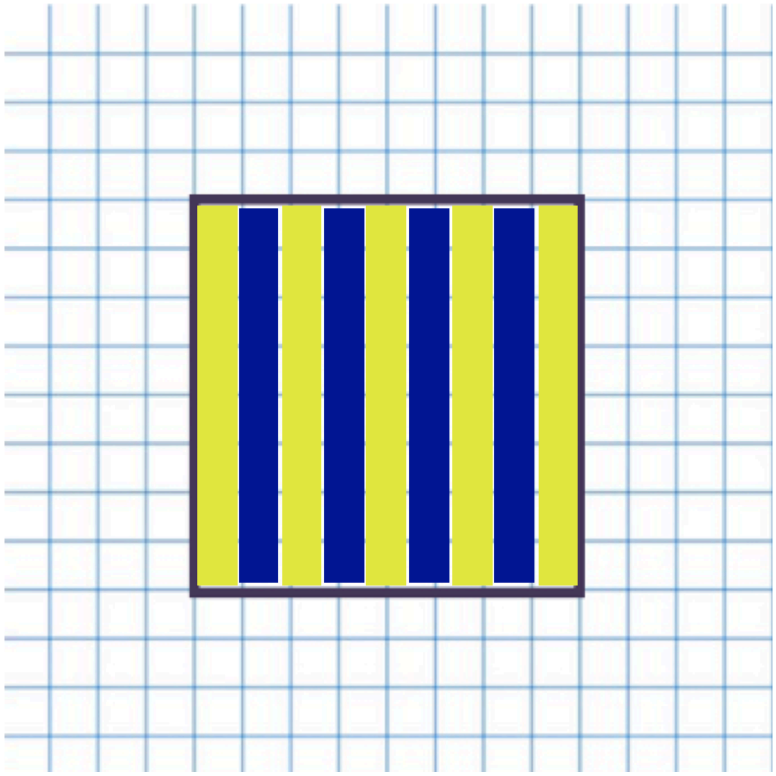
This tier does not provide the graph paper, so students will need to determine the best method for ensuring they are dividing the square into equal pieces, yet the result will be similar to the Whitson group's work.

1. *Students' responses will vary, but it should contain 8 equal parts. One example could be:*



2. *Students' responses will vary, but they should include that all 8 parts cover the same number of squares or the same area. The graph paper supports their efforts because each square on the graph paper is exactly the same size. They could also use other forms of evidence or reasoning.*

3. *Students' responses will vary, but the blue pieces should cover the same area as the yellow pieces. For example:*



4. *The key idea is that 1/2 is equivalent to 4/8, so both Li and Joe are correct. These students do not have a scaffolded answer but their response should address the following points: (a) Define: 1/2 means that there are 2 equal pieces and 1 of them is colored blue; 4/8 means that there are 8 equal pieces and 4 are colored blue. Both describe a fair flag divided in half. (b) Evidence 1: You could compare the two fractions using graph paper by counting the number of total squares that blue occupies. In both the 1/2 and 4/8 flags, blue occupies 32 squares exactly. This demonstrates 1/2 and 4/8 represent equal areas. OR Evidence 2: You could also demonstrate using folding or cutting that the blue pieces, whether 1 out of 2 or 4 out of 8 occupies the same area.*

Ellen Ochoa Tier 3 Lesson

Tier 3 is quite different from Tier 1 and Tier 2. Instead of exploring squares and eighths, these students are examining circles and sixths. They need to consider how to turn thirds into sixths to successfully create halves. (There are other ways they could approach this problem, but that is the simplest.)

1. *Students' responses will vary, but they should describe how they could not simply split the thirds fairly among two countries. They need to break it down further. One approach would be to simply cut each piece in half again, which yields 6 equal pieces. They could test the fairness using graph paper, cutting, folding, or other approaches.*

2. *Fractions could vary, but they will all be equivalent to 1/2. The most likely fraction would be 3/6.*

3. *The key idea is that 1/2 is equivalent to 3/6, so both Li and Joe are correct. These students do not have a scaffolded answer but their response should address the following points: (a) Define: 1/2 means that there are 2 equal pieces and 1 of them is colored blue; 3/6 means that there are 6 equal pieces and 3 are colored blue. Both describe a fair flag divided in half. (b) Evidence 1: You could compare the two fractions using graph paper by counting the number of total squares that blue occupies. In both the 1/2 and 3/6 flags, blue occupies the same number of squares. This demonstrates 1/2 and 3/6 represent equal areas. OR Evidence 2: You could also demonstrate using folding or cutting that the blue pieces, whether 1 out of 2 or 3 out of 6 occupies the same area.*

4. *The challenge question response should discuss even/odd numbers or how certain numbers cannot be divided perfectly into two pieces without additional fractions. However, you can make those additional fractions by dividing the odd numbers in half again. For example, each of the 9 pieces could be halved to make 18 pieces and that can easily be divided to make a fair flag.*

Hint Cards* for Developing an Argument

Hint 1 How might you add a definition of “one-half” into your explanation? How do you define 4/8?	**Hint 2** How might you use scissors to test your thoughts?
Hint 3 How might graph paper help demonstrate your thinking?	**Hint 4** How might folding the paper help to demonstrate your thoughts?

*These are already embedded within the Tier 1 page, but the other students may also find them useful. They do not necessarily need to be distributed in order.

Challenge Cards* for Developing Conceptual Understanding

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Challenge 1

Astronaut Li asks you to create a fair flag with 16 equal total pieces. How would you respond? Explain your thoughts with evidence.

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Challenge 2

Astronaut Joe asks you to create a fair flag with 9 equal total pieces. How would you respond? Explain your thoughts with evidence.

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Challenge 3

Is this a fair flag? What fraction of the flag is blue? Yellow? Explain your thinking.

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Challenge 4

One of the crew members suggests that all of the possible flag designs must have an even number of total pieces for the design to be fair. Is this true? How do you know?

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*The first two cards' concepts are already embedded within Tier 3 page, but other students may find it interesting after they finish their work.

Thinking Like Mathematicians:
Challenging All Grade 3 Students

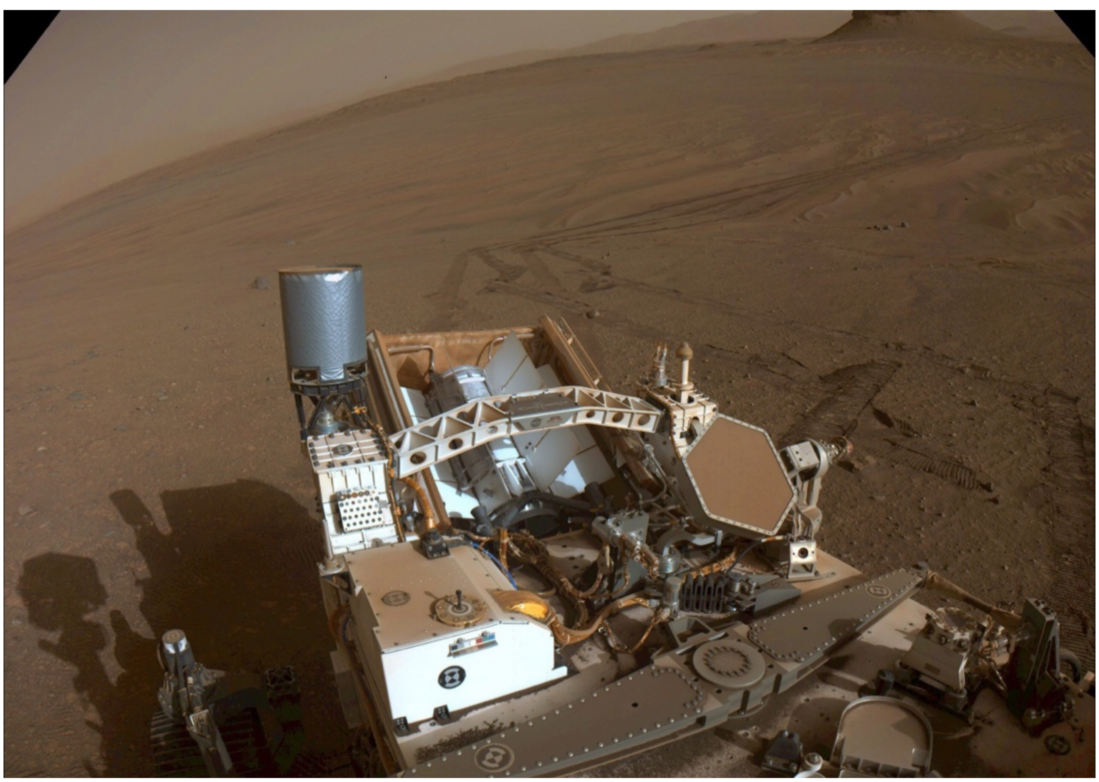
Common Core State Standard

Comparing Fractions

CCSS.MATH.CONTENT.3.NF.A.3
Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

> **CCSS.MATH.CONTENT.3.NF.A.3.D**
> Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Standards in Plain Language:
Students will compare two fractions that either have the same numerator (top number) or the same denominator (bottom number). They will recognize that they can only compare fractions that refer to the same-sized whole. Students will use appropriate symbols (>, =, or <) when comparing the size of these fractions.



Lesson Designer: Lisa DaVia Rubenstein

LESSON 9

Lesson 9: Comparing Fractions on Number Lines—

Walking on Mars: Activate—Mission Rover Recapture

Big Ideas

Numbers provide a consistent method to communicate a precise quantity being considered. Specifically, fractions are numbers that precisely describe a situation where a whole has been broken up into *equal* parts. The “denominator” communicates how many equal parts there are in the whole, and it is written on the bottom of the fraction. The “numerator” communicates how many of those equal parts are present, and it is recorded at the top of the fraction.

In this lesson series, we will be examining number lines that span from 0 to 1. In this case, [0 to 1] is the whole that will be split into equal parts. Fractions can be used to communicate the precise distance between 0 and 1. Precision is important to establish a common understanding of distance that most closely communicates the true or desired value. For example, let’s suppose you want to tell your friend how far you are to their house. A fraction can be used to communicate the relative distance from your starting point to their house, like “I am halfway there.” The more precise you are; your friend will be better able to estimate when to expect you. Generally, understanding fractions on number lines will support the measurement of distance, reading gauges, and generally comparing relative lengths. The big idea of this lesson is to develop mathematical reasoning about the relative size of fractions using a number line and to be able to compare these relative sizes.

Lesson Objectives	• Students will recognize and communicate that a fraction communicates when a whole unit is divided into equal parts. In this lesson, students will conceptualize the whole unit as the distance between 0 and 1 on a number line. • Students will define “denominator” as the bottom number in the fraction and recognize a denominator communicates how many total parts make up the whole. In this lesson, students will describe the denominator as the total parts of the

whole, when the whole is 0 to 1 on a number line.

- Students will define “numerator” as the top number in the fraction and recognize the numerator communicates how many parts of the whole are present. In this lesson, students will describe the how the numerator communicates the precise distance from 0, when the whole is equally divided into the denominator’s equal parts.
- Students will compare fractions using a number line to determine relative size.
- Students will use fractions to communicate precisely with others. Students provide precise explanations and definitions in their communication.

Common Core State Standard
Develop understanding of fractions as numbers.

<u>CCSS.MATH.CONTENT.3.NF.A.2</u>
Understand a fraction as a number on the number line; represent fractions on a number line diagram.

<u>CCSS.MATH.CONTENT.3.NF.A.2.A</u>
Represent a fraction 1/*b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size 1/*b* and that the endpoint of the part based at 0 locates the number 1/*b* on the number line.

<u>CCSS.MATH.CONTENT.3.NF.A.3.A</u>
Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.

<u>CCSS.MATH.CONTENT.3.NF.A.3.D</u>
Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Materials	Students should have access to graph paper, tiles, fraction rods, paper clips, scissors, scrap paper, and other materials teachers may have available. Students may not choose to build their responses using these tools, but in general, mathematicians have various tools at their disposal that they can use to test their hypotheses.
Mathematical Terms	• **Denominator:** bottom number in a fraction that identifies the number of pieces the whole has been divided into equal parts • **Equal:** the same portion, piece, or segment • **Fraction:** a number that represents part of a whole • **Number Line:** a line with numbers placed in their correct position • **Numerator:** top number in a fraction that identifies the number of equal pieces considered as part of the whole • **Precise:** describes responses that are exact, accurate, careful about details
Selected Mathematical Practices	• MP1: Make sense of problems and persevere in solving them. *I never give up on a problem and I do my best to get it right.* • MP3: Construct viable arguments and critique the reasoning of others. *I can explain my math thinking and talk about it with others.* • MP5: Use appropriate tools strategically. *I know how to choose and use the right tools to solve a math problem.* • MP6: Attend to precision. *I can work carefully and check my work.*
Differentiation	**Content** ***Guiding Questions*** • prior knowledge or learner readiness *What evidence do you have about students' current knowledge and skills?*

	• tiered activities *How will you design tiered activities on the same mathematical concept with varied levels of difficulty?* • formative assessment *What techniques will you use to assess students' prior knowledge and skills?* • varied levels of challenge *How will you vary the level of difficulty for each tiered activity?* • "teaching up" (aim high, provide scaffolding) *How will you increase the depth, breadth, complexity, and abstractness of lessons to challenge and support student learning?* **Process** ***Guiding Questions*** • questioning strategies *How will you pose and how will you encourage students to pose open-ended, closed-ended, lower-level, and higher-level questions to promote mathematical discourse?* • 4Cs (21st Century Skills) o creative thinking *How will you encourage students to "think outside the box" and synthesize information in new, different, and useful ways?* • 4Cs (21st Century Skills) o collaboration *How will you encourage students to work with other students and appreciate their contributions to solving problems or making connections to prior work?* • 4Cs (21st Century Skills) o communication *How will you promote students' opportunities to communicate face-to-face, in large and small groups, in online environments, and with print and non-print resources using their oral, written, and non-verbal skills?* • hands-on activities/manipulatives *How will you incorporate activities promoting the use of manipulatives to clarify or illustrate mathematical concepts?*

	Product ***Guiding Questions*** • oral, visual, and written opportunities *How will you encourage students to represent their thinking and problem solving using different communication strategies?* • summative assessment *How will you assess student learning upon completion of the lesson?* **Learning Environment** ***Guiding Questions*** • flexible grouping *How will you use your tiered lesson to support flexible grouping?* • whole group/small group/individual instruction *How will you incorporate different grouping plans to address students' learning needs?* • learning community *How will you support a positive learning community as students are encouraged to think, work, and communicate like mathematicians?*

Lesson Preview

This lesson builds on previous lessons' emphasis on fractions require **equal** parts. Within this lesson, students will use **number lines** to develop mathematical reasoning about the relative size of fractions. Students will be able to compare fractions with the same numerators but different denominators as well as fractions with the different numerators but the same denominators. Further, they will consider the concept of equivalent fractions.

They will explore these concepts within a Mars Mission. The mission premise is the astronauts must retrieve Mars Perseverance Rover and all the test tubes the rover buried in its previous mission. The rover is located exactly 1 mile from their dome home. Specifically, the astronauts need to collect the test tubes the rover missed and bring the rover back to the dome.

Launch

1. Thinking like Mathematicians: Centering Mission With the Mathematical Practice

CCSS.Math.Practice.MP6: Attend to Precision

Within this lesson, students will be developing the following mathematical practices:

- Precise communication by specifying how many equal parts are present.
- Precise use of tools and visual representations of equal parts.
- Determining the degree of precision appropriate for specific contexts.
- Use of clear definitions in discussion with others and in their own reasoning.

Explain the Current Task: Throughout this lesson series, we have learned that mathematicians and astronauts need to be precise in their answers. For today's mission, the astronauts have finally made it to Mars! One of their first tasks is to retrieve Mars Perseverance Rover and all the soil samples the rover buried in its previous mission. The Mars rover is located exactly 1 mile from their dome home. Thus, the astronauts need to collect the soil samples and bring the rover back to the dome. While walking on Mars and spacewalking are different, both require the space suits and similar precautions and levels of precision.

Actual Description of the Mars Perseverance Rover's Mission
"A big job for the rover is collecting carefully selected samples of Mars rock and soil. These samples will be sealed in tubes and left in a well-identified place, or more than one spot, on the surface of Mars. Detailed maps will be provided for any future mission that might go to Mars and pick up these samples for study by scientists… the samples are deposited on the surface of Mars at a spot that the team designates as a "sample cache depot." The depot location or locations must be well-documented by both local landmarks and precise coordinates from orbital measurements. The cache of Mars samples remains at the depot, available for pickup and potential return to Earth." (NASA Science, 2020).

Ask: Why might it be important for the astronauts to know the precise location the rover left the soil samples?

If students struggle to generate ideas, suggest comparing how the surface changes over time, requires knowing exactly where the first sample was taken from. Further, the Mars Perseverance Rover collected samples and left them in well-defined places on the surface of Mars (NASA Science, 2020), so to find those samples, you must know precisely where to look.

Resources
NASA Science Mars 2020 Mission (n.d.). *Sample handling.* https://mars.nasa.gov/mars2020/spacecraft/rover/sample-handling/

NASA Science Mars 2020 Mission (n.d.). *What is surface Operations?* https://mars.nasa.gov/mars2020/timeline/surface-operations/

Explore

2. Activate Mission: Rover Recapture

Use this opening task to determine which differentiated group may be most appropriate for each student.

Opening Task
In 2020, Mars Perseverance Rover cached (buried) two types of test tubes, including where it is currently located, 1 mile away from the dome home:

- **Soil Tubes:** These tubes tested the soil composition in specific locations on the surface of Mars. These were cached every ¼ of a mile, including in its final location.
- **Life Tubes:** These tubes ran specific tests looking for cellular life. These were cached every 1/8 of a mile, including in its final location.

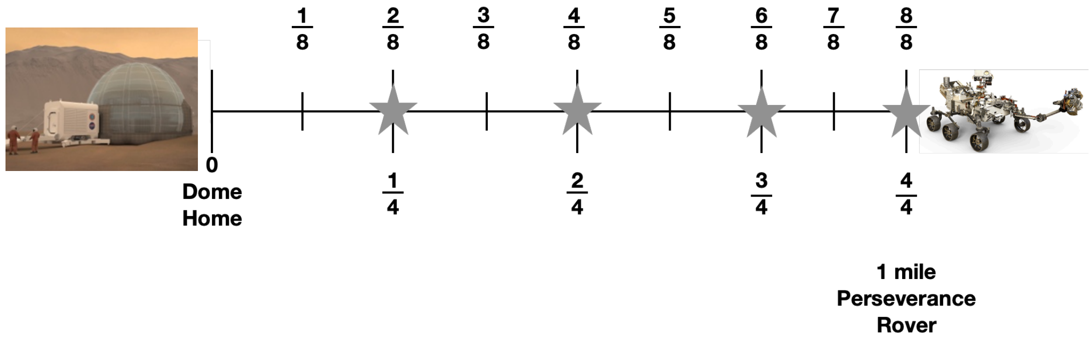
Mission: Rover Recapture

Is there a location where a Soil Tube Experiment is cached in the location as the Life Tube Experiment? Explain your answer.

Using the Opening Task to Determine Differentiated Groups
Teachers should evaluate students' responses using both students' knowledge of the content and their mathematical processes.

- **Fraction Concept:** The correct answer is yes. There will be four overlapping sites, as pictured below. It is most important that students display the concept of equal parts. They need to demonstrate that the number line needs to be broken into equal parts to determine, communicate, and compare fractions. Even though the instructions indicate tubes were buried in the final location (4/4 and 8/8), students may miss this instruction, which is not an indication of a lack of fractional understanding and should not be used for grouping placement.

Mission: Rover Recapture



- **Mathematical Practice 6:** Students should demonstrate the precise use of tools and visual representations of equal parts. They may use a variety of tools, anything from a ruler, paper clips, or folding the paper, yet anything used must be used as a method for communicating equal parts. If the number line is not precisely broken into components, students may not arrive at the correct answer.

In this investigation, students will be working on one of the Student Pages in their differentiated groups, based on readiness levels. The groups are based on teacher's observations of students' conceptual understanding and mathematical practice acumen as demonstrated in the introductory task.

- **Tier 1:** Students who do not demonstrate a conceptual understanding of fractions on number lines should be placed in Tier 1.
- **Tier 2:** If students demonstrate a vague sense of fractions (i.e., they try to establish equal parts) but do not demonstrate a specific strategy or fail to arrive at the correct answer, they should be in Tier 2. If students can plot one fraction accurately but not replicate with the other fraction OR they struggle to compare fractions, they should be placed in Tier 2.
- **Tier 3:** If students communicate the appropriate locations of equivalent fractions (e.g., both tubes will be buried at 1/4 of a mile, 2/4 of a mile, and 3/4 of a mile) and they used a specific strategy to precisely establish equal parts, they should be placed in Tier 3.

Groups Formed by Student Readiness		
Tier 1: Peggy Whitson	**Tier 2: Guion Bluford**	**Tier 3: Ellen Ochoa**
Student Names	**Student Names**	**Student Names**

Collaborate and Communicate

Have students record their ideas for on their individual worksheets or one for the small group. Help them clarify their ideas by asking questions like, "How might you be more precise?" and "How might you share that idea with the rest of the class?" Remind students that mathematicians use definitions, examples/non-examples, and various representations to help support their conclusions. Mathematician can demonstrate their precision by using appropriate tools and precise language, including units and fractions.

You may wish to record student responses in the blank boxes below to explore later in the debrief section of the lesson plan.

A. [Possible response] *This group . . .*	**B.** [Possible response] *This group . . .*	**C.** [Possible response] *This group . . .*

Examine and Elaborate

3. **Highlight Students' Mathematical Thinking**

Mathematicians think about possible solutions in a variety of ways. Therefore, it is important for students to realize that they, too, can approach problems using different strategies. Ultimately, students need to understand that a possible solution should be judged by the correctness of the mathematics, and there might even be some valid ideas within a solution when a student has an incorrect answer.

Share and Discuss

After the groups have an opportunity to explore their assigned tasks, bring the class back together for a full group discussion. In this discussion, it is important to stress that mathematicians explain their thoughts using mathematical definitions, examples/non-examples, and concrete evidence using tools.

Guiding MP questions for all groups should include:

- What tools could you use to make your conclusion? (MP5: Encourages appropriate use of tools)
- How precise are your number lines? How do you know? (MP6: Attend to precision)

In this discussion, it is important to stress that mathematicians decide to be either more or less precise in their responses based on the situation and materials. In this case, astronauts need to be very precise regarding where the test tubes are buried. If they dig in the wrong place, they not only will be unable to recover their tubes, but they also may run out of oxygen, water, or fuel in their space suits.

During this discussion, continue to discuss how these maps are number lines, how 0 to 1 is the whole that can be broken into equal parts, and then, fractions and fraction notation can be used to describe a location. In other words, fractions only describe when a whole is divided into EQUAL parts. As they share, connect back to fraction notation, demonstrating how the number line can be divided into equal parts, communicated as the denominator.

Teacher: Let's start with the Whitson group. You first located every Life Tube for Astronaut Kathryn. I am curious how did you know what the denominator would be for each location?

Monroe: The instructions said the Life Tubes were every eighth of a mile. I know eighth is 8, so I marked each location using 8 as the denominator.

Teacher: Good thinking! Can anyone add on to Monroe's thinking by incorporating numerators? *(Adding On talk move)*

Lilly: After I labeled all the denominators, I simply add 1 to each fraction to show how many eighths away from 0 the tube would be.

Teacher: Could you come to the board and show us your thinking? *(Adding on talk move)*
(Lilly shows her thinking on the board.)

Teacher: Interesting! Bluford and Ochoa groups, did you also define denominators in your responses? Check your work to make sure you included that information. Now, let's think about an additional situation that Bluford and Ochoa groups faced. Astronaut Kathryn reported she was a third of the way to the rover. How many tubes has she collected?

Avalyn: I said she collected 2 Life Tubes.

Teacher: Hmmm. I want everyone to look at a number line. Do you agree with Avalyn? *(Reasoning talk move)*
(Wait Time talk move)

Teacher: How did you check Avalyn's thinking?

Gerardo: First, I needed to label thirds on the same line with the eighths.

Teacher: Did anyone else do it that way? Is there a different way? How did you check to make sure you labeled the thirds accurately?

Differentiate Further as Needed

Please see the Hint and Challenge cards at the end of the lesson. The hint cards remind students to incorporate precise language and how fractions require equal parts. The challenge cards begin to probe students thinking on finding "improper" fractions on number lines as well as using different anchor points on lines to determine locations.

Debrief and Look Ahead

4.

Debrief Content and Skills

Remind students that the mathematical practice for this lesson focused on how mathematicians use precision to communicate amounts. Review how students were precise throughout this lesson. Ask them to recall all their approaches to increase precision. (Perhaps they used a number line, paper strips, rulers, graph paper, or perhaps they used precise language including denominators and numerators. In general, emphasize how they acted like mathematicians while they worked together during the lesson.) You may revisit your earlier notes of student responses in the Collaborate and Communicate section.

Assess

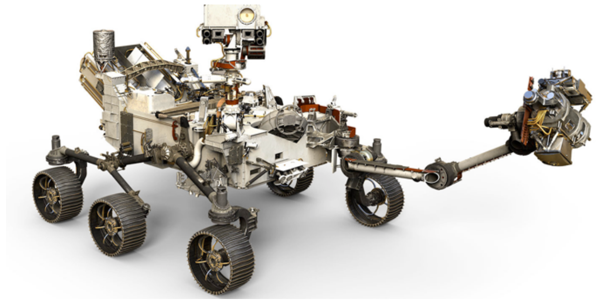
5.

What Students Learned

Use the following exit card to assess the degree to which students can transfer their learning from this lesson to new scenarios.

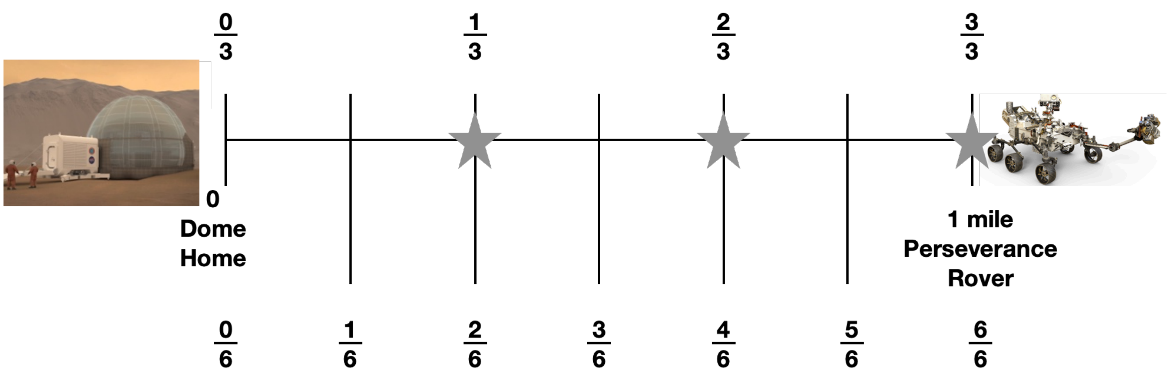
Exit Card

Imagine on Venus, a NEW rover cached Life Tubes every third of a mile and Soil Tubes every sixth of a mile. In one mile, where would you find BOTH tubes? Use a number line to explain your response.



Exit Card Answer Key

Answers may vary slightly. Students should develop a number line similar to the line below. There are 3 locations where both types of tubes will be located: 1/3, 2/3, and 3/3. By not providing a number line, students have the freedom to develop a line that will work for them.



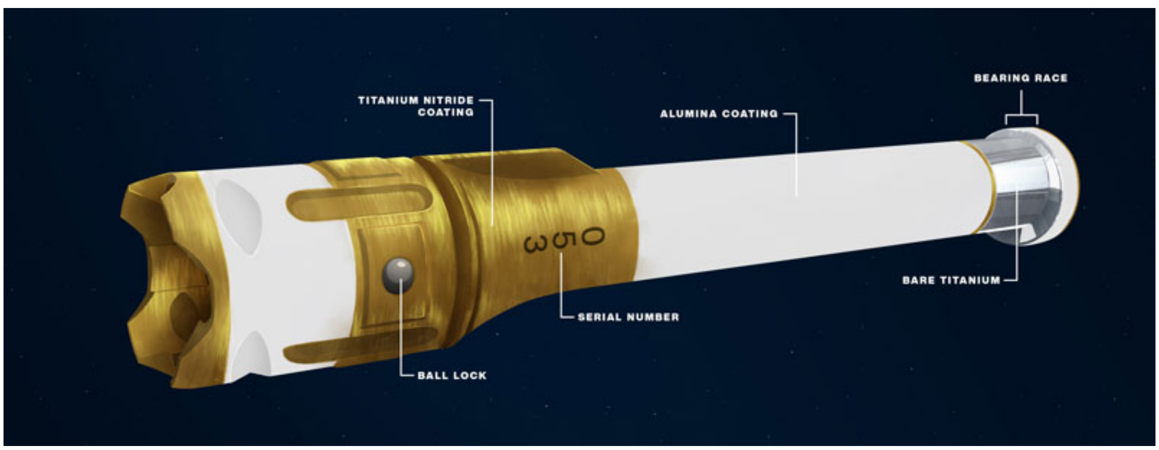
Rover Recapturer ______________________________ Date ___________________

Opening Task

In 2020, Mars Perseverance Rover cached (buried) several sample collection tubes, like the picture on the right.

The rover cached two types of tubes on its journey, including at its final resting location, 1 mile away from the astronauts' dome home. Those two tubes examined the samples for different types of soil and clues about life:

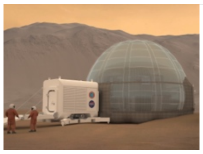
- *Soil Tubes:* These tubes tested the soil composition in specific locations on the surface of Mars. These were cached every 1/4 of a mile, including in its final location.
- *Life Tubes:* These tubes ran specific tests looking for cellular life. These were cached every 1/8 of a mile, including in its final location.

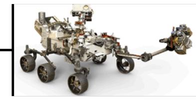


(Source: https://mars.nasa.gov/resources/25482/anatomy-of-a-sample-tube/)

The Soil Tubes are located every 1/4 of a mile from the dome, and the Life Tubes are located every 1/8 of a mile from the dome, including at the rover's final resting place. **Are there any locations where a Soil Tube is cached in the SAME location as the Life Tube? Explain your answer.**

Mission: Rover Recapture





0
Dome
Home

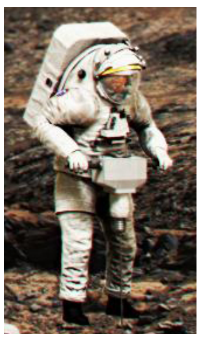
1 mile
Perseverance
Rover

Rover Recapturer ___________________________ Date _________________

Activate Mission Rover Recapture!

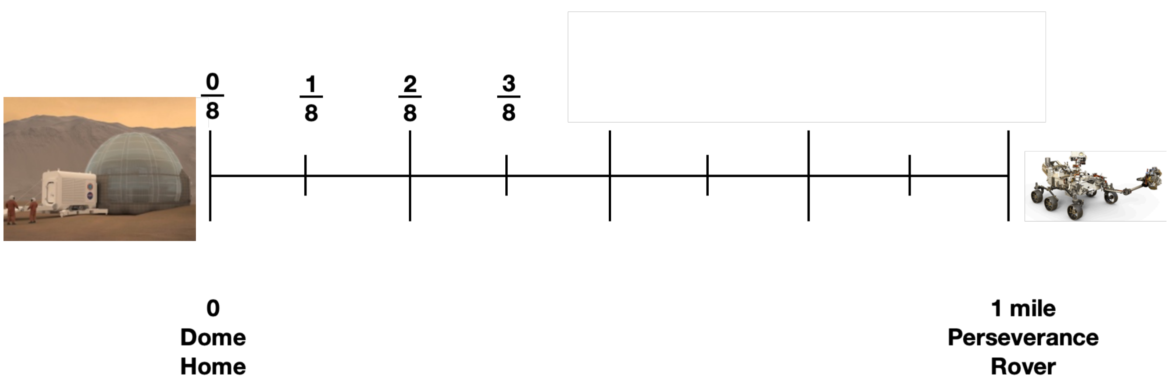
Peggy Whitson Tier 1 Lesson

In 2020, Mars Perseverance Rover cached (buried) two types of test tubes, including at its final resting location, 1 mile away from the dome home. Astronauts Kathryn and Chen are going out together to bring back the rover. Along the way, they will collect all the cached samples.



First, Astronaut Kathryn is collecting the **Life Tubes**. These tubes ran specific tests looking for cellular life (biosignatures).

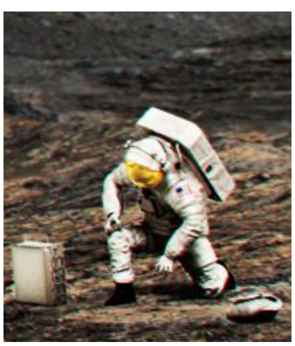
1. If these **Life Tubes were cached every 1/8 of a mile,** including at the rover's final, resting location, where should Kathryn look? Place an "L" (for Life Tubes) at every location Kathryn should stop and dig? Label each "L" using a fraction. The first few fractions have been labeled for you.



2. How did you know where to put an "L"? Use fractions in your response.

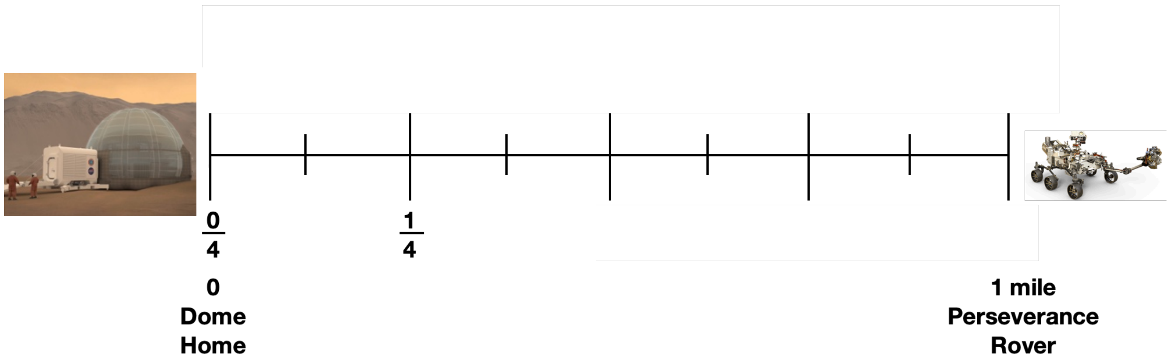
Guiding Questions	Your Response
In general-what is a fraction?	
How did you know what the denominator would be for each "K" location?	
How did you know what the numerator would be for each "K" location?	

3. How many total Life Tubes will Kathryn collect? __________



At the same time, Astronaut Chen is collecting the **Soil Tubes**. These tubes tested the soil composition in specific locations on the surface of Mars. These were cached every 1/4 of a mile, including in the rover's final, resting location.

4. If these **Soil Tubes were cached every 1/4 of a mile,** including at the rover's final, resting location, where should Chen look? Place an "S" (for Soil Tubes) at every location Chen should stop and dig. Label each "S" using a fraction. (Two have been labeled for you.)



5. How did you know where to put an "S"? Use fractions in your response.

Guiding Questions	Your Response
In general-what is a fraction?	
How did you know what the denominator would be for each "S" location?	
How did you know what the numerator would be for each "S" location?	

6. How many Soil Tubes will Chen collect? _________

7. Who is most likely to get to the rover first? Why? (Incorporate fractions into your response.)

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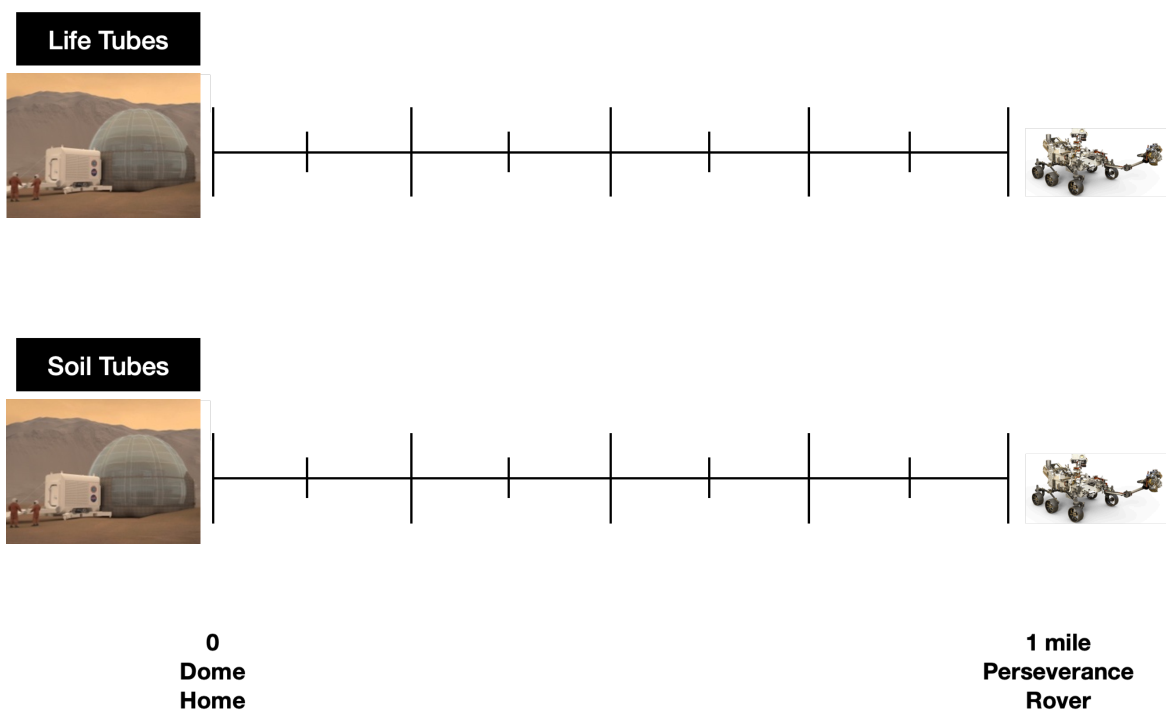
8. Returning to the Introductory Task, remember the Soil Tubes are located every 1/4 of a mile from the dome, and the Life Tubes are located every 1/8 of a mile from the dome, including at the rover's final resting place. **Are there any locations where a Soil Tube is cached in the SAME location as the Life Tube? Explain your answer using fractions and the number lines below.**

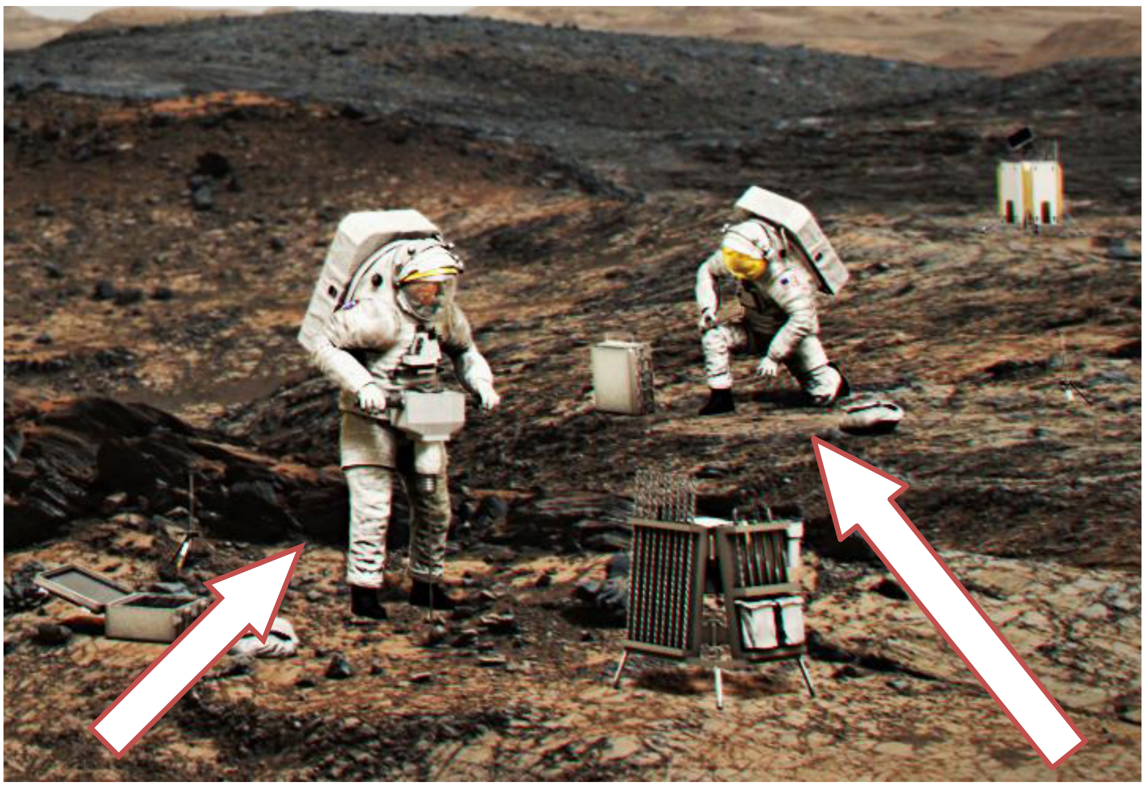


Rover Recapturer ___________________________ Date _________________

Activate Mission Rover Recapture!

Guion Bluford Tier 2 Lesson

In 2020, Mars Perseverance Rover cached (buried) two types of test tubes, including at its final resting location, 1 mile away from the dome home. Now, Astronauts Kathryn and Chen are going out together to bring back the rover. Along the way, they will collect all the cached samples.



(Source: https://images.nasa.gov/details-PIA24032)

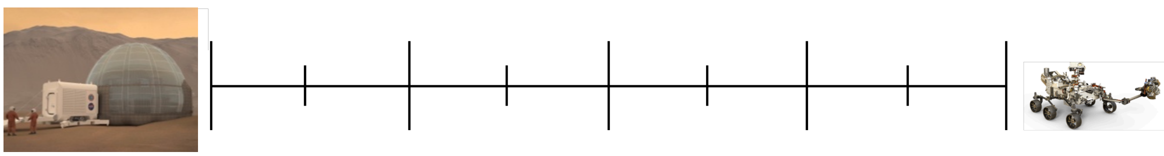
Astronaut Kathryn is collecting the **Life Tubes**. Again, these tubes ran specific tests looking for cellular life (biosignatures). **Life Tubes were cached every 1/8 of a mile,** including at the rover's final, resting location.

At the same time, Astronaut Chen is collecting the **Soil Tubes**. These tubes tested the soil composition in specific locations on the surface of Mars. **Soil Tubes were cached every 1.4 of a mile**, including in the rover's final, resting location.

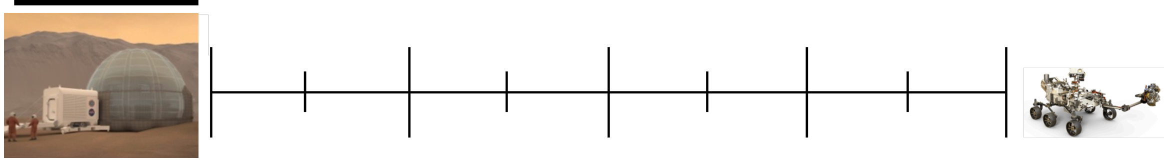
1a. On the number line below, place a “L” (for Life Tubes) where Kathryn will need to stop to collect a tube. Label each “L” with the appropriate fraction.

1b. On the number line below, place a “S” (for Soil Tubes) where Chen will need to stop to collect a tube. Label each “S” with the appropriate fraction.

Life Tubes



Soil Tubes



0
Dome
Home

1 mile
Perseverance
Rover

2. When they reach the rover, who will have collected the most tubes? ________

3. Imagine both Kathryn and Chen have collected 3 tubes.

   a. What fraction of tubes has Kathryn collected? _______

   b. What fraction of tubes has Chen collected? _______

   c. Who will reach the rover first? ______________

   d. Are fourths bigger or smaller than eighths? How do you know?

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4. Astronaut Kathryn radioed Mission Control: I am one third of the way to the rover. How many tubes has she collected? How do you know?

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5. Returning to the Introductory Task, remember the Soil Tubes are located every 1/4 of a mile from the dome, and the Life Tubes are located every 1/8 of a mile from the dome, including at the rover's final resting place. **Are there any locations where a Soil Tube is cached in the SAME location as the Life Tube? Explain your answer using fractions.** (You may want to use your number line above or make your own.)

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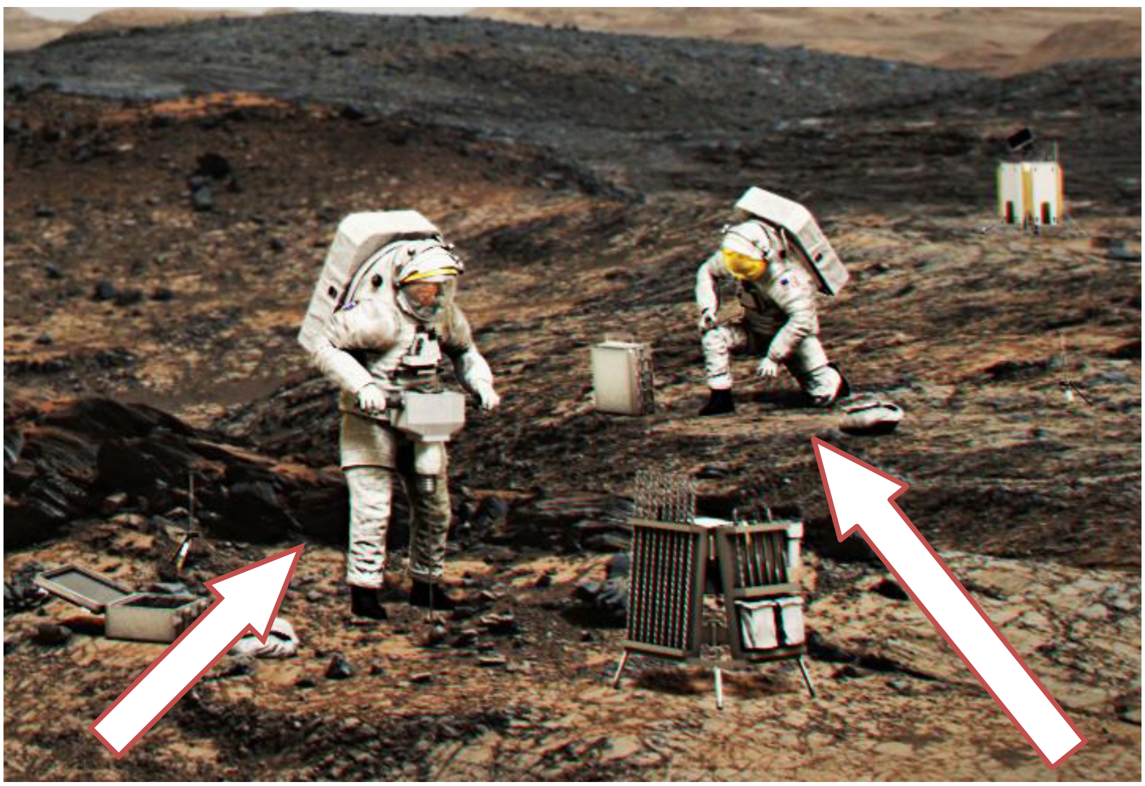
Rover Recapturer ____________________________ Date __________________

Activate Mission Recapture!

Ellen Ochoa Tier 3 Lesson

In 2020, Mars Perseverance Rover cached (buried) two types of test tubes, including at its final resting location, 1 mile away from the dome home. Now, Astronauts Kathryn and Chen are going out together to bring back the rover. Along the way, they will collect all the cached samples.



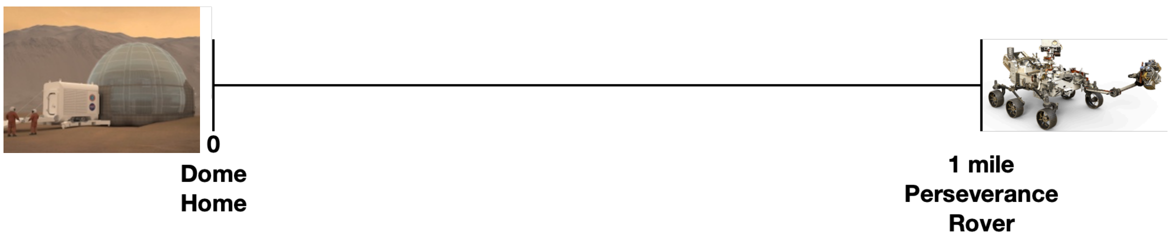
(Source: https://images.nasa.gov/details-PIA24032)

Astronaut Kathryn is collecting the **Life Tubes**. Again, these tubes ran specific tests looking for cellular life (biosignatures). **Life Tubes were cached every 1/8 of a mile,** including at the rover's final, resting location.

At the same time, Astronaut Chen is collecting the **Soil Tubes**. These tubes tested the soil composition in specific locations on the surface of Mars. **Soil Tubes were cached every 1/4 of a mile**, including in the rover's final, resting location.

2. Both Kathryn and Chen have collected 3 tubes. Who is most likely to get to the rover first? Why? (Incorporate fractions AND a number line into your response.)

Mission: Rover Recapture



3. Imagine Astronaut Kathryn radioed Mission Control to report: “I am one third of the way to the rover.” How many tubes has she collected? How do you know?

4. Which is bigger: thirds, fourths, or eighths? How do you know?

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5. What is *another* way you might demonstrate which is bigger: thirds, fourths, eighths?

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6. Astronaut Chen says when the numerator is the same, fractions with larger denominators are larger than fractions with smaller denominators. Is this general rule true? How could you explain your reasoning to Astronaut Chen?

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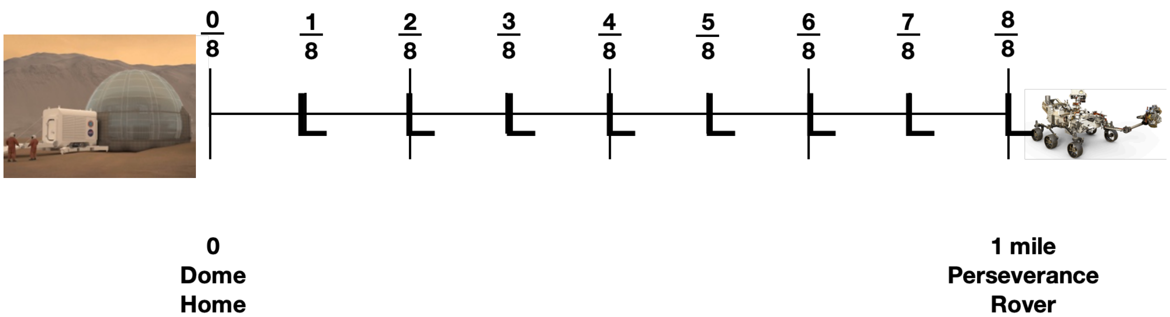
(Source: All rover images: https://mars.nasa.gov/resources/mars-2020-rover-artists-concept/, all Mars home images: https://www.nasa.gov/feature/langley/a-new-home-on-mars-nasa-langley-s-icy-concept-for-living-on-the-red-planet)

Peggy Whitson Tier 1 Lesson

1. *Students should finish labeling the number line with fractions and place an "L" in each Life Tube location as follows:*

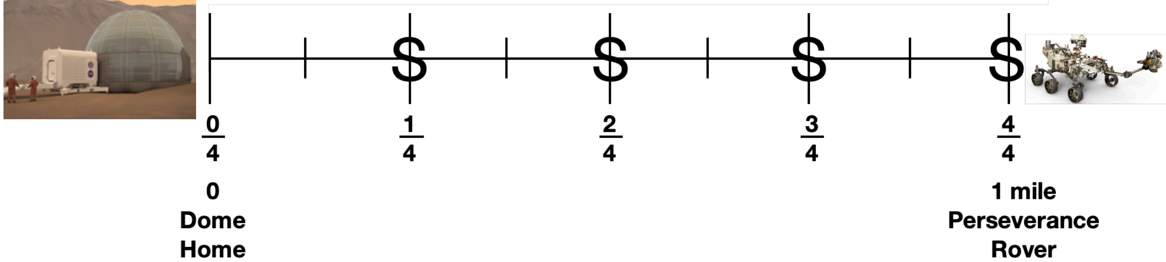


2. *Answers may vary, but a sample response is provided below:*

Guiding Questions	Your Response
In general-what is a fraction?	*A fraction is a part of a whole.*
How did you know what the denominator would be for each "K" location?	*The denominator is how many parts the whole is equally broken into, so the denominator is 8. I knew that because the Life Tubes are buried every eighth of a mile.*
How did you know what the numerator would be for each "K" location?	*The numerator is how many parts of the whole is being traveled, so the numerator starts with 1 and increases by 1 every eighth of a mile.*

3. *Kathryn will collect 8 Life Tubes.*

4. *Students should finish labeling the number line with fractions and place an "S" in each Soil Tube location as follows:*



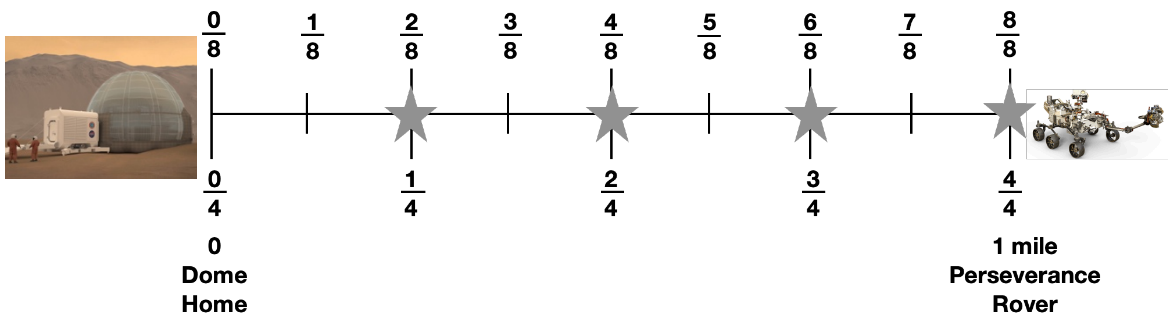
5 *Answers may vary, but a sample response is provided below:*

Guiding Questions	Your Response
In general-what is a fraction?	*A fraction is a part of a whole.*
How did you know what the denominator would be for each "K" location?	*The denominator is how many parts the whole is equally broken into, so the denominator is 4. I knew that because the Life Tubes are buried every fourth of a mile.*
How did you know what the numerator would be for each "K" location?	*The numerator is how many parts of the whole is being traveled, so the numerator starts with 1 and increases by 1 every fourth of a mile.*

6. *Chen will collect 4 Soil Tubes.*

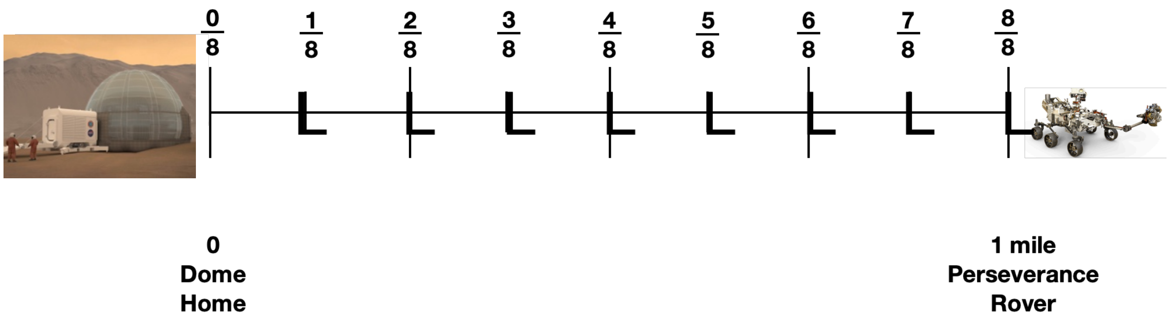
7. *Chen is likely to get to the rover first because he only must stop to collect tubes 4 times or every 1/4 of a mile, whereas Kathryn must stop 8 times or every 1/8 of mile.*

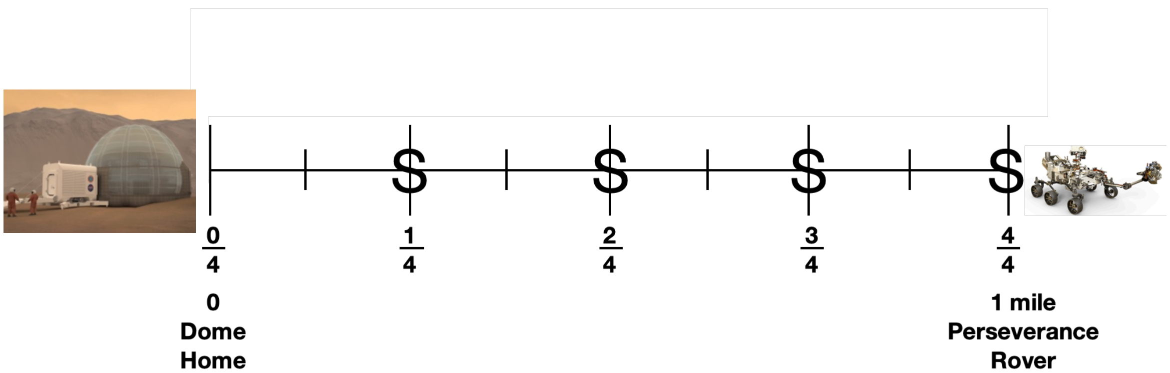
8. *Yes, there are 4 places in which both Kathryn and Chen will collect tubes. Students may show their answer on two separate lines and draw a line to highlight where the locations overlap. They could also use the same number line as illustrated below. Students should recognize that 2/8 is the same location as 1/4, similarly 4/8 is the same location as 2/4. This observation should be delineated in their response.*



Guion Bluford Tier 2 Lesson

1. *Students should finish labeling the number line with fractions and place an "S" in each Soil Tube location as follows:*





2. *Kathryn will collect the most tubes (8).*

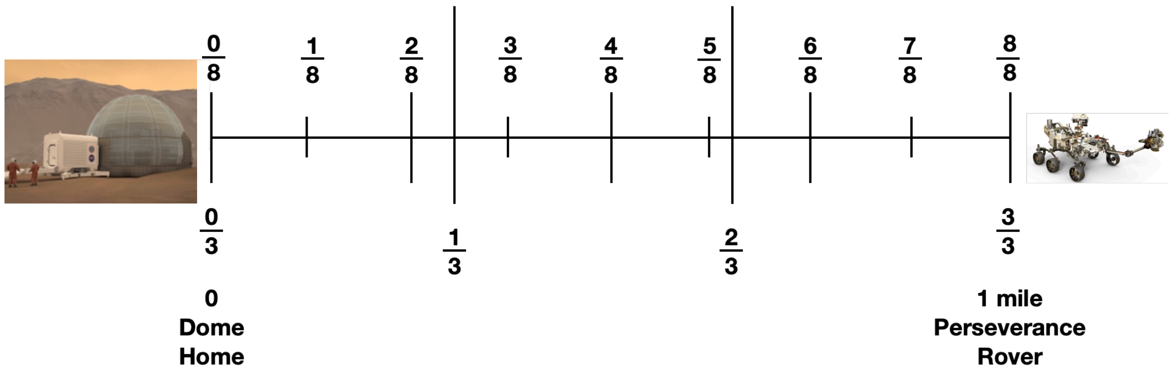
3A. Kathryn will have collected 3/8.

3B. Chen will have collected 3/4.

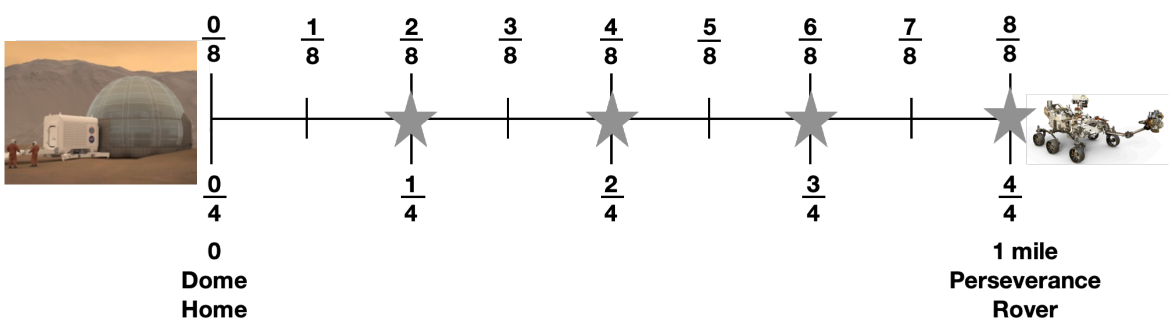
3C. Chen will likely reach the rover first. He only has 1/4 of a mile left, whereas Kathryn is not even halfway there.

3D. Answers will vary, but the key concept is that eighths are smaller than fourths. This is a tricky concept to understand since eight is larger than four. When numbers are used as a denominator, however, it indicates how many total equal pieces exist, so the more people you need to share with (or the more tubes you need to collect), the smaller the pieces are or the smaller the distance is. Students may explain their responses using the number line illustration as well as defining denominators.

4 *There are many correct approaches to solving this problem. This approach is how students may use what they have already explored in this lesson to find the answer. Kathryn will have collected 2 Life Tubes if she is a third of the way to the rover. To explain their response, students could create an additional number line or make additional marks on the existing number line. They need to be able to locate 1/3 on a number line in relationship to eighths.*

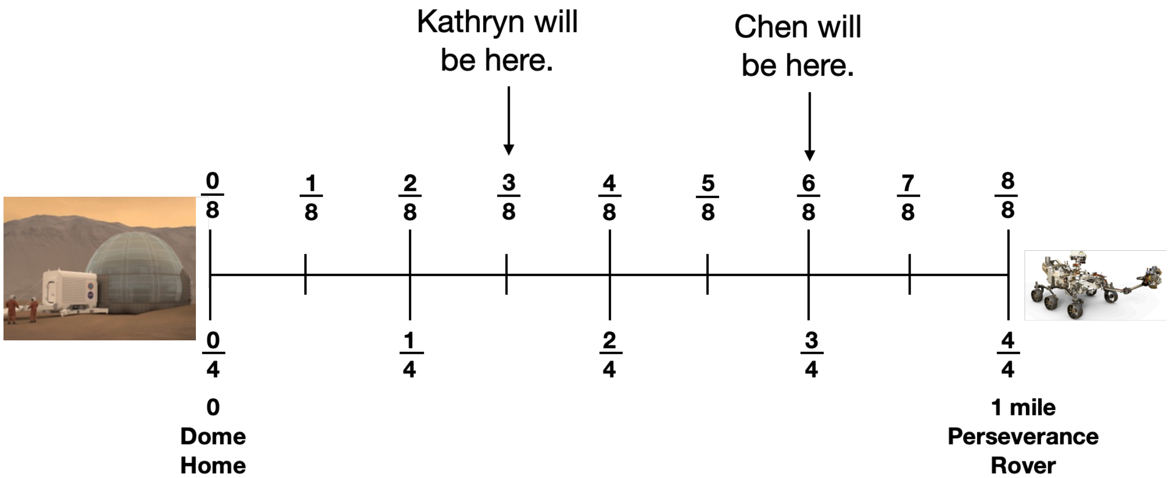


5 *Yes, there are 4 places in which both Kathryn and Chen will collect tubes. Students may show their answer on two separate lines and draw a line to highlight where the locations overlap. They could also use the same number line as illustrated below. Students should recognize that 2/8 is the same location as 1/4, similarly 4/8 is the same location as 2/4. This observation should be delineated in their response.*

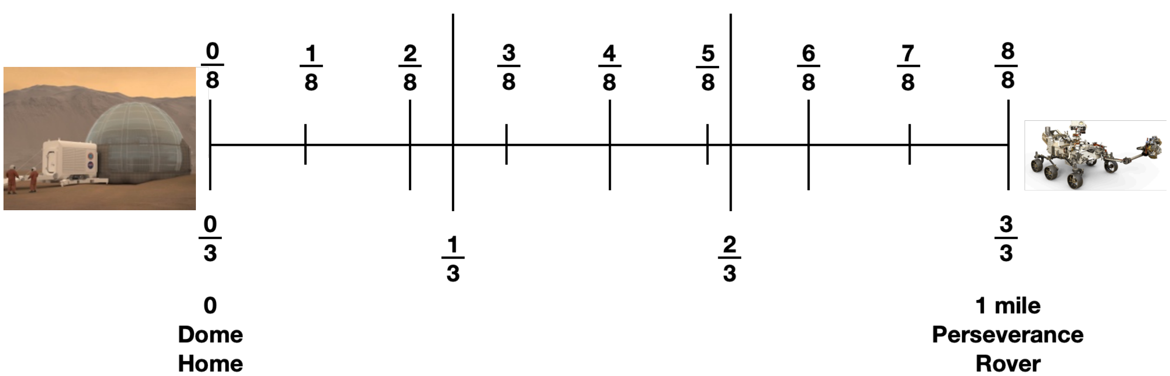


Ellen Ochoa Tier 3 Lesson

1. *If they have both collected 3 tubes, Kathryn will have collected 3/8. Chen will have collected 3/4. Chen will likely reach the rover first. He only has 1/4 of a mile left, whereas Kathryn is not even halfway there.*



2. *There are many correct approaches to solving this problem. This approach is how students may use what they have already explored in this lesson to find the answer. Kathryn will have collected 2 Life Tubes if she is a third of the way to the rover. To explain their response, students could create an additional number line or make additional marks on the existing number line. They need to be able to locate 1/3 on a number line in relationship to eighths.*



3. *Answers will vary, but the key concept is that eighths are smaller than fourths and thirds. This is a tricky concept to understand since eight is larger than four or three. When numbers are used as a denominator, however, it indicates how many total equal pieces exist, so the more people you need to share with (or the more tubes you need to collect), the smaller the pieces are*

or the smaller the distance is. Students may explain their responses using the number line illustration as well as defining denominators.

4. *The answer is the same as #3, but students need to show their response in a different way. They may choose to use paper strips, graph paper, rulers, or even unit/area models.*

5. *This question is asking students develop a general rule that encompasses their work in #3 and #4. The general rule is the larger denominators yield smaller parts when the numerator is the same. Students may provide different forms of evidence to support this claim such as number lines, unit models, or area models.*

Hint Cards* for Understanding Number Lines

Hint 1 What is the whole? The denominator tells you how many EQUAL pieces that whole is broken into.	**Hint 2** How might folding a number line help to be more precise?
Hint 3 How might using graph paper help you create a precise number line?	**Hint 4** If you had to share a whole pizza, would you rather it be broken into fourths or eighths? Could you use this to create a rule about denominator size?

* Hint #4 will be most helpful for Tier 3 students as they develop a rule about the relationship among denominators and fraction size.

Challenge Cards for Understanding Number Lines

Challenge 1

Imagine Mission Control informed Kathryn an additional Life Tube is buried at 9/8 of a mile from the Dome.

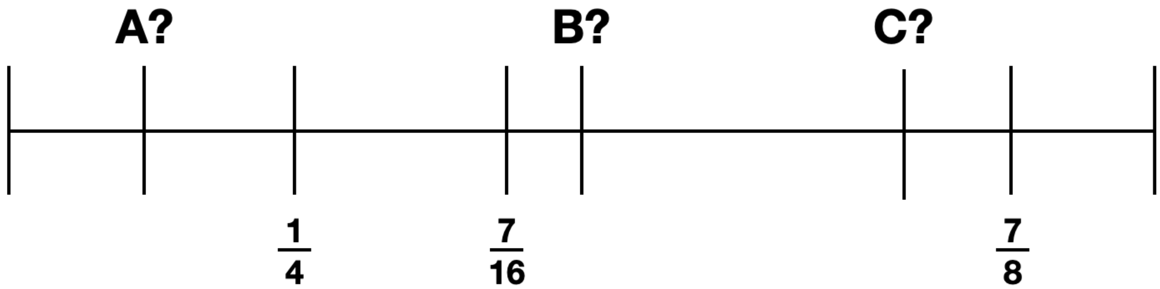
Could you draw her a map so she would know where to dig? Explain your process.

Include the Dome, Perseverance Rover, and the location of the newest Life Tube on your map.

Challenge 2

Kathryn and Chen found unique rocks and labeled them on their maps using letters.

What fractions would best mark where these rocks are located? Can you be more precise?


A?
B?
C?
1/4
7/16
7/8

Lesson Designer: Gregory Boldt

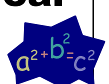
Lesson 10: Comparing Fractions—

Measuring and Mapping Mars

Big Ideas

We can better understand the relationships between fractions by comparing two fractions with the same top number (numerator) or the same bottom number (denominator). By doing this, we can learn to recognize that the numbers represent different relationships even though the numbers are the same. The easiest and most correct comparisons happen when fractions represent the same whole, like when we have two pies that are the same size, so we can begin by making these comparisons. Recognizing the relationship between the numerators and denominators becomes easier when we use a visual model to communicate the relationships between numbers. We can use the symbols >, =, or < to communicate these relationships between numbers with the same numerators or denominators. Understanding these relationships can help us complete many daily tasks, such as following recipes when cooking and recognizing good deals when stores mark items on sale (e.g., a $20 shirt is 1/4 off and a $30 shirt is 1/2 off; which shirt is cheapest?).

Lesson Objectives	• Students will be able to compare two fractions with the same numerator or the same denominator by reasoning about their size. • Students will understand that accurate comparisons must involve fractions representing the same whole. • Students will recognize the patterns involved when comparing fractions with the same numerator or the same denominator. • Students will be able to explain their mathematical thinking and respectfully critique the reasoning of others. **Common Core State Standards** Develop understanding of fractions as numbers.

	CCSS.MATH.CONTENT.3.NF.A.3.D Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.
Materials 	• student whiteboards with dry-erase markers OR scrap paper with pencils (one per student) • smartboard OR whiteboard OR projector for introductory lesson on comparing fractions • Mars Global Surveyor orbiter video (https://youtu.be/6M63GS9ya_U) • Mars Odyssey orbiter video (https://www.youtube.com/watch?v=jbQ4_Y1qWUc) • entrance ticket (one per student) • tiered student pages (with answer keys) • hint and challenge cards (approximately three of each)
Mathematical Terms 	• **Compare:** to examine two or more things and determine their similarities and differences (for example, whether a number is greater, equal to, or smaller than another number) • **Denominator:** bottom number in a fraction that identifies the number of pieces the whole has been divided into equal parts • **Equal:** shows the same amount • **Numerator:** top number in a fraction that identifies the number of equal pieces considered as part of the whole • **Pattern:** predictable, repeated way that something is done or presented • **Symbols:** less than <, equal to =, and greater than > • **Whole:** the entire unit that represents one • **Whole Number:** the set of numbers that include zero and natural numbers

Selected Mathematical Practices	• MP3: Construct viable arguments and critique the reasoning of others. *I can explain my math thinking and talk about it with others.* • MP5: Use appropriate tools strategically. *I know how to choose and use the right tools to solve a math problem.* • MP7: Look for and make use of structure. *I can use what I know to solve new problems.* • MP8: Look for and express regularity in repeated reasoning. *I can solve problems by looking for rules and patterns.*
Differentiation	**Content** ***Guiding Questions*** • learning objectives *What do you want students to know, understand, and be able to do?* • prior knowledge or learner readiness *What evidence do you have about students' current knowledge and skills?* • tiered activities *How will you design tiered activities on the same mathematical concept with varied levels of difficulty?* • formative assessment *What techniques will you use to assess students' prior knowledge and skills?* • varied levels of challenge *How will you vary the level of difficulty for each tiered activity?* • "teaching up" (aim high, provide scaffolding) *How will you increase the depth, breadth, complexity, and abstractness of lessons to challenge and support student learning?* • know (information, facts, vocabulary), understand (concepts, big ideas, connections), apply (skills, processes) *How will you ensure students have a deep understanding of mathematical concepts and skills?* • real-world application *What real-world connections will you make explicit about mathematical concepts and skills?*

	Process ***Guiding Questions*** • questioning strategies *How will you pose and how will you encourage students to pose open-ended, closed-ended, lower-level, and higher-level questions to promote mathematical discourse?* • 4Cs (21st Century Skills) ○ critical thinking *How will you promote a learning environment in which students question data and view issues or problems from multiple perspectives?* • 4Cs (21st Century Skills) ○ creative thinking *How will you encourage students to "think outside the box" and synthesize information in new, different, and useful ways?* • 4Cs (21st Century Skills) ○ collaboration *How will you encourage students to work with other students and appreciate their contributions to solving problems or making connections to prior work?* • 4Cs (21st Century Skills) ○ communication *How will you promote students' opportunities to communicate face-to-face, in large and small groups, in online environments, and with print and non-print resources using their oral, written, and non-verbal skills?* • hands-on activities/manipulatives *How will you incorporate activities promoting the use of manipulatives to clarify or illustrate mathematical concepts?* • connections *How will you use "big ideas" to emphasize connections between and among mathematical concepts and skills and their connections to real-world situations?* **Product** ***Guiding Questions*** • multiple ways to demonstrate knowledge, understanding, and skills *How will you encourage students to share their understanding of mathematical concepts and skills?*

	• summative assessment *How will you assess student learning upon completion of the lesson?* **Learning Environment** ***Guiding Questions*** • flexible grouping *How will you use your tiered lesson to support flexible grouping?* • whole group/small group/individual instruction *How will you incorporate different grouping plans to address students' learning needs?* • growth mindset *How will you promote the perspective that it is important to view abilities as malleable?* • learning community *How will you support a positive learning community as students are encouraged to think, work, and communicate like mathematicians?*

Lesson Preview

The Global Surveyor and Odyssey were both active between 2001 and 2006. During this time, they collected measurements of Mars' radiation levels and magnetic fields. Every day, each orbiter had a job that was divided into a number of measurements they were supposed to take. NASA (National Aeronautics and Space Administration) researchers were comparing what fraction of their total job each orbiter completed to see which orbiter was more successful on a given day.

In this lesson, students will work together to discover the structure and repeated reasoning comparing fractions involves. Students will use the symbols >, =, or < to compare different fractions representing the number of measurements completed (numerator) out of the total number of measurements needed that day (denominator), including whole numbers expressed as fractions.

Launch

1. **Thinking Like Mathematicians**

As a whole group, discuss how mathematicians react when they disagree about a solution. Consider writing a list of the students' ideas on a whiteboard or chart paper. Ask students:

- How do mathematicians react when they disagree about a solution?
- How might discussing disagreements help mathematicians?

If students struggle to generate ideas, explain that when mathematicians disagree, they listen respectfully to others' perspectives and think about why and how their answers are different. Explain that mathematicians learn through disagreements and from hearing different perspectives to strengthen their knowledge of mathematics.

After discussing how mathematicians settle disagreements, show your class these short videos describing histories of the Global Surveyor (https://youtu.be/6M63GS9ya_U) and Odyssey (https://www.youtube.com/watch?v=jbQ4_Y1qWUc) orbiters. From there, explain to your students that their mission for the day will be to help NASA (National Aeronautics and Space Administration) researchers compare fractions representing the number of measurements each orbiter completed (numerator) out of the total number of measurements they were assigned that day (denominator).

Explore

2. Measuring and Mapping Mars

Begin students' exploration with an overview of the mathematical concepts they will use in this lesson.

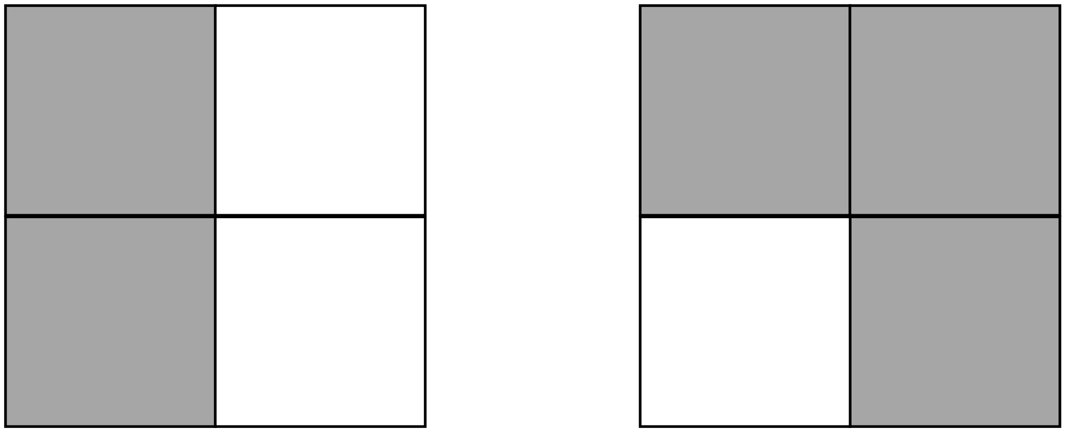
Ask:

- What does each of the following symbols represent: <, =, >
  - ***Sample response:*** *They mean less than <, equal to =, and greater than >.*
- Can everyone write down an expression that compares two numbers using one of these symbols?
  - ***Sample response:*** *5<10.*
- We have learned that we use these symbols to compare whole numbers, but how could we use them to compare fractions?
  - ***Sample response:*** *We could use them to say that half a pizza is less than a whole pizza.*

Explain:

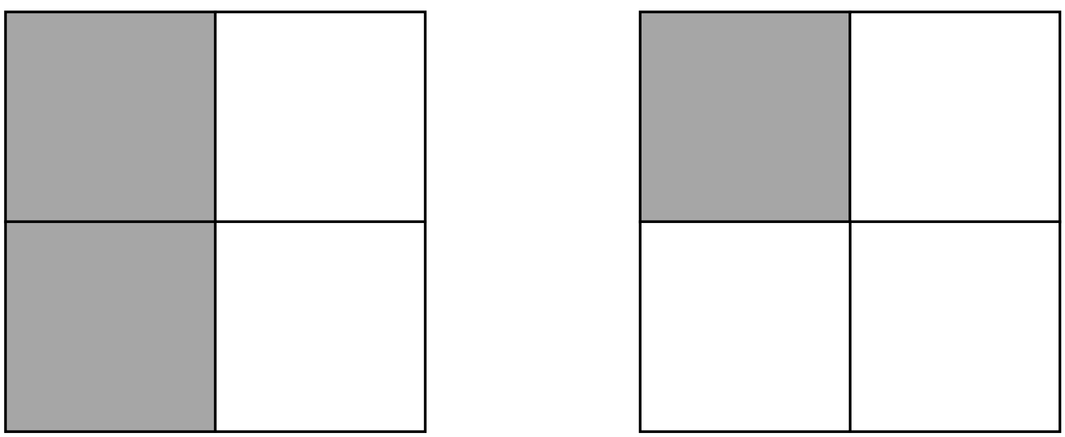
- We can use these symbols to compare fractions, just like we would with whole numbers.
- When we are comparing two fractions with the same denominator, the fraction with the larger numerator is larger because we have more parts of out the same whole.
  - Demonstrate this using the following fraction models (you can also have students participate in demonstrating this concept).

1/2 < 3/4



- When we are comparing two fractions with the same numerator, the fraction with the smaller denominator is larger because each part represents a smaller piece of the whole.
  - Demonstrate this using the following fraction models (you can also have students participate in demonstrating this concept).

1/2 > 1/4



- These comparisons are only valid when fractions represent the same whole (e.g., 3/4 of a day's measurements ≠ 3/4 of a week's measurements).

In the subsequent investigation, students will work on one set of the Student Pages based on their differentiated groups. The groups are based on teacher's observations of students' understanding.

To assess students' readiness for the tiered lesson activities, hand out a copy of the entrance ticket below to each student. When all students have finished, display the correct answers and ask students to self-grade their work. Students with 0–1 correct answers should be assigned to Tier 1: Peggy Whitson, students with 2–3 correct answers should be assigned to Tier 2: Guion Bluford, and students with 4–5 correct answers should be assigned to Tier 3: Ellen Ochoa.

Entrance Ticket

Between 2001 and 2006 the Global Surveyor and Odyssey took measurements of Mars' radiation levels and magnetic fields. Every day, each orbiter had a job that was divided into several planned measurements. NASA (National Aeronautics and Space Administration) researchers compared what fraction of their job each orbiter completed to see which orbiter was more successful.

For each question, circle the correct symbol (less than <; equal to =; greater than >) to compare the fractions.

(Source: https://mars.nasa.gov/mars-exploration/missions/mars-global-surveyor/)

(Source: https://mars.nasa.gov/odyssey/)

Global Surveyor				**Odyssey**
½	<	=	>	4/8
1/3	<	=	>	2/3
¼	<	=	>	1/8
½	<	=	>	3/4
1/3	<	=	>	4/9

Entrance Ticket Answer Key

(Source: https://mars.nasa.gov/mars-exploration/missions/mars-global-surveyor/)

Global Surveyor

(Source: https://mars.nasa.gov/odyssey/)

Odyssey

1/2	<	=	>	4/8
1/3	<	=	>	2/3
1/4	<	=	>	1/8
1/2	<	=	>	3/4
1/3	<	=	>	4/9

Groups Formed by Student Readiness		
Tier 1: Peggy Whitson	**Tier 2: Guion Bluford**	**Tier 3: Ellen Ochoa**
Student Names	**Student Names**	**Student Names**

Collaborate and Communicate

Have students record their ideas on their individual worksheets or on one for the small group. Help them clarify their ideas by asking questions like, “What do you mean here?” and “How might you share that idea with the rest of the class?” Point out that mathematicians use various representations to help explain their thoughts and use precise language to do so.

A. [Possible response]	**B.** [Possible response]	**C.** [Possible response]
This group . . .	*This group . . .*	*This group . . .*

Examine and Elaborate

3.

Highlight Students' Mathematical Thinking

Mathematicians think about possible solutions in a variety of ways. Therefore, it is important for students to realize that they, too, can approach problems using different strategies. Ultimately, students need to understand that a possible solution should be judged by the accuracy of the mathematics, and there might be some valid ideas within a solution when a student has an incorrect answer.

Share and Discuss

It is therefore important for students to begin by clearly sharing their ideas with others so their validity can be determined by the class.

Teacher: Today, we practiced comparing different fractions representing two Mars orbiters' measurements. What rule do we follow when we compare two fractions with the same denominator? For example, let's compare 1/4 and 3/4. Take some time to think carefully before answering. (*Wait Time talk move*)

Osmund: When the denominators are the same, the fraction with the bigger numerator is bigger: 3/4 is bigger than 1/4.

Teacher: Osmund is saying that when two fractions have the same denominator, the fraction with the larger numerator is greater than the fraction with the smaller numerator, which means that 3/4 is greater than 1/4 because it has the greater numerator. (*Revoicing talk move*) Do you agree with Osmund? Explain how we can show that 3/4 is greater than 1/4. (*Reasoning talk move*)

Neas: We can use a number line to show that 3/4 is more parts of a whole than 1/4.

Teacher: That makes sense to me. Does anyone have anything to add? (*Adding On talk move/Wait Time talk* move)

Dan: A fraction model could also show that we have more parts of the whole.

Teacher: Thank you, Dan. That is another helpful way to show that 3/4 is greater than 1/4. Now, how about when we are comparing fractions that have the same numerator? What rule do we follow? For example, let's compare 1/2 and 1/4.

Hewie: I think 1/4 is bigger than 1/2 because 4 is bigger than 2.

Athene: I don't think that's right. I think 1/2 is bigger than 1/4.

Teacher: Can someone add to Athena's response and tell me why they think 1/2 is bigger than 1/4? Remember that even when mathematicians disagree, they listen to one another and discuss things respectfully. (*Adding On talk move*)

Keefe: The rule is that when our numerators are the same, the fraction with the bigger denominator is smaller. So, 1/4 is smaller than 1/2. If you fill it in with a model you can see that 1/4 has less.

Teacher: Thank you, Keefe. Using a model can be a helpful strategy. Can someone else explain Keefe's answer using their own words? (*Repeat/Rephrase talk move*)

Kamilla: Fractions with bigger denominator are actually smaller.

Ruben: We can also use our fraction tools to make sure. (*Adding On talk move*).

Teacher: Excellent work, everyone. You shared your ideas like respectful mathematicians. Thank you!

Differentiate Further as Needed

Hint Cards

If students have difficulty completing the questions in their tiered activities, you may provide them with hint cards, which have been specifically designed for each question in each tier.

Tier 1: Peggy Whitson

Question 1:	**Question 2:**
1/4 and 3/4 are both fractions where the whole has been divided into 4 parts. Remember that the numerator (top number) represents how many parts of the whole we have.	Remember that the denominator (bottom number) represents how many parts the whole has been divided into. Look at the fraction model you made. What fraction shows more of the whole?
Question 3: Less<More More>Less Equal=Equal	**Question 4:** Remember that the numerator (top number) represents how many parts of the whole we have. The denominator (bottom number) represents how many parts the whole has been divided into.

Sample response *for Question 2: 1/4 shows more of the whole.*

Tier 2: Guion Bluford

Question 1:	**Question 2:**
Which is bigger, 1/2 of a pizza or 1/4 of a pizza? Draw a model if you are not sure.	Which is bigger, 3/4 of a pizza or 1/4 of a pizza? Draw a model if you are not sure.
Question 3: Try using fraction bars, a fraction model, or a number line to show each fraction. Remember: Less<More More>Less Equal=Equal	**Question 4:** Remember, if denominators are the same, the fraction with the bigger numerator is greater. If numerators are the same, the fraction with the smaller denominator is greater. Some of these questions might have more than one correct answer.

Sample response *for Question 1: 1/2 is bigger than 1/4.*

Sample response *for Question 2: 3/4 is bigger than 1/4.*

Tier 3: Ellen Ochoa

Question 1:	Question 2:
Think about whether each question is discussing a larger or smaller amount of the whole. When is it better to have more? When could it be better to have less?	It is easier to compare fractions when they have the same denominator. Can you make each fraction’s denominator equivalent?
Question 3: Each question will only have one correct answer. Try coming up with a few answers that make sense for each of the two comparisons. Which option works with both comparisons?	

Sample response *for Question 1: It is better to have more when we want to have more parts. It is better to have less if we want fewer parts.*

Challenge Cards

If students complete all questions in their tiered activities with time remaining, you may provide them with challenge cards, which have been specifically designed for each tier.

Tier 1: Peggy Whitson

Compare these different fractions with different numerators and denominators: 3/4 < = > 2/3 3/10 < = > 1/3 3/5 < = > 6/10 2/5 < = > 1/4	Look at the pictures of the Global Surveyor and the Odyssey orbiters. How many solar panels does each orbiter have? How could we represent these as fractions showing the number of operational solar panels for each orbiter? Compare these two numbers using the correct symbol.
Two scientists are comparing fractions. One says that a fraction cannot be greater than 4/4 because it represents a whole. The other says that the fraction 6/4 is greater than 4/4 because it represents more than one whole. Which scientist do you agree with? Find a partner and explain your reasoning.	Comparing fractions was important in this Mission to Mars, but we compare fractions in many other situations. What are some other situations where we may need to compare fractions? Come up with as many answers as you can.

Tier 2: Guion Bluford

Sometimes we need to compare fractions like 5/2 or 1 and 5/10. Compare these fractions: 3/4 < = > 2/3 20/5 < = > 35/10 10/8 < = > 2 & 1/2 7/3 < = > 10/4	Look at the pictures of the Global Surveyor and the Odyssey orbiters. How many solar panels does each orbiter have? If space junk damages 2 solar panels on the Global Surveyor and 1 solar panel on the Odyssey, what orbiter has a greater fraction of total operational solar panels?
Two scientists are comparing fractions. One says that there are fractions greater than 4/4. The other says there cannot be because 4/4 represents a whole. Which scientist do you agree with? Find a partner and explain your reasoning.	Comparing fractions was important in this Mission to Mars, but we compare fractions in many other situations. What are some other situations where we may need to compare fractions? Come up with as many answers as you can.

Tier 3: Ellen Ochoa

Sometimes we need to compare fractions like 5/2 or 1 and 5/10. Compare these fractions: 6/4 < = > 3/2 20/5 < = > 35/10 10/8 < = > 2 & 1/2 7/3 < = > 10/4	Look at the pictures of the Global Surveyor and the Odyssey orbiters. How many solar panels does each orbiter have? If space junk damages 1 solar panel on each orbiter, what orbiter has a greater fraction of total operational solar panels?
Two scientists are discussing the fraction 5/8. The first scientist says that there are only three fractions greater than 5/8: 6/8, 7/8 and 8/8. The second scientist says there are many more than three, and there are too many to list. Which scientist do you agree with? Find a partner and explain your reasoning.	Comparing fractions was important in this mission to Mars, but we compare fractions in many other situations. What are some other situations where we may need to compare fractions? Come up with as many answers as you can.

Challenge Cards (Answer Key)

Tier 1: Peggy Whitson

Compare these different fractions with different numerators and denominators: 3/4 < = > 2/3 3/10 < = > 1/3 3/5 < = > 6/10 2/5 < = > 1/4	Look at the pictures of the Global Surveyor and the Odyssey orbiters. How many solar panels does each orbiter have? How could we represent these as fractions showing the number of operational solar panels for each orbiter? Compare these two numbers using the correct symbol. *The Global Surveyor has 4 solar panels and the Odyssey has 3 solar panels.* *4/4=3/3*
Two scientists are comparing fractions. One says that a fraction cannot be greater than 4/4 because it represents a whole. The other says that the fraction 6/4 is greater than 4/4 because it represents more than one whole. Which scientist do you agree with? Find a partner and explain your reasoning. *The second scientist is right; 6/4>4/4 because 4/4 is a whole and 6/4 is more than a whole. Fractions can be greater than a whole.*	Comparing fractions was important in this Mission to Mars, but we compare fractions in many other situations. What are some other situations where we may need to compare fractions? Come up with as many answers as you can. *We sometimes compare fractions when baking, dividing food, talking about money, thinking about sales, or describing some sports statistics.*

Tier 2: Guion Bluford

Sometimes we need to compare fractions like 5/2 or 1 and 5/10. Compare these fractions: 3/4 < = > 2/3 20/5 < = > 35/10 10/8 < = > 2 & 1/2 7/3 < = > 10/4	Look at the pictures of the Global Surveyor and the Odyssey orbiters. How many solar panels does each orbiter have? If space junk damages 2 solar panels on the Global Surveyor and 1 solar panel on the Odyssey, what orbiter has a greater fraction of total operational solar panels? *The Global Surveyor has 4 solar panels and the Odyssey has 3 solar panels. If 2 of the Global Surveyor's solar panels were damaged, it would have 2/4 remaining. If 1 of the Odyssey's solar panels was damaged it would have 2/3 remaining.* *2/4<2/3*
Two scientists are comparing fractions. One says that there are fractions greater than 4/4. The other says there cannot be because 4/4 represents a whole. Which scientist do you agree with? Find a partner and explain your reasoning. *The first scientist is correct. Fractions can be greater than 1 whole. For example, 1 & 1/2 or 5/4.*	Comparing fractions was important in this Mission to Mars, but we compare fractions in many other situations. What are some other situations where we may need to compare fractions? Come up with as many answers as you can. *We sometimes compare fractions when baking, dividing food, talking about money, thinking about sales, or describing some sports statistics.*

Tier 3: Ellen Ochoa

Sometimes we need to compare fractions like 5/2 or 1 and 5/10. Compare these fractions: 6/4 < = > 3/2 20/5 < = > 35/10 10/8 < = > 2 & 1/2 7/3 < = > 10/4	Look at the pictures of the Global Surveyor and the Odyssey orbiters. How many solar panels does each orbiter have? If space junk damages 1 solar panel on each orbiter, what orbiter has a greater fraction of total operational solar panels? *The Global Surveyor has 4 solar panels and the Odyssey has 3 solar panels. If 1 of the Global Surveyor's solar panels were damaged, it would have 3/4 remaining. If 1 of the Odyssey's solar panels was damaged it would have 2/3 remaining.* *3/4>2/3*
Two scientists are discussing the fraction 5/8. The first scientist says that there are only three fractions greater than 5/8: 6/8, 7/8 and 8/8. The second scientist says there are many more than three, and there are too many to list. Which scientist do you agree with? Find a partner and explain your reasoning. *The second scientist is correct. We can have fractions that are greater than 1 whole, such as 9/8, or 2 and 3/8. We can also have fractions with different denominators that are greater than 5/8, such as 5/7 or 9/10.*	Comparing fractions was important in this mission to Mars, but we compare fractions in many other situations. What are some other situations where we may need to compare fractions? Come up with as many answers as you can. *We sometimes compare fractions when baking, dividing food, talking about money, thinking about sales, or describing some sports statistics.*

Debrief and Look Ahead

4.

Debrief Content and Skills

Reiterate to students that in this lesson, fractions represented the number of measurements completed out of the total number of measurements needed that day. Remind students that the <, =, and > symbols can be used to compare different numbers, including fractions.

Ask:

- How do we compare fractions with the same denominator?
  - ***Sample response:*** *When two fractions have the same denominator, the fraction with the larger numerator is greater.*
- How do we compare fractions with the same numerator?
  - ***Sample response:*** *When two fractions have the same numerator, the fraction with the smaller denominator is greater.*

Debrief Thinking Like Mathematicians

Remind students that the mathematical practice for this lesson focused on how mathematicians solve problems and work together. Review some of the ideas that students brainstormed at the beginning of class and have students offer examples of how they acted like mathematicians while they worked together during the lesson.

Ask:

- How can we find patterns in mathematical problems?
  - ***Sample response:*** *We can pay attention when we are solving problems and think critically about how they are similar to other problems.*
- How can we use these patterns to solve new problems?
  - ***Sample response:*** *Based on these patterns, we can make rules that describe how the problems are solved. We can then use these rules to solve new problems.*
- How can we respectfully disagree with other mathematicians and offer new arguments?
  - ***Sample response:*** *Instead of saying "you are wrong," or "that is a bad answer," you can say "I don't agree." You can explain why you are thinking about things differently and explain the different math that you are using.*

Assess

5.

What Students Learned

Formative Assessment

The entrance ticket distributed prior to tiered lesson activities will assess students' initial level of understanding, which will help determine what lesson tier is most appropriate for each student.

Summative Assessment
At the end of the lesson, students will hand in their completed tiered lesson worksheets. Students' responses to these questions will indicate whether they met the lesson's objectives (specifically, CCSS.MATH.CONTENT.3.NF.A.3.D).

Resources
AP Archive. (2018, April 7). *Mars Odyssey launch - 2001 — Today in history* [video]. YouTube. https://www.youtube.com/watch?v=jbQ4_Y1qWUc

VideoFromSpace. (2018, November 7). *OTD in space - Nov. 7: Mars Global Surveyor launches to the red planet* [video]. YouTube. https://youtu.be/6M63GS9ya_U

Mission to Mars Student Pages

Mars Explorer ______________________________ Date _________________

Peggy Whitson Tier 1 Lesson

NASA (National Aeronautics and Space Administration) researchers work as a team. Form groups of 3–5 students and work together to complete these learning activities.

1. Use a model to show the fractions 1/4 and 3/4.

   a) Which fraction has the larger numerator? _______

   b) When our denominator is the same, does a larger numerator mean we have more or less?

   ____

   ____

   ____

2. Use a model to show the fractions 1/4 and 1/8.

   a) Which fraction has the larger denominator? _______

   b) When our numerator is the same, does a larger denominator mean we have more or less?

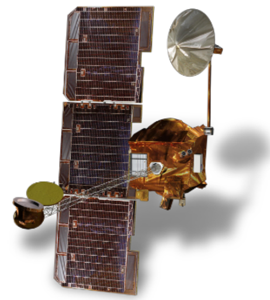
   ____

   ____

   ____

3. NASA researchers want to compare how many jobs the Global Surveyor and Odyssey have completed, but they are having a difficult time. For each question, circle the correct symbol (less than <; equal to =; greater than >) to compare the fractions.

(Source: https://mars.nasa.gov/mars-exploration/missions/mars-global-surveyor/)



(Source: https://mars.nasa.gov/odyssey/)

Global Surveyor		Odyssey
3/4	< = >	2/4
2/3	< = >	2/10
2/2	< = >	2/5
1/3	< = >	3/3
3/8	< = >	5/8

4. The NASA researchers lost some of the Odyssey's measurements! Use the fraction models to find the missing numbers and circle the correct symbol to compare the fractions.

(Source: https://mars.nasa.gov/mars-exploration/missions/mars-global-surveyor/)

Global Surveyor

(Source: https://mars.nasa.gov/odyssey/)

Odyssey

Global Surveyor		Odyssey
1/2	< = >	_/_
3/4	< = >	_/_
2/3	< = >	_/_
6/8	< = >	_/_
3/5	< = >	_/_

5. The NASA researchers want to find a way to easily represent and compare the fraction of jobs completed by each orbiter on a certain day. Each day, the researchers want each orbiter to complete 4 jobs. Design a mathematical tool that might allow the researchers to more easily compare these fractions.

Mission to Mars Student Pages

Mars Explorer ______________________________ Date _________________

Guion Bluford Tier 2 Lesson

NASA (National Aeronautics and Space Administration) researchers work as a team. Form groups of 3–5 students and work together to complete these learning activities.

1. Imagine two fractions that have the same numerator, but different denominators.

   a. Which will be bigger–the fraction with the smaller denominator or the fraction with the bigger denominator?

   ____

   ____

   ____

   b. How do you know?

   ____

   ____

   ____

2. Imagine two fractions that have the same denominator, but different numerators.

   a. Which will be bigger–the fraction with the smaller numerator or the fraction with the bigger numerator?

   ____

   ____

   ____

b. How do you know?

____

____

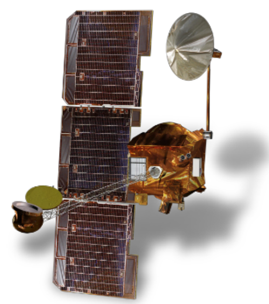
____

3. NASA researchers want to compare how many jobs the Global Surveyor and Odyssey have completed, but they are having a difficult time. Use different fraction tools and models to show them how they can figure out the problems.

   a. For each question, circle the correct symbol (less than <; equal to =; greater than >) to compare the fractions.

(Source: https://mars.nasa.gov/mars-exploration/missions/mars-global-surveyor/)

Global Surveyor



(Source: https://mars.nasa.gov/odyssey/)

Odyssey

3/4	< = >	2/4
3/8	< = >	3/10
2/5	< = >	2/2
2/3	< = >	1/3
1/2	< = >	3/5

   b. What tools worked best to solve these problems?

____

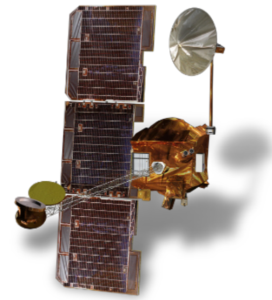
____

____

4. The NASA researchers lost some of the Odyssey's measurements! Help them figure out what fractions are missing.

(Source: https://mars.nasa.gov/mars-exploration/missions/mars-global-surveyor/)

Global Surveyor



(Source: https://mars.nasa.gov/odyssey/)

Odyssey

1/2	<	_/2
3/4	>	3/_
2/3	>	_/3
6/8	<	6/_
4/5	<	_/10

5. The NASA researchers want to find a way to easily represent and compare the fraction of jobs completed by each orbiter on a certain day. Design a mathematical tool that might allow the researchers to more easily compare fractions with:
   a. A numerator of 1
   b. A denominator of 10

Mission to Mars Student Pages

Mars Explorer ______________________________ Date _________________

Ellen Ochoa Tier 3 Lesson

NASA (National Aeronautics and Space Administration) researchers work as a team. Form groups of 3–5 students and work together to complete these learning activities.

1. Think critically about the following questions and provide your best argument.
   a. When would it be better to have a fraction with a larger denominator?

   ____

   ____

   ____

   b. When would it be better to have a fraction with a larger numerator?

   ____

   ____

   ____

   c. When would it be better to have a fraction with a smaller denominator?

   ____

   ____

   ____

   d. When would it be better to have a fraction with a smaller numerator?

   ____

   ____

   ____

2. NASA researchers want to compare how many jobs the Global Surveyor and Odyssey have completed, but they are finding it difficult because the fractions do not have similar numerators or denominators and the researchers do not have models to represent the fractions. How can they solve these problems without using a model?

(Source: https://mars.nasa.gov/mars-exploration/missions/mars-global-surveyor/)

Global Surveyor

(Source: https://mars.nasa.gov/odyssey/)

Odyssey

3/4	< = >	2/3
3/10	< = >	1/3

a. Explain how the researchers can solve the problems without using models.

____

____

____

b. Without using a model, solve each question above by circling the correct symbol (less than <; equal to =; greater than >) to compare the fractions.

3. The NASA researchers lost some of the Odyssey's measurements! Help them figure out what fractions are missing.

(Source: https://mars.nasa.gov/odyssey/)

Odyssey

1/2	<	_/4	<	1
1/4	=	6/_	=	5/20
4/4	>	7/_	>	3/4
2/3	>	_/3	<	1/3
7/8	<	_/2	<	13/10
5/3	<	11/_	<	200/100

4. The NASA researchers want to find a way to easily represent and compare the fraction of jobs completed by each orbiter on a certain day. Sometimes, the denominators of these fractions will not be the same, which can make them difficult to compare. Design a mathematical tool that might allow the researchers to more easily compare these fractions.

____

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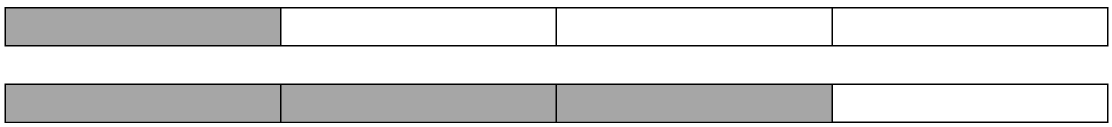
____

Mission to Mars Student Pages with Answer Keys

Peggy Whitson Tier 1 Lesson

NASA (National Aeronautics and Space Administration) researchers work as a team. Form groups of 3–5 students and work together to complete these learning activities.

1. Use a model to show the fractions 1/4 and 3/4.

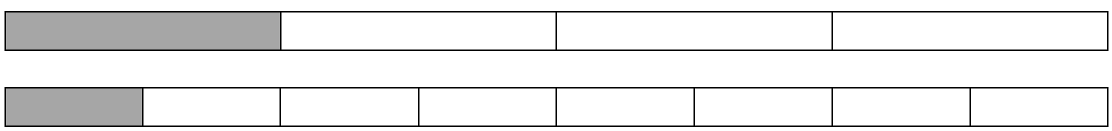


   a. Which fraction has the larger numerator? *1/4*

   b. When our denominator is the same, does a larger numerator mean we have more or less?

   When denominators are the same, a larger numerator means we have more.

2. Use a model to show the fractions 1/4 and 1/8.



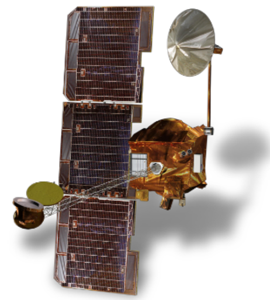
   a. Which fraction has the larger denominator? *1/8*

   b. When our numerator is the same, does a larger denominator mean we have more or less?

   When numerators are the same, a larger denominator means we have less.

3. NASA researchers want to compare how many jobs the Global Surveyor and Odyssey have completed, but they are having a difficult time. For each question, circle the correct symbol (less than <; equal to =; greater than >) to compare the fractions.

(Source: https://mars.nasa.gov/mars-exploration/missions/mars-global-surveyor/)



(Source: https://mars.nasa.gov/odyssey/)

Global Surveyor		Odyssey
3/4	< = >	2/4
2/3	< = >	2/10
2/2	< = >	2/5
1/3	< = >	3/3
3/8	< = >	5/8

4. The NASA researchers lost some of the Odyssey's measurements! Use the fraction models to find the missing numbers and circle the correct symbol to compare the fractions.

(Source: https://mars.nasa.gov/mars-exploration/missions/mars-global-surveyor/)

(Source: https://mars.nasa.gov/odyssey/)

Global Surveyor

Odyssey

Global Surveyor		Odyssey
1/2	< = >	*2/2*
3/4	< = >	*3/8*
2/3	< = >	*1/3*
6/8	< = >	*6/10*
3/5	< = >	*4/5*

5. The NASA researchers want to find a way to easily represent and compare the fraction of jobs completed by each orbiter on a certain day. Each day, the researchers want each orbiter to complete 4 jobs. Design a mathematical tool that might allow the researchers to more easily compare these fractions.

Because the purpose of this activity is to promote student creativity, responses should vary widely.

Sample response: *Students might create a set of bars divided into four sections and use paper sections representing 1/4 to cover the appropriate sections.*

Guion Bluford Tier 2 Lesson

NASA (National Aeronautics and Space Administration) researchers work as a team. Form groups of 3–5 students and work together to complete these learning activities.

1. Imagine two fractions that have the same numerator, but different denominators.

   a. Which will be bigger–the fraction with the smaller denominator or the fraction with the bigger denominator?

   The fraction with the smaller denominator.

   b. How do you know?

   ***Sample response:** Because the same numerator and a smaller denominator means we have the same number of parts, but the parts are bigger, so we have more of the whole. We can use a fraction model to show this.*

2. Imagine two fractions that have the same denominator, but different numerators.

   a. Which will be bigger–the fraction with the smaller numerator or the fraction with the bigger numerator?

   The fraction with the bigger numerator.

   b. How do you know?

   ***Sample response:** Because the same denominator and a bigger numerator means we have more parts of the same whole. We can use a fraction model to show this.*

3. NASA researchers want to compare how many jobs the Global Surveyor and Odyssey have completed, but they are having a difficult time. Use different fraction tools and models to show them how they can figure out the problems.

   a. For each question, circle the correct symbol (less than <; equal to =; greater than >) to compare the fractions.

(Source: https://mars.nasa.gov/mars-exploration/missions/mars-global-surveyor/)

Global Surveyor

(Source: https://mars.nasa.gov/odyssey/)

Odyssey

3/4	<	=	>	2/4
3/8	<	=	>	3/10
2/5	<	=	>	2/2
2/3	<	=	>	1/3
1/2	<	=	>	3/5

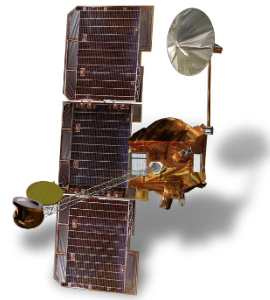
b. What tools worked best to solve these problems?

Sample response: *You can use fraction models or number lines to solve them.*

4. The NASA researchers lost some of the Odyssey's measurements! Help them figure out what fractions are missing.

(Source: https://mars.nasa.gov/mars-exploration/missions/mars-global-surveyor/)

Global Surveyor



(Source: https://mars.nasa.gov/odyssey/)

Odyssey

1/2	<	2/*2*
3/4	>	3/*8* *(Multiple correct answers)*
2/3	>	*1*/3 *or* ***0***/3
6/8	<	6/*6* *(Multiple correct answers)*
4/5	<	*9*/10

5. The NASA researchers want to find a way to easily represent and compare the fraction of jobs completed by each orbiter on a certain day. Design a mathematical tool that might allow the researchers to more easily compare fractions with:
   a. A numerator of 1
   b. A denominator of 10

   Because the purpose of this activity is to promote student creativity, responses should vary widely.

   Sample response *for question a: Students might cut out 20 equally-sized squares, color 2 squares, and leave the rest of the squares uncolored. By* *placing different numbers of uncolored squares beside the colored squares, they can then represent and compare fractions with denominators ranging from 1 to 10.*

Sample response *for question b: Students might create a set of bars divided into ten sections and use paper sections representing 1/10 to cover the appropriate number of sections.*

Ellen Ochoa Tier 3 Lesson

NASA (National Aeronautics and Space Administration) researchers work as a team. Form groups of 3–5 students and work together to complete these learning activities.

1. Think critically about the following questions and provide your best argument.
   a. When would it be better to have a fraction with a larger denominator?

   Sample response: *When you want to have the whole divided into more parts. You might need to divide a cake into more parts if you are sharing it with a large group.*

   b. When would it be better to have a fraction with a larger numerator?

   Sample response: *When you want to have more parts of the whole. It is better to have 4/4 of a tank of gas than 1/4 of a tank of gas.*

   c. When would it be better to have a fraction with a smaller denominator?

   Sample response: *When you need the whole divided into fewer parts. If you are sharing a pizza and you are hungry, you would want 1/2 instead of 1/4.*

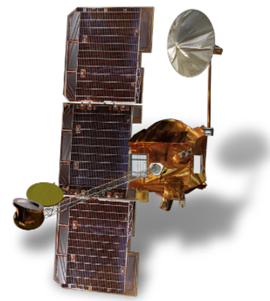
   d. When would it be better to have a fraction with a smaller numerator?

   Sample response: *When you want to have fewer parts of the whole. It is better to have 1/8 of your chores left to do than to have 7/8 of your chores left to do.*

2. NASA researchers want to compare how many jobs the Global Surveyor and Odyssey have completed, but they are finding it difficult because the fractions do not have similar numerators or denominators and the researchers do not have models to represent the fractions. How can they solve these problems without using a model?

(Source: https://mars.nasa.gov/mars-exploration/missions/mars-global-surveyor/)

Global Surveyor



(Source: https://mars.nasa.gov/odyssey/)

Odyssey

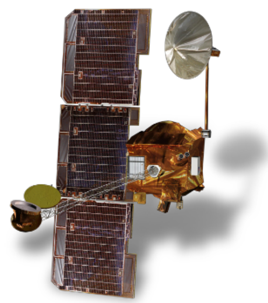
3/4	< = >	2/3
3/10	< = >	1/3

a. Explain how the researchers can solve the problems without using models.

You can create equivalent fractions sharing their lowest common denominator and then compare them: 3/4=9/12>8/12=2/3; 3/10=9/30<10/30=1/3.

b. Without using a model, solve each question above by circling the correct symbol (less than <; equal to =; greater than >) to compare the fractions.

3. The NASA researchers lost some of the Odyssey's measurements! Help them figure out what fractions are missing.



(Source: https://mars.nasa.gov/odyssey/)

Odyssey

1/2	<	*3*/4	<	1
1/4	=	6/*24*	=	5/20
4/4	>	7/*8*	>	3/4
2/3	>	*0*/3	<	1/3
7/8	<	*2*/2	<	13/10
5/3	<	11/*6*	<	200/100

4. The NASA researchers want to find a way to easily represent and compare the fraction of jobs completed by each orbiter on a certain day. Sometimes, the denominators of these fractions will not be the same, which can make them difficult to compare. Design a mathematical tool that might allow the researchers to more easily compare these fractions.

Because the purpose of this activity is to promote student creativity, responses should vary widely.

Sample response: *students might create two parallel panels where each side can be slid to adjust numerators and denominators.*

Lesson Designer: Elizabeth Canavan

Lesson 11: Comparing Fractions—

Launch Into the Whole Mars Pathfinder Landing!

LESSON 11

Big Ideas

We already know what whole numbers are: 1, 2, 3, ... We also learned that fractions show us the relationship between the parts out of a whole (numerator) and the whole itself (denominator). We can combine our knowledge of *whole numbers* and *fractions* and describe the relationship between the two. If we have the number 3, we can think of that as 3 wholes. So, we can write whole numbers as fractions with a denominator of 1. This means that the whole number 3 can be written equivalently as 3/1; so, 3=3/1. This can be applied to ANY whole number, not only small ones like 2 and 3. We know that 100=100/1 and 184937=184937/1. It is helpful for us to plot whole numbers on a number line with other fractions so that we can understand the sizes of fractions. Also, it is important for us to think about this mathematically and use whole numbers as fractions to practice mathematical reasoning. We can use this in real life too–if you want to make a cake that uses 2 cups of flour, you will need to use the 1 cup measuring scoop TWO times. We can think of this as 2 cups of flour or 2/1 cups of flour!

<table>
<tr>
<td>Lesson Objectives</td>
<td>

Students will express whole numbers as fractions.
Students will recognize and write fractions that are equivalent to whole numbers.
Students will describe the relationship between whole numbers and fractions.
Students will plot whole numbers and fractions equivalent to whole numbers on a number line.
Students will reason abstractly and quantitatively when describing whole numbers as fractions.

Common Core State Standards
Develop understanding of fractions as numbers.
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	<u>CCSS.MATH.CONTENT.3.NFA.3</u> Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. <u>CCSS.MATH.CONTENT.3.NF.A.3.C</u> Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram*.
Materials 	• colored pencil sets (one per student) • highlighters (one per student) • student white boards (one per student) • dry-erase markers (one per student) • whiteboard eraser (one per student) • smartboard or similar projector with computer and audio/visual setup • Pathfinder Landing video (https://www.youtube.com/watch?v=KGNV2Pu9Ms0) • Interactive 3D Airbag Model (https://sketchfab.com/3d-models/nasa-pathfinder-airbags-4f3f10f0fca74407846dccdaa06c9991) • entrance ticket (one per student) • tiered student pages (with answer keys) • hint and challenge cards
Mathematical Terms 	• **Denominator:** bottom number in a fraction that identifies the number of pieces the whole has been divided into equal parts • **Equal:** shows the same amount • **Equivalent:** equal in value • **Equivalent Fractions:** fractions with different numerators and denominators that represent the same value • **Fraction:** a number that represents part of a whole • **Integer:** a whole number that is not a fraction • **Number Line:** a line with numbers placed in their correct position • **Numerator:** top number in a fraction that identifies the number of equal pieces considered as part of the whole • **Unit:** an individual, single component of a larger or more complex whole • **Whole:** the entire unit that represents one

	• **Whole Number:** the set of numbers that include zero and natural numbers
Selected Mathematical Practices	• MP1: Make sense of problems and persevere in solving them. *I never give up on a problem and I do my best to get it right.* • MP2: Reason abstractly and quantitatively. *I can solve problems in more than one way.* • MP3: Construct viable arguments and critique the reasoning of others. *I can explain my math thinking and talk about it with others.* • MP7: Look for and make use of structure. *I can use what I know to solve new problems.*
Differentiation	**Content** ***Guiding Questions*** • learning objectives *What do you want students to know, understand, and be able to do?* • prior knowledge or learner readiness *What evidence do you have about students' current knowledge and skills?* • tiered activities *How will you design tiered activities on the same mathematical concept with varied levels of difficulty?* • formative assessment *What techniques will you use to assess students' prior knowledge and skills?* • varied levels of challenge *How will you vary the level of difficulty for each tiered activity?* • "teaching up" (aim high, provide scaffolding) *How will you increase the depth, breadth, complexity, and abstractness of lessons to challenge and support student learning?* • real-world application *What real-world connections will you make explicit about mathematical concepts and skills?*

	Process ***Guiding Questions*** • questioning strategies *How will you pose and how will you encourage students to pose open-ended, closed-ended, lower-level, and higher-level questions to promote mathematical discourse?* • 4Cs (21st Century Skills) o critical thinking *How will you promote a learning environment in which students question data and view issues or problems from multiple perspectives?* • 4Cs (21st Century Skills) o collaboration *How will you encourage students to work with other students and appreciate their contributions to solving problems or making connections to prior work?* • 4Cs (21st Century Skills) o communication *How will you promote students' opportunities to communicate face-to-face, in large and small groups, in online environments, and with print and non-print resources using their oral, written, and non-verbal skills?* • connections *How will you use "big ideas" to emphasize connections between and among mathematical concepts and skills and their connections to real-world situations?* **Product** ***Guiding Questions*** • oral, visual, and written opportunities *How will you encourage students to represent their thinking and problem solving using different communication strategies?* • multiple ways to demonstrate knowledge, understanding, and skills *How will you encourage students to share their understanding of mathematical concepts and skills?* • multiple models and representations *What techniques of lesson design will you include to support students' deep understanding and the ability to apply mathematical concepts and skills?*

	• summative assessment *How will you assess student learning upon completion of the lesson?* **Learning Environment** ***Guiding Questions*** • whole group/small group/individual instruction *How will you incorporate different grouping plans to address students' learning needs?* • growth mindset *How will you promote the perspective that it is important to view abilities as malleable?* • learning community *How will you support a positive learning community as students are encouraged to think, work, and communicate like mathematicians?*

Lesson Preview

In this lesson, students will learn about whole numbers and how to represent them as equivalent fractions. They will do this while engaging with information about the Mars Pathfinder Landing. As a group, they will learn background information about the mission as well as recall prior knowledge about fractions and whole numbers. Then, the group will discuss how whole numbers can be written as fractions within a mini-lesson. After this discussion, the students will complete an entrance ticket (as a type of formative assessment) and will be assigned to a tiered group based on their responses. Students will then split into their groups and complete their assigned student pages. This assignment will be turned in to the teacher and will serve as a summative assessment for the lesson. The lesson will culminate with a class discussion to debrief content and skills learned as well as a discussion about how students were thinking like mathematicians within the lesson.

Launch

1. Thinking Like Mathematicians

As a whole group, discuss how mathematicians support their answers. Consider writing a list of the students' ideas on a whiteboard or chart paper. Ask the students:

- How do mathematicians support their answers?
- What type of information do mathematicians give in support of their answers?

If students struggle to generate ideas, share that mathematicians support their answers by explaining the steps they used to arrive at a solution.

They also use precise mathematical language, pictures, charts, tools, manipulatives, or other examples.

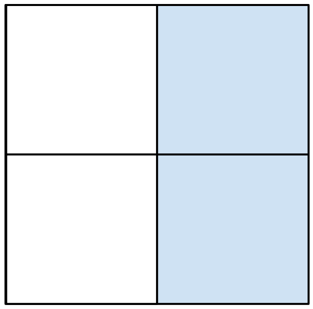
Pathfinder Launch
Ask students what they know about robotic rovers and Mars. Explain to students that the first-ever robotic rover to land on Mars was in 1997 and was a part of the Mars Pathfinder Mission. Describe the purpose of the mission and how the lander and rover were sent there to collect information about Mars using scientific instruments. During the mission, Pathfinder sent **2.3 billion** bits of information back to the scientists on Earth! But there was a lot that went into getting the Pathfinder to land on Mars. Play this video [https://www.youtube.com/watch?v=KGNV2Pu9Ms0] that gives an overview of the process to send Pathfinder to Mars for the class. You can also show the students the Pathfinder airbags in 3D using this link [https://sketchfab.com/3d-models/nasa-pathfinder-airbags-4f3f10f0fca74407846dccdaa06c9991] (Note: you do not have to purchase this model, this is just a website with a visual aid for students). Tell the students that today they are going to use their knowledge about fractions and numbers to describe the Pathfinder Landing.

Explore

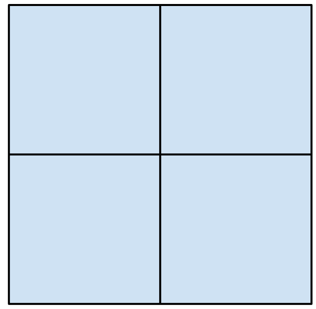
2. How Do Whole Numbers Connect to Fractions and to the Pathfinder Landing?

Ask students to brainstorm examples of fractions and give them 1 minute of independent think-time. After that, have students share out and write their examples on the classroom board. Most likely, students will generate numbers such as 1/2, 3/4, 2/6, etc. Next, ask students what it means for a number to be a fraction. After they respond, write the fraction definition on the board (e.g., a number that represents part of a whole).

Now, add the number 1 to the list of fractions on the board and ask the students if 1 is a fraction. Some students may disagree and say that there is not a denominator. Some students may agree and explain that the number one is a fraction because it is the whole part of a whole! Elaborate on student thinking by explaining that the number 1 is indeed a fraction because it represents a part of a whole. It can be written as 1/1 where 1 is the whole. Then, display this picture on the board for students:



Shape 1



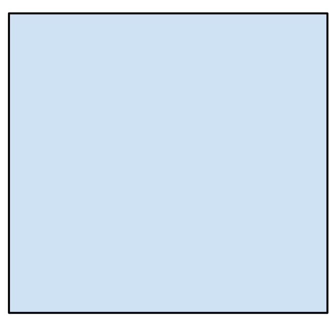
Shape 2

Ask students to write a fraction that describes the blue portion of each shape above.

Sample student responses: 2/4 or 1/2 for Shape 1 and 4/4 or 1/1 for the Shape 2

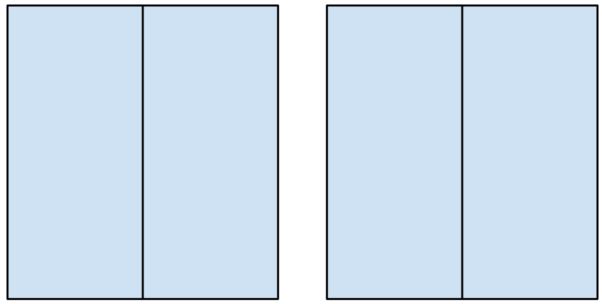
Ask students to share their responses to the class and emphasize the fact that 4/4 is equivalent to 1/1 and 1. Refresh students on the meaning of numerator and denominator. Explain that the denominator represents the equal number of pieces the whole has and the numerator is the selected number of equal pieces. Explain the importance of structure in mathematics and how mathematicians use their prior knowledge of mathematical concepts to help them solve new problems.

Now add this picture to the board and ask students what number the blue represents:



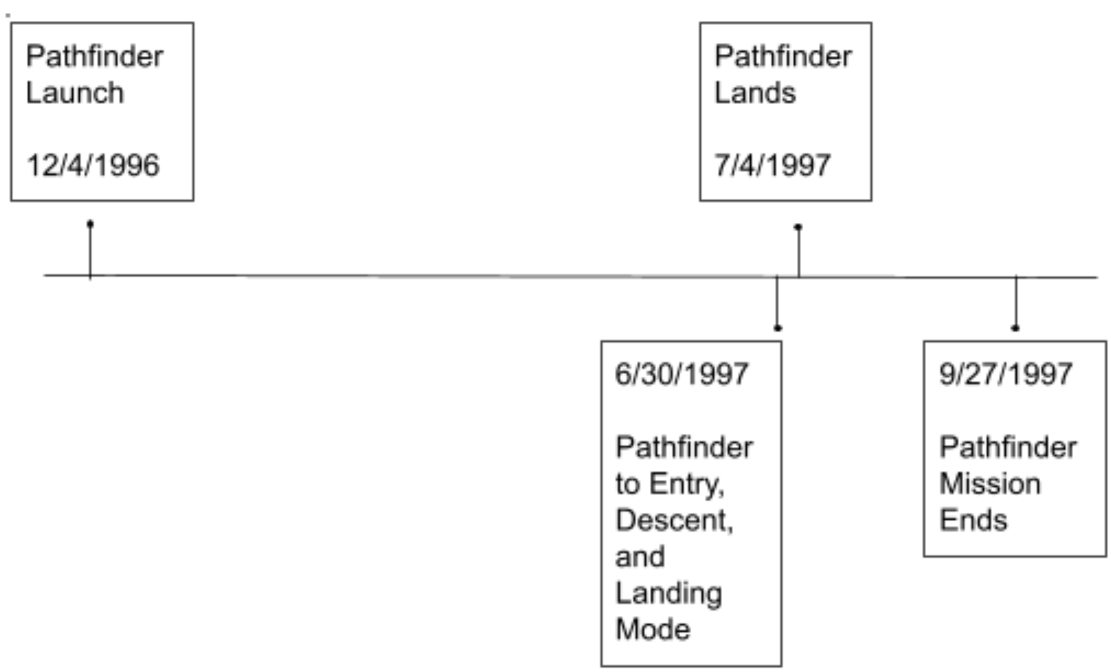
Students will likely respond with 1 or 4/4 or 1/1. Explain to students that all of these answers are correct and are equal to 1.

Now display this image to the class:



Ask how many equal pieces **each** whole is being split into. *Sample student response: 2 equal pieces within each whole*. Draw a fraction bar on the board and put the number 2 as the denominator after the students answer. Next, ask students how many of these pieces are shaded within the picture. *Sample student response: 4 pieces shaded.* Write 4 in the numerator of the fraction on the board. Ask students what fraction this whole image represents. Students will likely respond with 4/2 because that is written on the board. Ask students if there is another way that this can be written. *Sample student response: 2 or 8/4 or 2/1*. Explain to students that all these answers are correct, are all equivalent to 4/2, and are all fractions because they represent part of a whole.

Display this timeline of the Pathfinder mission that connects to the video shown at the beginning of class:



Ask the students to describe the number of days between when Pathfinder went into Entry, Descent, and Landing Mode and when Pathfinder Landed. Write as a fraction. Ask them what the denominator is in this situation (how many equal pieces the whole is split into).

Sample student responses: 4 days = 4/1 days and the denominator is one day.

Ask students to share answers with the class. Ensure that students understand that any whole number is also a fraction with a denominator of 1.

After this class discussion, distribute the entrance ticket to students and have them complete it independently. The teacher will use student responses to the entrance ticket (see below) and observations of students' understanding based on class discussion to split the group into 3 differentiated groups. In the following investigation, students will be working on one of the Student Pages based on their differentiated groups.

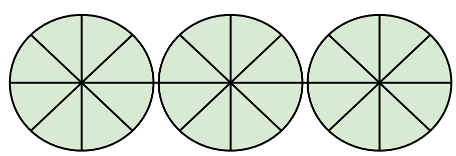
While students are working on their student pages, ask students to regroup with their peers completing the same tier as them and, as a group, discuss their answers for questions 1 and 2. This will encourage students to communicate mathematically with each other and potentially engage in mathematical discourse. This will also help strengthen student understanding of the content in the *middle* of their assignment, instead of at the end of the activity.

Entrance Ticket

1. Highlight all the numbers below that are fractions:

   1/2 1 3/4 100 7/7 8/4 9

2. Look at the picture below, describe what part of the shapes is green with **three** different fractions.



3. What does the denominator of a fraction represent?

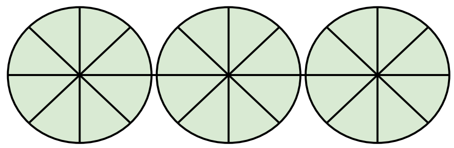
4. What does the numerator of a fraction represent?

Entrance Ticket Key

1. Highlight all the numbers below that are fractions:

   1/2 1 3/4 100 7/7 8/4 9

2. Look at the picture below, describe what part of the shapes is green with **three** different fractions.

   Sample student responses: 3, 3/1, 24/8, 12/4, 6/2

3. What does the denominator of a fraction represent?

   Sample student response: The denominator of a fraction represents the whole (or the number of equal pieces that the whole is split into).

4. What does the numerator of a fraction represent?

   Sample student response: The numerator of a fraction represents the number of equal pieces of a whole that are being considered.

Groups Formed by Student Readiness		
Tier 1: Peggy Whitson	**Tier 2: Guion Bluford**	**Tier 3: Ellen Ochoa**
Student Names	**Student Names**	**Student Names**

Collaborate and Communicate

Have students record their ideas on their individual worksheets or one for the small group. Help them clarify their ideas by asking questions like, “What do you mean here?” and “How might you share that idea with the rest of the class?” Point out that mathematicians use various representations to help explain their thoughts and use precise language to do so.

A. [Possible response]	**B.** [Possible response]	**C.** [Possible response]
This group . . .	*This group . . .*	*This group . . .*

Examine and Elaborate

3. Highlight Students' Mathematical Thinking

Mathematicians think about possible solutions in a variety of ways. Therefore, it is important for students to realize that they, too, can approach problems using different strategies. Ultimately, students need to understand that a possible solution should be judged by the correctness of the mathematics, and there might be some valid ideas within a solution when a student has an incorrect answer.

Share and Discuss

It is therefore important for students first to clearly share their ideas with others so their validity can be determined by the class.

Teacher: I see that everyone has learned about the Pathfinder airbag structure that keeps the rover safe when it lands. Who can tell me the fractions that they worked with today?
Giovanni: I used the fraction 4/4 on my worksheet.
Teacher: Nice, Giovanni! Can you elaborate on how you used that fraction? *(Reasoning talk move)*
Giovanni: Well, the scientists needed 4 airbags to complete the structure and the final structure had 4 airbags present. So, both the numerator and denominator for the fraction are 4.
Teacher: Great job! Did anyone use that fraction or a fraction equivalent to Giovanni's fraction?
Dayshia: I used the same fraction as Giovanni!
Cris: My fraction was 24/24 to represent the number of spheres that were needed for the whole structure. This is equal to Giovanni's fraction because both our fractions equal 1.
Teacher: Can anyone restate what Cris said in their own words? *(Repeat/Rephrase talk move)*

Jamilec: Cris said that both his fraction and Giovanni's fraction are equal to 1 and they each represent 1 whole. Giovanni was talking about the number of airbags there were for the complete structure and Cris was talking about the number of spheres there were for the complete structure.

Glen: Wait, Jami. How do you know that they are equal to 1?

Jamilec: For Giovanni's fraction, the whole was split into 4 equal pieces, so the denominator was 4. The scientist had 4 airbags so the numerator is also 4. That means that the scientist had 4 out of 4 or one complete whole so 4/4 is equal to 1. Cris's fraction does the same thing except instead the whole is split into 24 equal pieces because the scientists need 24 spheres and they have 24 spheres, so 24/24 equals one whole.

Teacher: I love that you are explaining your mathematical thinking and helping each other when you need it! Can everyone create a fraction that was not used in today's activity but is equal to Giovanni and Cris's fractions and write it on their whiteboard? I will give you a minute to think and write. *(Wait Time talk move)*

Teacher*:* What fractions did you write on your whiteboard?

Donovan: I wrote 2/2.

Victoria: I wrote 1/1.

Ace: I wrote 10/10.

Trinity: I wrote 100/100.

Teacher: Nice work, everyone! You all created fractions that are equivalent to 1 whole.

Differentiate Further as Needed

If students are struggling with the material and need more scaffolding, present them with an appropriate hint card and/or offer them manipulatives so that they can show their thinking in the best way possible for them. If students complete their tiered lesson and need more activities to engage them, you may present them with a challenge card to keep them thinking about the concept of whole numbers as fractions.

Hint Cards

This card is specifically useful for Tier 2 #3c:

Hint Card
The researcher says there are 2 airbags. What is the whole?
What if the whole was split into 6 equal parts?

This card is specifically useful for Tier 2 #4:

Hint Card
How many hours are there in one day?
How many days was the Pathfinder mission?

This card is specifically useful for Tier 1 #3d:

Hint Card
What does it mean for a number to be “out of” another?
How does this connect to the parts of a fraction?

This card is specifically useful for Tier 1 #4b:

Hint Card
How many spheres are there if you have 2 airbags?
How many spheres are there if you have 3 airbags?
How many spheres are there if you have 4 airbags?

Sample Responses

This card is specifically useful for Tier 2 #3c:

Hint Card
The researcher says there are 2 airbags. What is the whole?
When the researcher says that, the whole is 1 airbag. This means that there are 2 wholes out of the 1 whole present and the denominator is 1.
What if the whole was split into 6 equal parts?
If the whole was split into 6 equal parts (spheres) then the denominator would become 6 because that is the number of equal pieces of the whole and the numerator would be the number of spheres there are in 2 airbags (12).

This card is specifically useful for Tier 2 #4:

Hint Card

How many hours are there in one day?

There are 24 hours in one day.

How many days was the Pathfinder mission?

The Pathfinder mission was 297 days long.

This card is specifically useful for Tier 1 #3d:

Hint Card

What does it mean for a number to be "out of" another?

When a number is "out of" another it means how many of the equal parts of the whole are being discussed.

How does this connect to parts of a fraction?

This connects to the parts of a fractions because the two parts of a fraction are numerators and denominators. When people read a fraction out loud, they usually say the numerator "out of" the denominator. The numerator represents the selected number of parts that are being discussed and the denominator represents the total number of equal parts of the whole.

This card is specifically useful for Tier 1 #4b:

Hint Card

How many spheres are there if you have 2 airbags?

If there are 2 airbags, there are 12 spheres because each airbag has 6 airbags and 6+6=12 OR 6x2=12.

How many spheres are there if you have 3 airbags?

If there are 3 airbags, there are 18 spheres because each airbag has 6 airbags and 6+6+6=18 OR 6x3=18.

How many spheres are there if you have 4 airbags?

If there are 4 airbags, there are 24 spheres because each airbag has 6 airbags and 6+6+6+6=24 OR 6x4=24.

Challenge Cards

Challenge Card

During today's lesson, you and your classmates created many fractions that were equivalent to the number 1. List a few here and see if you can come up with a **rule** to describe fractions equivalent to 1.

Challenge Card

Create a number line and plot the following points on it: 4/1, 1/4, 8/4, 44/11, 4/2, 4/4, 4. Then, complete the following statements with the symbols: <, >, or = to make the following statements correct.

4/1 ______ 1/4

4 ______ 4/1

4/2 ______ 4

44/11 ______ 4/4

4/4 ______ 4

8/4 ______ 4/2

Challenge Card

Draw three different diagrams that all represent the value 6. These diagrams should all be related to a different fraction equivalent to 6.

Sample Responses

Challenge Card

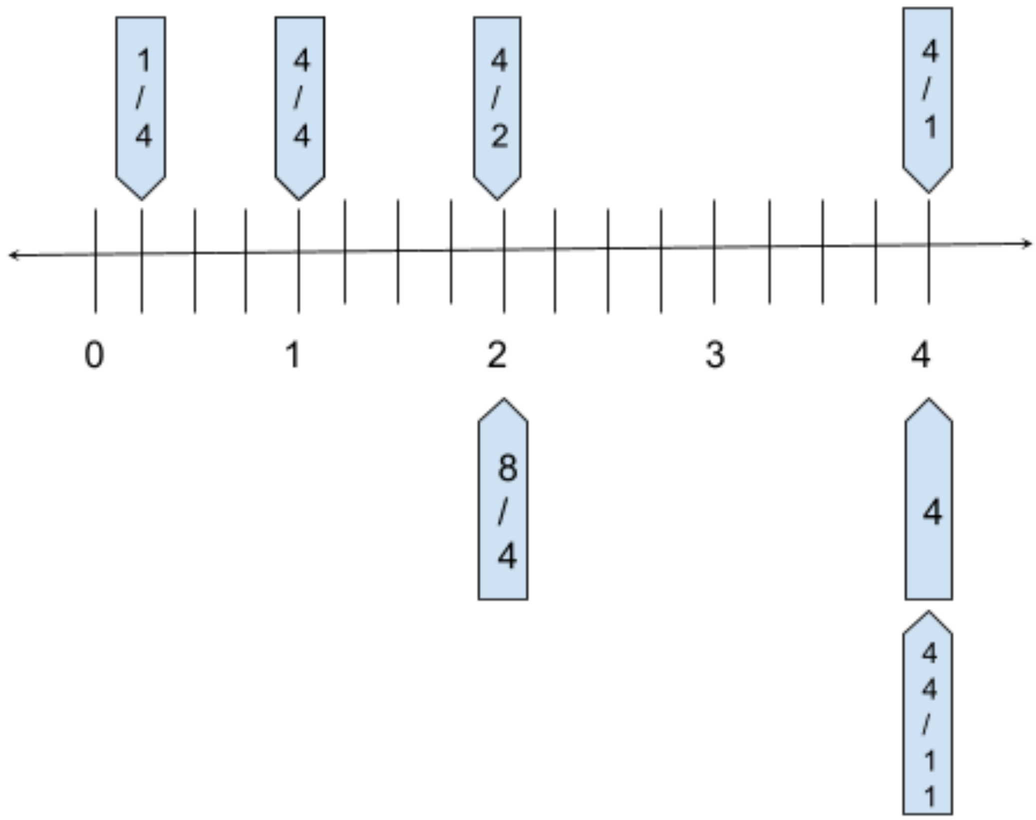
During today's lesson, you and your classmates created many fractions that were equivalent to the number 1. List a few here and see if you can come up with a **rule** to describe fractions equivalent to 1.

Students answers will vary: i.e., 4/4, 6/6, 1/1, 18/18, 2/2, 3/3

A rule to describe fractions equivalent to 1 is that any fraction whose numerator and denominator are the same is equivalent to 1.

Challenge Card

Create a number line and plot the following points on it: 4/1, 1/4, 8/4, 44/11, 4/2, 4/4, 4. Then, complete the following statements with the symbols: <, >, or = to make the following statements correct.



4/1 ___ > ___ 1/4

4 ___ = ___ 4/1

4/2 ___ < ___ 4

44/11 ___ > ___ 4/4

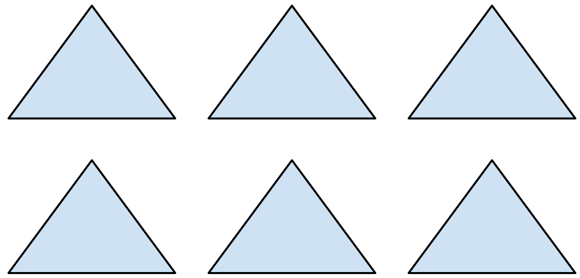
4/4 ___ < ___ 4

8/4 ___ = ___ 4/2

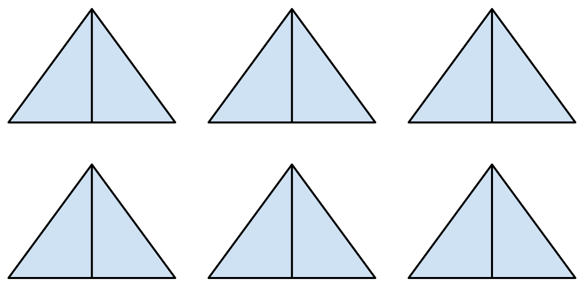
Challenge Card

Draw three different diagrams that all represent the value 6. These diagrams should all be related to a different fraction equivalent to 6.

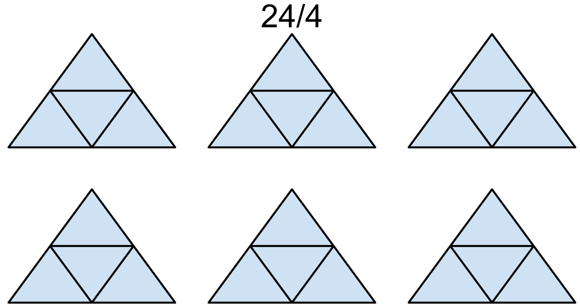
6/1



12/2



24/4



Debrief and Look Ahead

4.

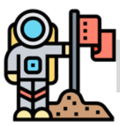
Debrief Content and Skills

Remind students that they learned about whole numbers in a new way. Review some of the ways that they represented whole numbers during the lesson and ask students to provide examples of when they wrote whole numbers as fractions. Ask the students to describe how a whole number can be represented as a fraction and how to write equivalent fractions with whole numbers. *Sample student response: Any whole number can be a fraction with a denominator of 1. If you change the number of equal pieces* *that the whole is being split into you can write fractions that represent the same amount of a whole but with a different numerator and denominator.*

Debrief Thinking Like Mathematicians

Remind students that the mathematical practice for this lesson focused on how mathematicians solve problems and work together. Review some of the ideas that students brainstormed at the beginning of class and have students offer examples of how they acted like mathematicians while they worked together during the lesson. Ask the students: How did you represent mathematics in different ways today? Why was it useful to present these numbers in different ways?

Sample student response: Today, I represented fractions in many different ways. I drew diagrams to represent fractions, rewrote whole numbers as fractions, and used number lines to represent the numbers. It was useful to see numbers in different ways because on a number line you can compare different fractions and with a diagram you can visualize the numbers that you have.



Assess

5.

What Students Learned

Prior to assignment of students into tiered groups, they will complete an entrance ticket. This will serve as a type of formative assessment outlining students' level of understanding and the teacher will use the results to split students into the groups. Students will submit their tiered student pages into the teacher at the end of the lesson. These student pages will serve as a type of summative assessment and will show the teacher if the students achieved the lesson objectives.

Resources

NASA. (1997). *Mars Pathfinder landing: Press kit.* https://mars.nasa.gov/internal_resources/816/

NASA Archives. (2014, January 7). Space history photo: Pathfinder air bags. *Space.com* https://www.space.com/24326-pathfinder-air-bags.html

NASA Science Mars Exploration Program (n.d.). *Mars Pathfinder.* https://mars.nasa.gov/mars-exploration/missions/pathfinder/

Placecom Learning. (2018, January 31). *Mars Pathfinder landing, robot on the red planet (1996-1997)* [Video]. YouTube. https://www.youtube.com/watch?v=KGNV2Pu9Ms0

Mission to Mars Student Pages

Landing Gear Designer ________________________ Date __________________

Peggy Whitson Tier 1 Lesson

The Mars Pathfinder Mission took a lot of people, time, and equipment to send the lander to Mars so that we could learn about the planet. There was a very specific process that needed to happen for Pathfinder to get from Earth to Mars safely without anything breaking. We can use whole number fractions to discuss the Mars Pathfinder Landing.

(Source: https://picryl.com/media/pathfinder-air-bags)

1. As you saw in the video [https://www.youtube.com/watch?v=KGNV2Pu9Ms0] earlier during class, right before the Pathfinder's landing, the airbags inflated. The airbags [https://sketchfab.com/3d-models/nasa-pathfinder-airbags-4f3f10f0fca74407846dccdaa06c9991] had to be an interesting shape to protect Pathfinder. The scientists used **6 equal spheres** to make *each airbag* to keep Pathfinder from breaking during landing. The scientists needed **4 airbags** total.

a) How many spheres are needed for 1 airbag?

____

b) How many airbags are needed in total?

____

c) How many spheres are needed, **in total**, to create the airbags?

____

2. Your friend, Toby, says that the number you wrote in (1a) cannot be written as a fraction. Use your knowledge of fractions, numerators, and denominators to defend your answer.

   a) What does the denominator of a fraction represent?

   ____

   ____

   ____

   b) If we want to write your answer to (1a) as a fraction and our unit is in spheres, what will be the denominator?

   ____

   ____

   ____

   c) What does the numerator of a fraction represent?

   ____

   ____

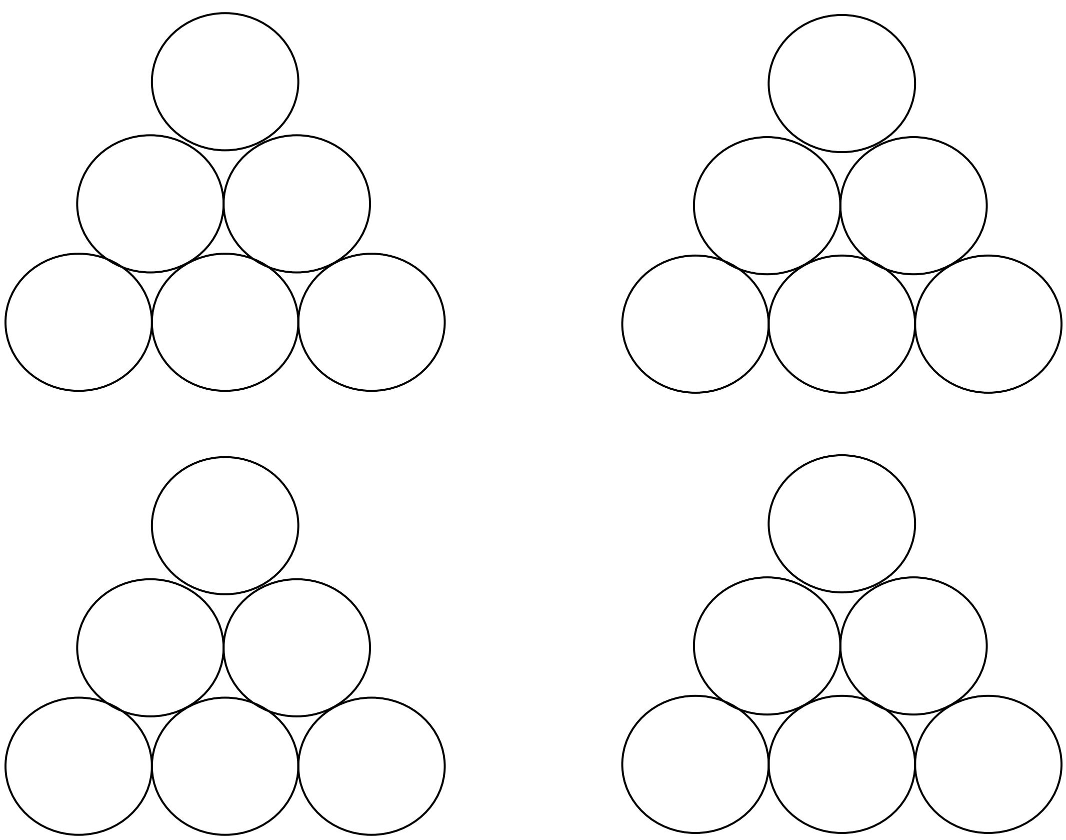
   ____

d) As you write your answer for (1a) as a fraction, what will your numerator be?

e) Write the number in (1a) as a fraction to challenge Toby’s answer.

3. One scientist on the Pathfinder team drew this diagram for the total structure (where each circle represents one sphere for the airbags).

a) Shade in the diagram below to represent how many spheres are needed for **2 complete airbags**.



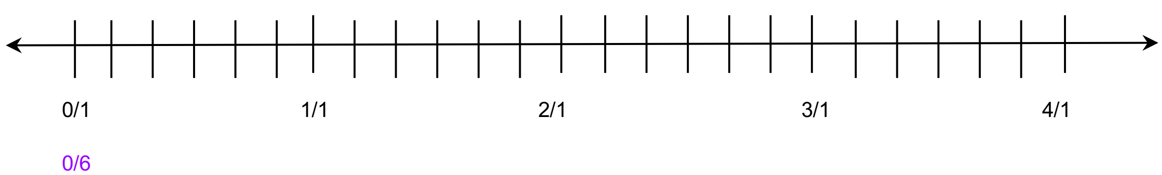
b) How many equal parts is each airbag split into?

c) How many spheres were shaded? Explain why you shaded that many.

d) Use these numbers to write a fraction to represent the number of spheres that the scientists have right now out of the number of spheres needed for one airbag.

e) Write the number in (1a) as a fraction to challenge Toby's answer.

4. The number line below labels the number of **airbags** the scientists built for the Pathfinder landing structure.



a) If you have 1 airbag, how many spheres do you have?

b) Using your purple colored pencil, rewrite 1/1, 2/1, 3/1, and 4/1 where the whole is being split into *6 equal parts* instead of 1 (see the example for 0).

c) One day, the builders finished making **8 spheres** before their lunch break. Write this number as a fraction of the number of airbags built and plot it on your number line in red.

Landing Gear Designer ______________________ Date ________________

Guion Bluford Tier 2 Lesson

The Mars Pathfinder Mission took a lot of people, time, and equipment to send the lander to Mars so that we could learn about the planet. There was a very specific process that needed to happen for Pathfinder to get from Earth to Mars safely without anything breaking. We can use whole number fractions to discuss the Mars Pathfinder Landing.

(Source: https://picryl.com/media/pathfinder-air-bags)

1. As you saw in the video [https://www.youtube.com/watch?v=KGNV2Pu9Ms0] earlier during class, right before the Pathfinder's landing, the airbags inflated. The airbags [https://sketchfab.com/3d-models/nasa-pathfinder-airbags-4f3f10f0fca74407846dccdaa06c9991] had to be an interesting shape to protect Pathfinder. The scientists used **6 equal spheres** to make *each airbag* to keep Pathfinder from breaking during landing. The scientists needed **4 airbags** total.

a) How many spheres are needed for 1 airbag?

b) How many airbags are needed in total?

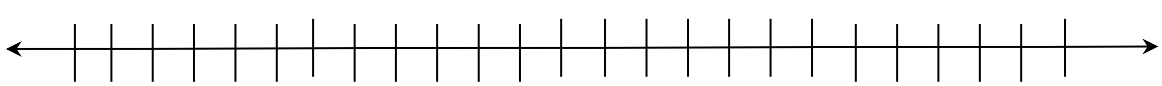
2. One of the builders assembled the complete safety structure for Pathfinder. They said that they had 1 complete structure.

    a) The researchers want this number written in terms of fractions. Think about the number of **airbags** the scientists needed and used. Write a fraction equivalent to 1.

    b) The researchers want this number written in another way using fractions! Think about the number of **spheres** the scientists needed and used. Write the number 1 as a fraction in a new way.

3. The scientists are trying to describe the number of airbags made for the mission.

    a) Label the number line below from 0 to 4 where the number 1 means that the scientists have 1 airbag. *(Hint: How many spheres are needed for each airbag?)*

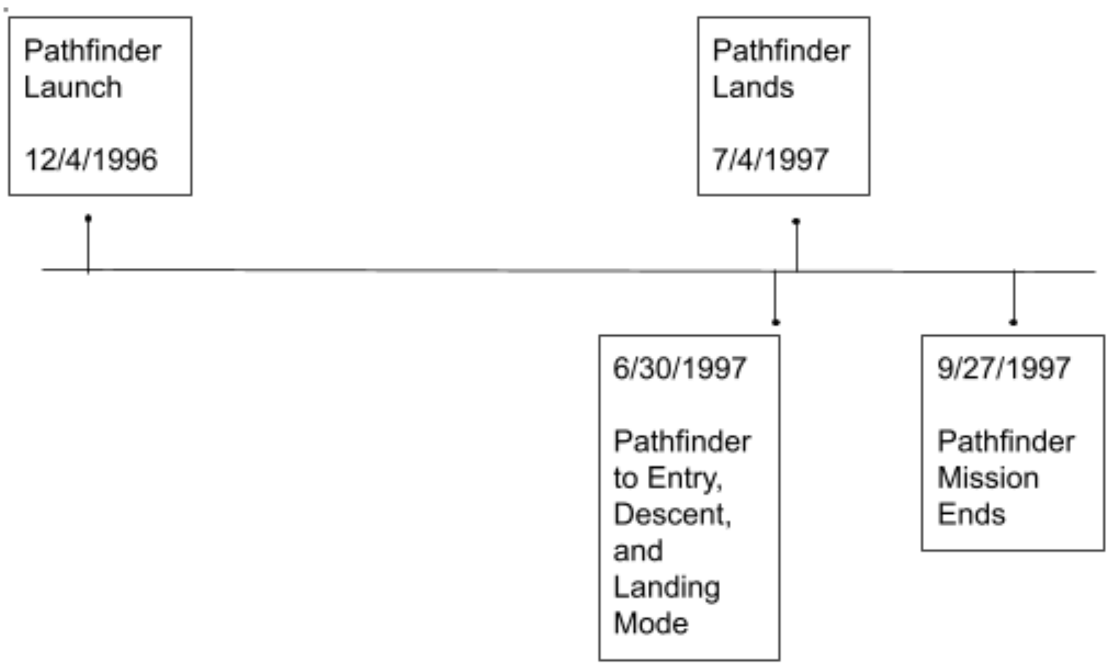


    b) Plot the point when the scientists have 12 **spheres** in green.

c) Plot the point when the scientists have 2 **airbags** in red and write two different equivalent fractions that represent this value.

d) What do you notice about the red and green points that you plotted on the number line?

4. Looking back at the timeline that your teacher showed the class, we can see how many days the Pathfinder was in space. We know that the total mission length (from launch to ending) was 297 days.



a) How can you represent the number of days in space as a fraction?

b) We want to split the whole of 1 day into hours. What would be the new denominator?

c) How many total hours was the Pathfinder mission? How do you know?

d) Write a fraction that represents the number of mission hours out of the number of hours in one day.

e) Your classmate, Jermaine, says that the fractions you wrote in (4a) and (4d) are equivalent. Is he right?

Mission to Mars Student Pages

Landing Gear Designer ______________________ Date ________________

Ellen Ochoa Tier 3 Lesson

The Mars Pathfinder Mission took a lot of people, time, and equipment to send the lander to Mars so that we could learn about the planet. There was a very specific process that needed to happen for Pathfinder to get from Earth to Mars safely without anything breaking. We can use whole number fractions to discuss the Mars Pathfinder Landing.

(Source: https://picryl.com/media/pathfinder-air-bags)

1. As you saw in the video [https://www.youtube.com/watch?v=KGNV2Pu9Ms0] earlier during class, right before the Pathfinder's landing, the airbags inflated. The airbags [https://sketchfab.com/3d-models/nasa-pathfinder-airbags-4f3f10f0fca74407846dccdaa06c9991] had to be an interesting shape to protect Pathfinder. The scientists used **6 equal spheres** to make *each airbag* to keep Pathfinder from breaking during landing. The scientists needed **4 airbags** total.

a) How many spheres are needed in total for the whole structure? Explain your reasoning.

b) Name two operations that you could have used to find that number.

2. One of the builders assembled the complete safety structure for Pathfinder. The researchers said that they had 1 complete structure.

   a) How can you write the number **1** in a different way using fractions? *(Hint: Think about how many airbags are needed for the whole structure.)*

   b) Rewrite this fraction in another way so it is equal in value but has a different numerator and denominator. *(Hint: Think about how many spheres are needed for the whole structure.)*

3. Create a number line where the whole represents the number of **airbags** used.

Label the following numbers on your number line:

a) In red, label and write a fraction that represents the number of airbags made when the scientists have 3 **airbags** made.
b) In green, label and write a fraction that represents the number of airbags made when the scientists have 12 **spheres** made.
c) In blue, label and write a fraction that represents the number of airbags made when the scientists have 1/4 of a **complete structure** made.
d) In purple, label and write a fraction that represents the number of airbags made when the scientists have 2 **airbags** made.
e) What do you notice about the points on your number line and the fractions you wrote?

4. Looking back at the Pathfinder timeline that your teacher showed the class, we can see how many days the Pathfinder was in space. We know that the total mission length (from launch to ending) was 297 days.

    a) Write 297 as a fraction where the whole is split into days.

    b) Write 297 as a fraction where the whole is split into hours.

    c) Write 297 as a fraction where the whole is split into minutes.

Peggy Whitson Tier 1 Lesson

The Mars Pathfinder Mission took a lot of people, time, and equipment to send the lander to Mars so that we could learn about the planet. There was a very specific process that needed to happen for Pathfinder to get from Earth to Mars safely without anything breaking. We can use whole number fractions to discuss the Mars Pathfinder Landing.

(Source: https://picryl.com/media/pathfinder-air-bags)

1. As you saw in the video [https://www.youtube.com/watch?v=KGNV2Pu9Ms0] earlier during class, right before the Pathfinder's landing, the airbags inflated. The airbags [https://sketchfab.com/3d-models/nasa-pathfinder-airbags-4f3f10f0fca74407846dccdaa06c9991] had to be an interesting shape to protect Pathfinder. The scientists used **6 equal spheres** to make *each airbag* to keep Pathfinder from breaking during landing. The scientists needed **4 airbags** total.

a) How many spheres are needed for 1 airbag?

6 spheres are needed for 1 airbag.

b) How many airbags are needed in total?

4 airbags total are needed.

c) How many spheres are needed, **in total**, to create the airbags?

24 spheres are needed in total because we need 4 airbags and each one has 6 spheres. (6+6+6+6=24 spheres OR 6x4=24 spheres)

2) Your friend, Toby, says that the number you wrote in (1a) cannot be written as a fraction. Use your knowledge of fractions, numerators, and denominators to defend your answer.

a) What does the denominator of a fraction represent?

The denominator of a fraction represents the whole (or the number of total equal parts of the whole).

b) If we want to write your answer to (1a) as a fraction and our unit is in spheres, what will be the denominator?

The denominator will be 1 because each whole is worth 1 sphere.

c) What does the numerator of a fraction represent?

The numerator of a fraction represents the selected number of equal parts of the whole.

d) As you write your answer for (1a) as a fraction, what will your numerator be?

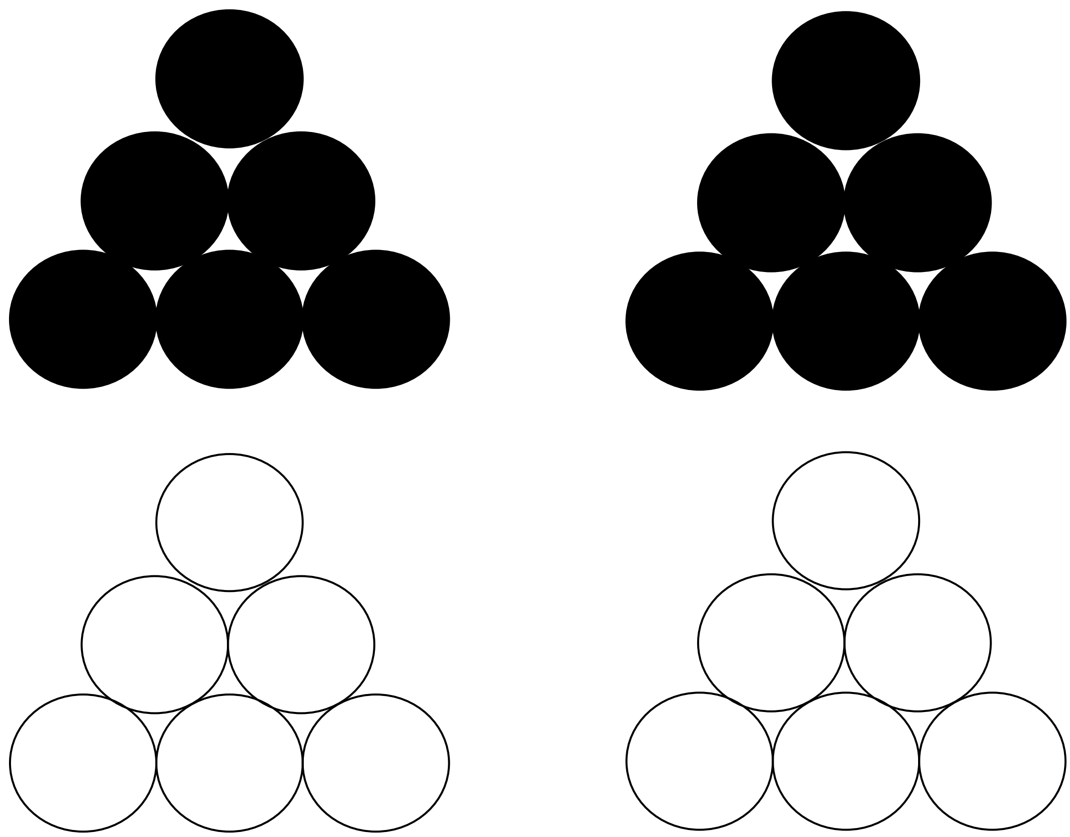
The numerator will be 6 because there are 6 spheres within each airbag.

e) Write the number in (1a) as a fraction to challenge Toby's answer.

The fraction equivalent to 6 is 6/1.

3) One scientist on the Pathfinder team drew this diagram for the total structure (where each circle represents one sphere for the airbags).

   a) Shade in the diagram below to represent how many spheres are needed for **2 complete airbags**.



   b) How many equal parts is each airbag split into?

Each airbag is split into 6 equal parts.

   c) How many spheres were shaded? Explain why you shaded that many.

I shaded in 12 spheres because there were 2 completed airbags and each airbag has 6 spheres so 6+6=12 spheres OR 6x2=12 spheres.

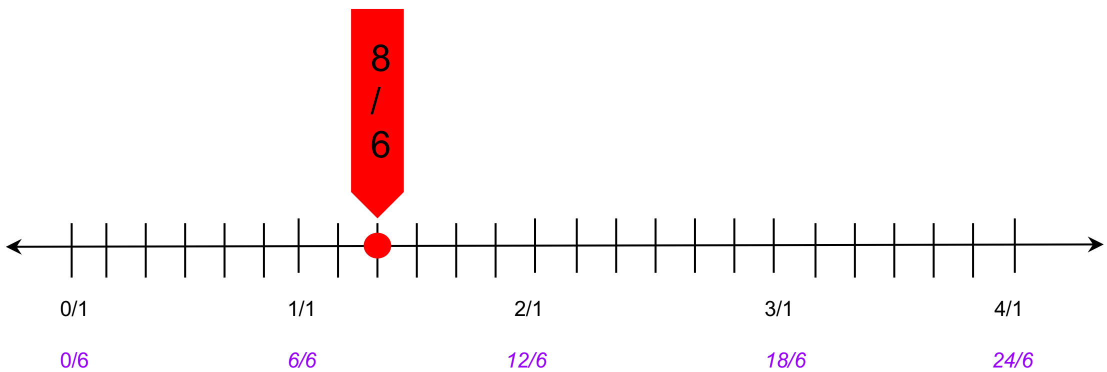
   d) Use these numbers to write a fraction to represent the number of spheres that the scientists have right now out of the number of spheres needed for one airbag.

12/6 because the scientists have 12 spheres and each airbag is split into 6 equal parts.

e) Write the number in (1a) as a fraction to challenge Toby's answer.

Yes, 12/6 is equivalent to the number 2. 2=2/1 because each whole is 1 therefore 2/1 represents 2 wholes and 12/6 also represents 2 wholes, each whole just has 6 equal parts each.

4) The number line below labels the number of **airbags** the scientists built for the Pathfinder landing structure.



a) If you have 1 airbag, how many spheres do you have?

If you have 1 airbag, you have 6 spheres.

b) Using your purple colored pencil, rewrite 1/1, 2/1, 3/1, and 4/1 where the whole is being split into *6 equal parts* instead of 1 (see the example for 0).

c) One day, the builders finished making **8 spheres** before their lunch break. Write this number as a fraction of the number of airbags built and plot it on your number line in red.

8/6 airbags are built before lunch. This is because there are 8 spheres and the whole (airbags) are each split into 6 equal spheres

Guion Bluford Tier 2 Lesson

The Mars Pathfinder Mission took a lot of people, time, and equipment to send the lander to Mars so that we could learn about the planet. There was a very specific process that needed to happen for Pathfinder to get from Earth to Mars safely without anything breaking. We can use whole number fractions to discuss the Mars Pathfinder Landing.

(Source: https://picryl.com/media/pathfinder-air-bags)

1. As you saw in the video [https://www.youtube.com/watch?v=KGNV2Pu9Ms0] earlier during class, right before the Pathfinder's landing, the airbags inflated. The airbags [https://sketchfab.com/3d-models/nasa-pathfinder-airbags-4f3f10f0fca74407846dccdaa06c9991] had to be an interesting shape to protect Pathfinder. The scientists used **6 equal spheres** to make *each airbag* to keep Pathfinder from breaking during landing. The scientists needed **4 airbags** total.

   a) How many spheres are needed for 1 airbag?

   24 total spheres were needed to protect the lander.

b) How many airbags are needed in total?

4 airbags are needed and each one has 6 spheres. (6+6+6+6=24 spheres OR 6x4=24 spheres)

2. One of the builders assembled the complete safety structure for Pathfinder. They said that they had 1 complete structure.

   a) The researchers want this number written in terms of fractions. Think about the number of **airbags** the scientists needed and used. Write a fraction equivalent to 1.

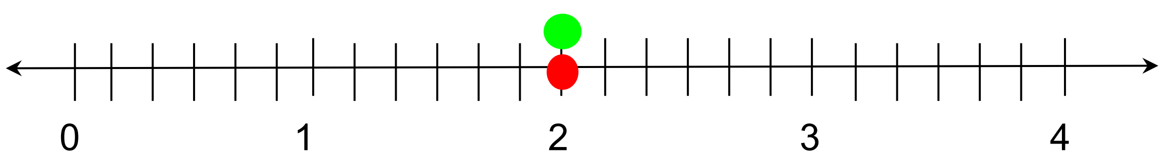
   4/4=1. The whole structure is being split into 4 equal parts (the airbags) and when the complete structure is built it has all 4 airbags.

   b) The researchers want this number written in another way using fractions! Think about the number of **spheres** the scientists needed and used. Write the number 1 as a fraction in a new way.

   24/24=1. The whole structure is being split into 24 equal parts (the spheres) and when the complete structure is built it has all 24 spheres.

3. The scientists are trying to describe the number of airbags made for the mission.

   a) Label the number line below from 0 to 4 where the number 1 means that the scientists have 1 airbag. *(Hint: How many spheres are needed for each airbag?)*



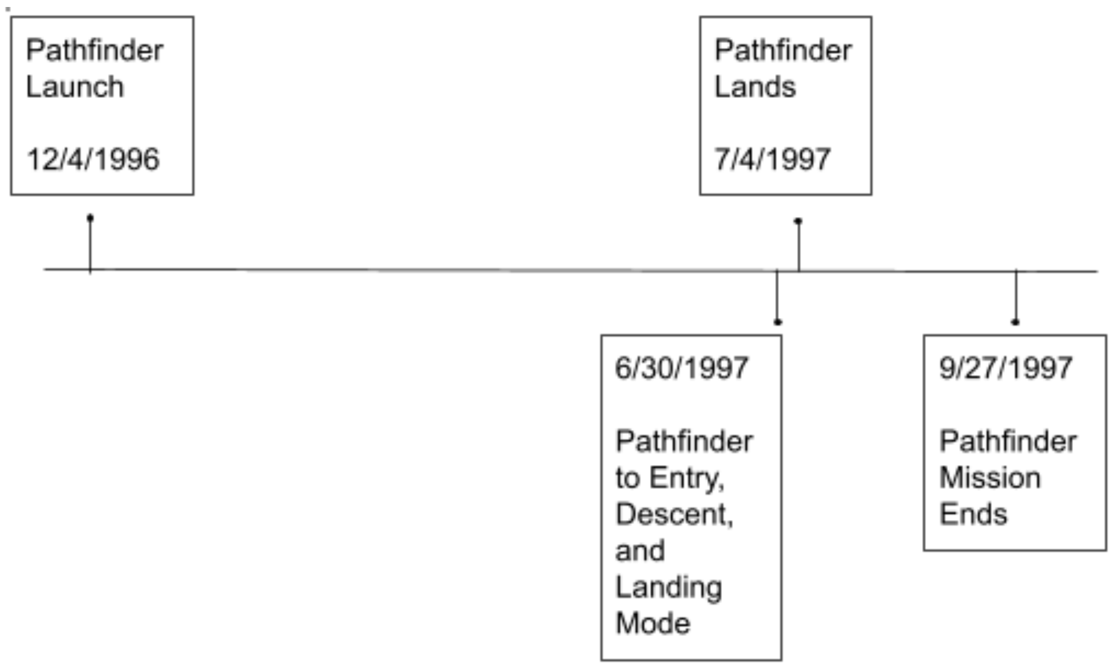
   b) Plot the point when the scientists have 12 **spheres** in green.
   c) Plot the point when the scientists have 2 **airbags** in red and write two different equivalent fractions that represent this value.

   2=2/1 because the whole is 1 airbag and the scientists have 2 of them. This fraction is also equivalent to 12/6 because when the whole (the airbag) is split into 6 equal parts (the spheres) there are 12 of them.

d) What do you notice about the red and green points that you plotted on the number line?

The red and green points are at the same point on the number line and this makes sense because 12/6=2/1.

4. Looking back at the timeline that your teacher showed the class, we can see how many days the Pathfinder was in space. We know that the total mission length (from launch to ending) was 297 days.



a) How can you represent the number of days in space as a fraction?

297/1 where the whole is 1 day and there are 297 days in space.

b) We want to split the whole of 1 day into hours. What would be the new denominator?

The new denominator would be 24 because there are 24 hours in a day. The whole is being split from 1 day to 24 equal hours.

c) How many total hours was the Pathfinder mission? How do you know?

The Pathfinder mission was 7,128 hours long. There are 24 hours in each day and the mission was 297 days and 297x24=7,128 hours.

d) Write a fraction that represents the number of mission hours out of the number of hours in one day.

7128/24

e) Your classmate, Jermaine, says that the fractions you wrote in (4a) and (4d) are equivalent. Is he right?

Jermaine is correct because 297/1 and 7128/24 are both equivalent to 297. This is because to find 7128/24, we split the whole of 1 day into 24 equal parts and counted how many of those parts were in 297 days.

Ellen Ochoa Tier 3 Lesson

The Mars Pathfinder Mission took a lot of people, time, and equipment to send the lander to Mars so that we could learn about the planet. There was a very specific process that needed to happen for Pathfinder to get from Earth to Mars safely without anything breaking. We can use whole number fractions to discuss the Mars Pathfinder Landing.

(Source: https://picryl.com/media/pathfinder-air-bags)

1. As you saw in the video [https://www.youtube.com/watch?v=KGNV2Pu9Ms0] earlier during class, right before the Pathfinder's landing, the airbags inflated. The airbags [https://sketchfab.com/3d-models/nasa-pathfinder-airbags-4f3f10f0fca74407846dccdaa06c9991] had to be an interesting shape to protect Pathfinder. The scientists used **6 equal spheres** to make *each airbag* to keep Pathfinder from breaking during landing. The scientists needed **4 airbags** total.

   a) How many spheres are needed in total for the whole structure? Explain your reasoning.

   24 spheres were needed for the whole structure. Each airbag has 6 equal spheres and the structure contains 4 airbags.

6x4=24 spheres OR 6+6+6+6=24 spheres

b) Name two operations that you could have used to find that number.

The two operations that you could have used to find 24 spheres is with addition and multiplication (6x4=24 spheres OR 6+6+6+6=24 spheres).

2. One of the builders assembled the complete safety structure for Pathfinder. The researchers said that they had 1 complete structure.

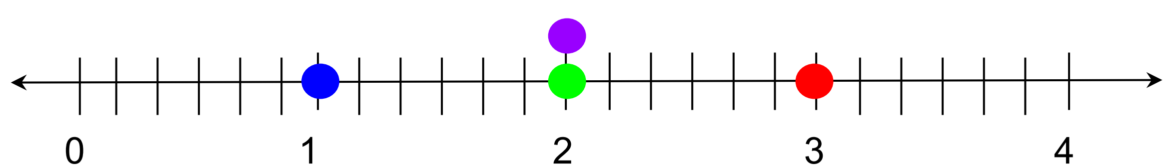
   a) How can you write the number **1** in a different way using fractions? *(Hint*: *Think about how many airbags are needed for the whole structure.*)

   *1=1/1=**4/4** because the whole structure can be split into 4 equal parts (the airbags) and the complete structure has 4 airbags.*

   b) Rewrite this fraction in another way so it is equal in value but has a different numerator and denominator. *(Hint: Think about how many spheres are needed for the whole structure.*)

   1=1/1=24/24 because the whole structure can be split into 24 equal parts (the spheres) and the complete structure has 24 spheres.

3. Create a number line where the whole represents the number of **airbags** used.



Label the following numbers on your number line:

a) In red, label and write a fraction that represents the number of airbags made when the scientists have 3 **airbags** made.
b) In green, label and write a fraction that represents the number of airbags made when the scientists have 12 **spheres** made.
c) In blue, label and write a fraction that represents the number of airbags made when the scientists have 1/4 of a **complete structure** made.
d) In purple, label and write a fraction that represents the number of airbags made when the scientists have 2 **airbags** made.

e) What do you notice about the points on your number line and the fractions you wrote?

I notice that the green and purple points are at the same point on the number line. This means that 12/6 and 2/1 are equivalent.

4. Looking back at the Pathfinder timeline that your teacher showed the class, we can see how many days the Pathfinder was in space. We know that the total mission length (from launch to ending) was 297 days.

a) Write 297 as a fraction where the whole is split into days.

297/1 because the whole is already 1 day.

b) Write 297 as a fraction where the whole is split into hours.

7,128/24 The whole of 1 day is being split into 24 equal pieces because there are 24 hours in a day so 24 is the denominator. In 297 days there are 7,128 hours because 297x24=7,128 therefore, 7,128 is the numerator.

c) Write 297 as a fraction where the whole is split into minutes.

427,680/1,440 The whole of 1 day is being split into 1,440 equal pieces because there are 1,440 minutes in a day (24x60=1,440) so 1,440 is the denominator. In 297 days, there are 427,680 minutes because 297x1,440=427,680 so, 427,680 is the numerator.

Supplementary Information

NCTM Curriculum Standards

These lessons are aligned with contemporary standards for mathematics development (see Table 2). The following tables outline the standards upon which the Mission to Mars curriculum was based.

Table 2

NCTM Curriculum Standards

FRACTIONS
1. Develop understanding of fractions as parts of unit wholes, as parts of a collection, as locations on number lines, and as divisions of whole numbers
2. Use models, benchmarks, and equivalent forms to judge the size of fractions
3. Recognize and generate equivalent forms of commonly used fractions
4. Develop and use strategies to estimate computations involving fractions and decimals in situations relevant to students' experience
5. Use visual models, benchmarks, and equivalent forms to add and subtract commonly used fractions and decimals

Common Core State Standards

The Common Core State Standards (CCSS, see Table 3) were developed in 2009 to establish consistent and relevant learning objectives for students across the United States (Common Core State Standards Initiative, 2022).

Sections of the Mission to Mars introduction and supplementary information built upon the *If Aliens Taught Algebra: Multiplication and Division Would be out of This world!* curriculum (2019), authored by Shelbi Cole, Lisa DaVia Rubenstein, Tutita M. Casa, Cindy M. Gilson, Micah N. Bruce-Davis, E. Jean Gubbins, Alexis Melendez, Hannah F. Brown, Siamak Vahidi, Rachel R. McAnallen, and Stacy M. Hayden, University of Connecticut, Storrs, Connecticut.

Table 3

Common Core State Standards

Develop understanding of fractions as numbers
CCSS.MATH.CONTENT.3.NF.A.1 Understand a fraction 1/*b* as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction *a*/*b* as the quantity formed by *a* parts of size 1/*b*
CCSS.MATH.CONTENT.3.NF.A.2 Understand a fraction as a number on the number line; represent fractions on a number line diagram
CCSS.MATH.CONTENT.3.NF.A.2.A Represent a fraction 1/*b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size 1/*b* and that the endpoint of the part based at 0 locates the number 1/*b* on the number line
CCSS.MATH.CONTENT.3.NF.A.2.B Represent a fraction *a*/*b* on a number line diagram by marking off a lengths 1/*b* from 0. Recognize that the resulting interval has size *a*/*b* and that its endpoint locates the number *a*/*b* on the number line
CCSS.MATH.CONTENT.3.NF.A.3 Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size
CCSS.MATH.CONTENT.3.NF.A.3.A Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line
CCSS.MATH.CONTENT.3.NF.A.3.B Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3. Explain why the fractions are equivalent, e.g., by using a visual fraction model
CCSS.MATH.CONTENT.3.NF.A.3.C Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.*
CCSS.MATH.CONTENT.3.NF.A.3.D Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model

21st Century Skills

The Partnership for 21st Century Learning's (2019) framework describes the knowledge, skills, and pedagogical considerations essential to students' development as 21st century citizens. A central component of this framework is the collection of learning and innovation skills known as the 4Cs: creativity, critical thinking, collaboration, and communication (see Table 4). For success in their personal and professional lives, today's learners must learn to think creatively, work creatively in groups, implement innovations, reason effectively, solve problems, communicate clearly, and work well with others.

Table 4

21st Century Skills (4Cs: Creativity, Critical Thinking, Collaboration, Communication)

CREATIVITY
Use a wide range of idea creation techniques (such as brainstorming)
Create new and worthwhile ideas
Elaborate, refine, analyze, and evaluate their own ideas in order to improve and maximize creative efforts
Demonstrate originality and inventiveness in work and understand the real-world limits to adopting new ideas
CRITICAL THINKING
Use various types of reasoning as appropriate to the situation
Analyze how parts of a whole interact with each other to produce overall outcomes in complex systems
Analyze and evaluate alternative points of view
Synthesize and make connections between information and/or arguments
Interpret information and draw conclusions based on the best analysis
Reflect critically on learning experiences and processes
Solve different kinds of non-familiar problems in both conventional and innovative ways
COLLABORATION
Demonstrate ability to work effectively and respectfully with diverse teams
Exercise flexibility and willingness to be helpful in making necessary compromises to accomplish a common goal
Assume shared responsibility for collaborative work
Value the individual contributions made by each team member
COMMUNICATION
Articulate thoughts and ideas effectively using oral, written, and nonverbal communication skills
Listen effectively to decipher meaning, including knowledge, values, attitudes, and intentions
Use communication for a variety of purposes (e.g., to inform, instruct, motivate, and persuade)

Differentiation and Enrichment Models

Efforts to differentiate learning for students often exclude students with gifts and talents because it is often presumed that these students will succeed without specific interventions (Rubenstein et al., 2015). In reality, students with gifts and talents often display concerning patterns of underachievement (Siegle et al., 2017; Snyder & Linnenbrink-Garcia, 2013). To counter underachievement and provide levels of challenge suitable for all learners, the lessons include specific adaptations designed for students whose needs for greater levels of challenge are unmet by traditional grade-level curriculum. According to the NMAP's (2008) final report, students with gifts and talents benefit from opportunities for acceleration, enrichment, and individualized instruction. To provide such opportunities, these lessons draw upon the body of research supporting three relevant educational models: the Differentiation of Instruction Model (DIM), the Depth and Complexity Model (DCM), and the Schoolwide Enrichment Model (SEM).

The DIM is based on the premise that no educational environment, content, process, or product determines whether curriculum is appropriate for students with gifts and talents, but rather "it is the intensity with which those elements are applied to learning experiences in response to a child's readiness, interests, or learning profile" (Tomlinson, 1996, p. 171). Overarching principles of effective teaching and learning are similar regardless of a student's ability level, but meeting a specific student's need can be achieved by differentiating elements of learning content, processes, products, and/or learning environments; for example, by modifying relative levels of complexity, abstractness, or open-endedness (Tomlinson, 2001). Successful differentiation is a cyclical process in which teachers (a) base expectations on essential curricular concepts, (b) assess students' specific learning needs, (c) modify instruction and provide feedback to support students in mastering core concepts, (d) assess students' progress, and (e) differentiate further when necessary (Tomlinson, 2005). Although many teachers report having difficulties differentiating, they also hold positive beliefs about differentiation and frequently implement it in practice, particularly when organizational support and self-efficacy is high (Whitley et al., 2019). Studies on differentiation have revealed corresponding increases in student engagement (Bondie et al., 2019) and modest, positive effects on students' academic performance (Deunk et al., 2018).

The DCM describes strategies for reinforcing core curricula with increased levels of depth and complexity that appropriately challenge students with gifts and talents (Kaplan, 2009). For all students, learning is organized around themes (universal concepts) and big ideas (generalizable principles), which intersect with content, processes, and products that are differentiated to provide levels of depth and complexity appropriate for different students. Increasing depth can involve introducing more sophisticated discipline-specific language, details, patterns, rules, trends, unanswered questions, generalizations, principles, theories, big ideas, and ethics, whereas increasing complexity can involve viewing content over time, from multiple perspectives, or across disciplines (Kaplan & Gould, 2005).

The SEM is a whole-school talent development intervention (Renzulli & Reis, 1985, 2014). In the SEM, the regular curriculum remains central to students' learning, but students are also given opportunities to participate in enrichment clusters based on their interests. Depending on levels of interest and readiness, students may engage in Type I activities where they are exposed to a variety of experiences and activities not covered in the regular curriculum, Type II activities where they develop the cognitive and affective skills needed to develop talents in a specific domain, or Type III activities where they engage in self-directed, domain-specific inquiries in a manner similar to discipline professionals. The SEM uses broader identification processes to identify a talent pool including 15–20% of the student population, and students in this pool are given further opportunities to compact the regular curriculum and participate in strengths-based, active, authentic, deep, and complex learning experiences. Diverse evidence has demonstrated that the SEM increases students' engagement, attitudes, and achievement, generating interests that commonly persist into their professional careers (Reis & Peters, 2021). The SEM has also been found to benefit students with both gifts and disabilities (i.e., students identified as twice exceptional; Baum et al., 2014).

Independently, the DIM, DCM, and SEM are effective and widely used, but evidence also suggests that there are benefits to integrating these three models. For example, combining enrichment and differentiation in a literacy intervention improved reading comprehension and fluency (Reis et al., 2011). Language arts units based on the CLEAR curriculum model, which borrows core elements of the DIM, DCM, and SEM, resulted in improved scores on a test measuring reading, writing, communicating, and knowledge skills (Callahan et al., 2015). CLEAR units were particularly effective when implemented with fidelity (Azano et al., 2011) and supplemented with formative assessment (Missett et al., 2014). Studies involving pre-differentiated and enriched mathematics units have revealed positive effects for high-achieving students (McCoach et al., 2014), and implementing teachers reported that these units helped them to better understand and accommodate students' varied needs (Gubbins et al., 2013; Rubenstein et al., 2015). Drawing from this research, these lessons leverage the many advantages afforded by these curriculum models and their combination, resulting in a series of pre-differentiated, enriched fractions lessons specifically designed to meet the needs of the diverse grade 3 student population.

Astronaut Biographies

Peggy Whitson (pronounced peg-ee, wit-sun) 1960 c.e. to present
On February 9, 1960, Peggy Whitson was born in Beaconsfield, Iowa, where she grew up on a farm. The first moon landing in 1969 inspired Peggy to be an astronaut. She received a degree in chemistry and biology from Iowa Wesleyan College in 1981 and then her doctorate in biochemistry from Rice University in 1986, where she was also a Robert A Welch Post-doctoral Fellow in 1986. Once this was done, she worked as a National Research Council Resident Research Associate at Johnson Space Center. Whitson worked as an adjunct assistant

professor at the University of Texas, a project scientist, and a deputy division chief at the Johnson Space Center until she was selected as an astronaut candidate in 1996.

While training as an astronaut, Whitson worked in various technical positions at the National Aeronautics and Space Administration (NASA) Operation Planning branch. On June 5, 2002, she embarked on her first space flight as a flight engineer on the Endeavor space shuttle. Whitson became the first NASA ISS science offer, conducting spacewalks to install a service module shielding, among other objectives. After 185 days, Whitson returned to Earth. In October 2007, she flew back to space aboard the Soyuz TMA-11. Whitson served as the first ISS female commander during her time on this mission. This mission lasted 6 months, ending in a dangerous and difficult journey back to Earth. Whitson served as the chief of the Astronaut Office after this flight, the first female and non-pilot to do so, before she returned to activity flight status in 2012. Whitson embarked on a third flight into space in 2016, returning to Earth on September 3, 2017, which is still the longest single spaceflight that a female astronaut has completed, with a total of 289 days spent in space.

Students can watch Peggy Whitson discuss her work and her legacy in a video posted by Vegas Film Critic (2020), listed in the references below. The running time of this video is 5 minutes and 40 seconds, making it an excellent introduction to a lesson. It may also be helpful to post Whitson's quotes about her space travel. When asked about what she will miss about space, she said that she will miss floating and moving with extremely light touch, as well as the view of Earth from space. She mentioned that she will miss the sense of satisfaction, gratitude and pride that comes from working with NASA. Her direct quotes can be found in the Garcia (2017) resource, listed below.

For further information, please consult the following sources:

Garcia, M. (2017, September 1). *Peggy Whitson shares thoughts on extended mission, returning to earth*. NASA. https://www.nasa.gov/feature/nasa-astronaut-peggy-whitson-shares-thoughts-on-extended-mission-returning-to-earth/

KidsKonnect. (2020, October 20). *Peggy Whitson facts, worksheets, early life, career & awards for kids*. https://kidskonnect.com/people/peggy-whitson/

Vegas Film Critic. (2020, November 12). *Dr. Peggy Whitson interview–space launch live; crew-1 lift off (SpaceX)* [Video]. YouTube. https://www.youtube.com/watch?v=kfAtPYp8fU8

Guion Bluford (pronounced g-ee-on, bloo-furd) 1942 c.e. to present
Guion Bluford had a successful career in the Air Force prior to his involvement with NASA. He was assigned to the 557th Tactical Fighter Squadron at Cam Ranh Bay, Vietnam and flew 144 combat missions in Southeast Asia as a F4C fighter pilot. At Sheppard Air Force Base in Texas, Bluford trained future Air

Force and West German fighter pilots. He graduated from the Air Force Institute of Technology in 1974 and was assigned to the U.S. Air Force Flight Dynamics Laboratory as Deputy for Advanced Concepts in the Aeromechanics Division before serving as the Branch Chief of the Aerodynamics and Airframe Branch.

Bluford was chosen, alongside two other African American candidates, for the astronaut program and designated a NASA astronaut in 1978. Originally, Bluford expected to be the second African American in space. However, out of the three African American astronauts at the time, Guion Bluford was selected to be the first African American to fly in space in 1983. He was the first to receive the U.S. Air Force Command Pilot Astronaut Wings. The other two astronauts met very different fates. Ron McNair lost his life aboard the Challenger, while Fred Gregory went on to become NASA's first African American Deputy Administrator. Bluford, meanwhile, logged more than 688 hours in space over four different flights, between 1985 and 1992.

Bluford stated that he felt immense responsibility because of his position as the first African American in space. He sought to emphasize his role as a scientist and astronaut, not merely as an African American. Bluford believed that he opened the door for the African American community to enter space and claimed that the people who follow him will be exceptional for their accomplishments more than their ethnicity. His thoughts are recorded by Today in Science History (n.d.), listed in the resources below. Future generations should remember him for his historic accomplishments as an extraordinary scientist, pilot, and astronaut rather than focusing solely on his role as the first African American in space.

For further information, please consult the following sources:

Space Center Houston. (2019, February 26). *Astronaut friday: Guion "Guy" Bluford, Jr.* https://spacecenter.org/astronaut-friday-guion-guy-bluford-jr/

Today in Science History. (n.d.). *Scientist quotes by Guion S. Bluford, Jr.* https://todayinsci.com/B/Bluford_Guion/BlufordGuion-Quotations.htm

The History Makers. (2013, May 9). *Guion Bluford's biography*. https://www.thehistorymakers.org/biography/guion-bluford

Ellen Ochoa (pronounced el-en, oh-cho-uh) 1958 c.e. to present
Ellen Ochoa grew up in La Mesa, California and attended San Diego State University. As a classically trained flutist, Ochoa thought she would pursue a music major in college. Instead, she obtained a degree in physics in 1980. While pursuing her doctorate at Stanford University, she received the student soloist award from the Stanford Symphony Orchestra. During her time at Stanford University, she witnessed Sally Ride becoming the first American woman to go into space and decided that she wanted to become an astronaut. She earned her doctorate in electrical engineering and applied to NASA's competitive astronaut program shortly after.

Ochoa was not accepted into NASA's program immediately. In fact, she applied to the astronaut program three times before finally being accepted. While she waited, she obtained her pilot's license and worked as a researcher in optical information systems at the Energy Department's Sandia National Lab and the NASA Ames Research Center. She worked on ways to process images for robotic manufacturing, holding patents for three optical devices that are useful for space exploration. Developing these skills in robotics made her a valued member of the NASA team and she was admitted to NASA's astronaut program in 1990 as the first Hispanic American female astronaut and became a mission specialist. She made her first of four space shuttle missions in 1993, where Ochoa released a research satellite using the shuttle's robotic arm. She logged nearly 1,000 hours in space, using robotic arms on multiple missions, even playing her flute aboard the space shuttle. Four schools are named after her, two in California, one in Texas, and one in Washington.

Students can watch Ellen Ochoa discuss teamwork, diversity, and her journeys to space in a video with a running time of two minutes and 53 seconds, which was posted by the Harvard Kennedy School in a resource listed below. This video may serve well as an introductory activity to a lesson involving Ellen Ochoa's work. Ochoa's quotes about education may also be included in an introductory activity to a larger lesson about her work. Ellen Ochoa says she tells students "that the opportunities I had were a result of having a good educational background" and claims that education is what allows an individual to stand out. This quote may motivate students and encourage them to think about their education in a different light. Brainy Quote (n.d.) listed her direct quotes in their website, referenced below.

For further information, please consult the following sources:

Scholastic. (n.d.). *Biography of Ellen Ochoa.* https://www.scholastic.com/teachers/articles/teaching-content/biography-ellen-ochoa/

Lantero, A. (2016, March 24). Five fast facts about astronaut Ellen Ochoa. *Department of Energy*. https://www.energy.gov/articles/five-fast-facts-about-astronaut-ellen-ochoa

Harvard Kennedy School. (2019, April 25). *3 questions for NASA pioneer Ellen Ochoa* [Video]. YouTube. https://www.youtube.com/watch?v=Iz95O6Q4ZY4

Brainy Quote. (n.d.). *Ellen Ochoa quotes*. https://www.brainyquote.com/quotes/ellen_ochoa_185237

Physical and Affective Learning Environment

The physical environment and social climate in a classroom can strongly influence students' attitudes and behaviors (Flutter, 2006; Nair, 2014; Pearlman, 2010), and both factors should be considered when attempting to establish a successfully differentiated classroom and build a community of mathematical thinkers.

The **physical learning environment** can be thought of as ***the concrete structure and layout of the learning space, as well as the placement of classroom furniture, tools, manipulatives, books, posters, and any other materials to support student learning***. Traditional classroom designs can impede differentiated learning, and it is important to design spaces that accommodate varied activities (Nair, 2014). The physical learning environment is not restricted to the classroom, as it could include the school library, science lab, computer room, resource room, or other spaces within the school that would further promote student learning of mathematics. Establishing and utilizing diverse spaces both within and outside schools can enhance efforts to implement more innovative and effective learning (Waks, 2014).

Because the goal of the lessons is to promote talent development among all students, learning environment modifications should aim to support all members of a diverse classroom. Another important consideration is the classroom's **affective environment**, or ***the social and emotional climate or culture of the classroom***. Students must believe they will be supported in their learning and their efforts are worthwhile (Siegle et al., 2017), and when students see concrete evidence that they are represented and connected to the class, in turn, they will feel emotionally safe to take risks, explore new ideas, think creatively, and communicate their ideas. The affective learning environment includes the positive learning space in which students' beliefs, values, ideas, feelings, mindsets, and personalities are shared and respected. It also includes the respectful relationships between and among the students, teacher, and support staff.

Essential guiding questions teachers should ask themselves before a new group of students arrive for the school year include the following:

- How can I design a physical and affective environment that supports the range of activities needed in differentiated learning?
- How can I create a physical and affective environment that supports students' ability to think, feel, and act like young practicing mathematicians?
- How can I design a physical and affective environment that will help create a safe classroom culture that is open to student discovery, collaboration, and ownership of their learning?

Here are some practical suggestions:

Tip #1: Promoting the Affective Learning Environment
As mentioned previously, the physical learning environment can support a positive affective environment when students see and feel that they are an important member of the classroom. What happens in the first few weeks of school can be crucial in setting the stage for a safe and positive emotional climate and successful school year for both the teacher and the students. Before expecting students to work in groups, take on challenges, or complete tiered

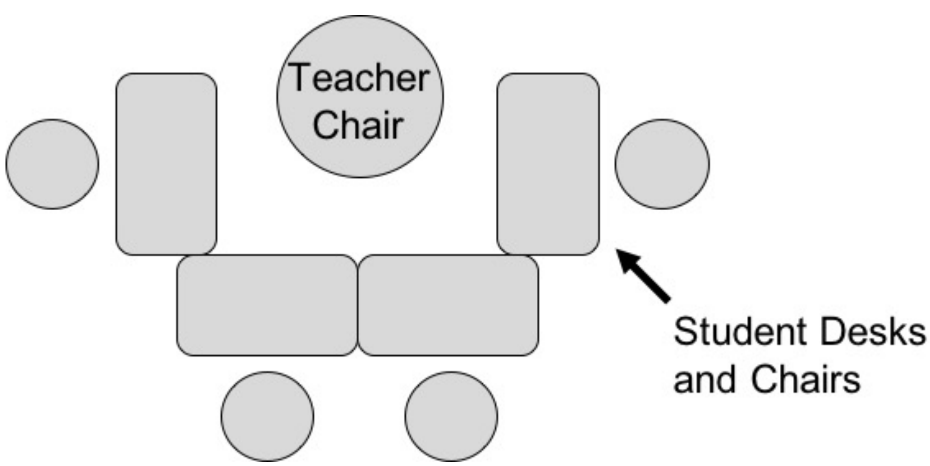
tasks, start by getting to know students and planning activities for students to know each other. Consider conducting activities that communicate, “This classroom is safe for students to have different interests, strengths, and weaknesses.” Celebrate when students learn from mistakes by creating an interactive bulletin board that displays new learning as a result of unanticipated mistakes. Ultimately, students learn best when they feel a sense of competence, relatedness, and autonomy (Ryan & Deci, 2000), and effective learning environments foster these feelings when they send welcoming, positive messages and promote active, social, and varied learning activities (Haythornthwaite, 2015; Nair, 2014).

Tip #2: Flexible Grouping of Desks

The tiered lessons allow teachers to group students by prior knowledge or learner readiness. At different times, students may work on the tiered lessons independently, gather with a small group to discuss what they learned, or come together to participate in whole-class activities. Therefore, the classroom should include options for flexibly grouping students for independent, small-group, and whole-class work, which better supports the diverse activities characteristic of 21st century learning in *all* subjects (Davies et al., 2013; Nair, 2014; Pearlman, 2010). For example, rather than placing desks into straight rows, consider grouping them to facilitate small group discussions. Small u-shaped arrangements of desks allow a teacher to place a chair in the open space, thus communicating to students that the teacher is a part of the discussion (see Figure 1). This also allows easy access to students to monitor, support, and extend student learning. A common area in the classroom for all students to gather, such as a rug for meeting spaces, would support a larger group discussion. Another important consideration is safety. The placement of desks and tables in the room should allow students to move safely and freely from one space to another.

Figure 1

Example Desk Arrangement to Facilitate Small Group Discussions



Tip #3: Accessibility of Materials
In the differentiated classroom, the materials, or tools, to support student learning, communication, thinking, processing, and discovery of mathematics should be accessible and varied in complexity. The placement of the materials should be intentional. Consider the height of your grade 3 students. Where can the materials be stored so that they can easily and safely be reached and returned by students? Materials should also be visible, if space permits, and clearly labeled, so the students will be more likely to use them when needed. For example, plastic, gallon-sized storage bags are a convenient option to store a variety of materials, needed for a single lesson for each student or small group. These can be placed in small storage bins on the lower shelves of a bookcase in plain sight for the students. Alternatively, materials might be placed on a countertop, at a mathematics station, or on a movable cart. Materials may include, but are not limited to the following: highlighters, sticky notes, manipulatives, computers, laptops, or other mobile devices.

Tip #4: Introducing Students to the Physical Learning Environment
To support classroom management, students should know the protocols and procedures for moving about the classroom, as well as accessing, using, and returning the materials. These routines should be explicitly communicated and practiced early in the school year and reviewed periodically as needed. Involving students in developing classroom expectations can contribute to a participatory, democratic classroom culture and help students internalize these expectations (Halverson et al., 2016; Woolner et al., 2014). A fun way to introduce students to the materials is to allow them to informally explore each learning tool, and then share with each other what they learned and experienced. In addition, have students practice moving about the classroom from one learning space to another. Doing so will allow you to communicate the classroom protocols for moving safely and when they are allowed to move about the classroom freely.

Tip #5: Making Mathematics Visible
Be intentional about the visuals on walls or what is displayed in different areas throughout the room. These visuals can communicate to students the importance of mathematics in your classroom. For example, to build a language rich classroom, display the authentic language of mathematicians that you want students to use. For classrooms with English learners, include mathematical language both in English and their home language, along with visual cues. Google Translate (https://translate.google.com) or a similar online translation tool can help you prepare visuals such as word walls or posters to display mathematical terms in different languages (see Figure 2).

Another way to be responsive to the diversity of your students is to include images, books, or other reading materials of mathematicians from diverse cultural or ethnic backgrounds. In this way, you will help to develop students' self-efficacy, or belief system that they, too, have the potential to become creative mathematical thinkers. Further, to promote an inclusive physical learning environment, display original student work or student-made learning posters.

Figure 2

Example Word Poster

Word Bank – Lesson 1

English	Spanish	Symbol/Símbolo
Addend	Sumand	$\boxed{4} + \boxed{5} = 9$
Benchmark	Punto de Referencia	$\ldots 19\ \boxed{20}\ 21\ 22\ldots.$
Decomposition	Descomposición	27 = 25 + 2 27 = 30 - 3
Equation	Ecuación	44 - 6 = 38 59 = 50 + 9 20 + 2 = 4 + 18
Inequality Sign	Signo de Desigualdad	$\neq$
Number Sentence	Frase Numerica	$42 \neq 45 - 7$ $40 + 2 = 42$

Conclusion

Together, the standards upon which these lessons are based outline knowledge and skills that will help students develop deeper understandings of mathematical concepts and procedures that can be applied in 21st century contexts. The Mission to Mars theme was integrated to engage students' interest and make visible mathematics' many applications in real-world situations. Instructional models emphasizing differentiation and enrichment served as the basis for these lessons, and the information provided in each lesson and this supplementary section were designed to build implementing teachers' understanding of these important pedagogical considerations. To further optimize students' learning, it is also important to establish a safe, inclusive, supporting physical and affective learning environment. It is our hope that this resource will ultimately facilitate meaningful and engaging mathematics learning, build your own capacity as an educator, and help you build a learning community in which students are supported and challenged to think like mathematicians.

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Glossary

- **1/2:** when a shape has been divided into two equal parts, each part is called 1/2 or one-half; it is important to note this both ways, as students will call it one-two

- **Benchmark Fraction:** a fraction that is used as a reference point. It is a common or familiar fraction that can be used to identify and measure more unfamiliar or less common fractions

- **Compare:** to examine two or more things and determine their similarities and differences (for example, whether a number is greater, equal to, or smaller than another number)

- **Criteria:** standard or principle for judging, evaluating, or selecting something

- **Denominator:** bottom number in a fraction that identifies the number of pieces the whole has been divided into equal parts

- **Distance:** number of units between the starting point and ending point on a number line

- **Divide:** split into equal parts or groups

- **Division:** the action of separating something into parts or the process of being separated

- **Eighths:** one part of eight equal parts into which something can be divided

- **End Point:** a point at which a line segment ends

- **Equal:** shows the same amount

- **Equal Parts:** the same portion, piece, or segment

- **Equal Shares:** parts that are the same size

- **Equivalence:** two values, numbers or quantities which are the same

- **Equivalent:** equal in value

- **Equivalent Fractions:** fractions with different numerators and denominators that represent the same value

- **Fourths:** the whole is divided into four equal parts

- **Fraction:** a number that represents part of a whole
- **Fraction Bar:** as a visual representation of fractions which helps in comparing fractions and carrying out operations with fractions
- **Geoboard:** square boards with pegs to which students attach rubber bands to form various shapes
- **Half:** one of two *equal* parts of a whole
- **Halves:** one of two equal parts into which something can be divided
- **Hypothesis:** an idea that might be true but needs to be tested
- **Integer:** a whole number that is not a fraction
- **Interval:** set of numbers including whole numbers and fractions. Also, an interval is a break or a pause
- **Math Talks:** activity designed to elicit multiple strategies and provide opportunities for students to reason about the relationships in the numbers and make connections in mathematics
- **Mathematical Model:** a representation of a math problem or concept. This may be an equation, drawing, or table
- **Number Line:** a line with numbers placed in their correct position
- **Numerator:** top number in a fraction that identifies the number of equal pieces considered as part of the whole
- **Partition:** splitting a whole into equal parts
- **Partitioned:** divided into parts
- **Pattern:** predictable, repeated way that something is done or presented
- **Precise:** describes responses that are exact, accurate, careful about details
- **Precision:** the quality, condition, or fact of being exact and accurate
- **Quadrant:** a section representing one quarter of the whole area (i.e., a garden section)
- **Sixths:** the whole is divided into six equal parts

- **Symbols:** less than <, equal to =, and greater than >
- **Thirds:** one part of three equal parts into which something can be divided
- **Unit:** an individual, single component of a larger or more complex whole
- **Unit Fraction:** a fraction that represents one equal part of a whole
- **Whole:** the entire unit that represents one
- **Whole Number:** the set of numbers that include zero and natural numbers

TLM Research Team

Thinking Like Mathematicians: Challenging All Grade 3 Students

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