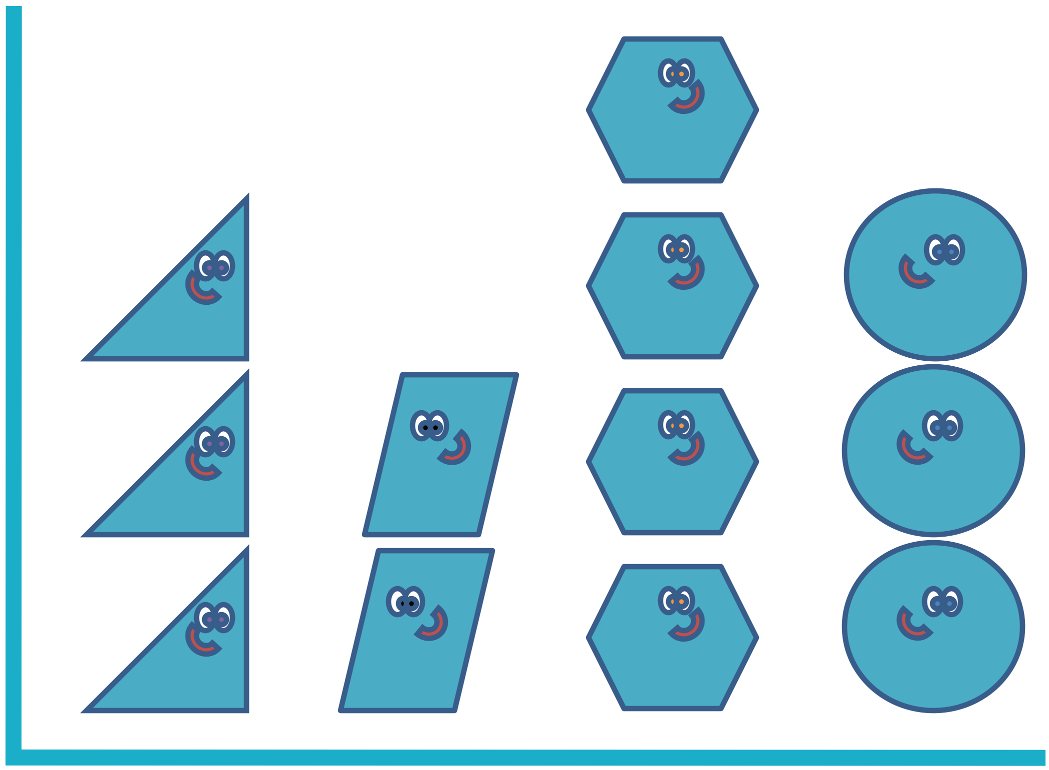
GREENING UP WITH GRAPHING: RECYCLE, REDUCE, & REUSE

Third Edition



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The National Research Center on the Gifted and Talented

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THE NATIONAL RESEARCH CENTER ON THE GIFTED AND TALENTED

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The University of Connecticut and the University of Virginia are collaborating on a 5-year research study of identification, curriculum interventions, and assessment to determine What Works in Gifted Education: Excellence and Equity in Educating Gifted Students.

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Mathematical language is a thinking tool that helps us use reason to create and communicate ideas about concepts, numbers, and shapes to solve problems in the real world.

- NRC/GT Research Team, University of Connecticut, 2008

GREENING UP WITH GRAPHING

Introduction

For centuries mathematicians and scientists have worked with data in the form of numbers and words to solve problems related to the world around us. They use data analysis techniques to determine answers to relevant questions such as whether global warming is occurring or how prevalent certain diseases are, so data analysis is often viewed as something related only to research. However, we also use data analysis in our everyday lives, such as when we ask advice from a number of people to choose a course of action. We may not be graphing the responses, but we are surveying, collecting data of sorts, analyzing, and taking action based on that information. For these reasons, it is important that students understand the fundamentals of data analysis, and therefore data analysis is one of the five major content strands outlined by the National Council of Teachers of Mathematics (NCTM) in *Principles and Standards for School Mathematics* (2000).

Students in this unit are encouraged to collect, interpret, and represent data in different formats. Because the end result of any data analysis is most appropriately to determine a course of action, the data analysis activities in the unit are embedded in a theme that encourages students to find ways to protect their environment. When we help students become researchers/practitioners who collect their own data for a practical and important purpose, we empower them to use mathematics as a tool for change in their world.

The *Greening Up With Graphing* unit is divided into three sections: *Line Graphs (Part 1 & Part 2), Bar Graphs,* and *Line Plots*. *Line Graphs* has been divided into two parts because students collect data throughout a multi-day intervention. They measure the amount of recycling that is happening at a location in their school, then plan an intervention to increase that amount. They conduct the intervention and display the results as a line graph. While data is being collected related to this intervention, students begin learning about bar graphs. They learn to sort and categorize data using foam shape manipulatives, then apply that knowledge to the implementation of a clothing, book, or crayon drive. In *Line Plots,* students explore ways of displaying data for questions that begin with the words "*How many*."

Research skills and ways to represent data are embedded in the unit. Research skills include data sampling, as well as observation and survey techniques. Data is represented using three graphing formats: line graphs, bar graphs, and line plots. Rather than simply asking students to graph numbers, the unit requires students to think deeply about how data is represented and how that representation changes if elements of the graph change. For example, students are asked to change the way a given data set gets categorized.

By exposing students to these techniques embedded in a real-world, meaningful context, we hope that students become excited, engaged, and enthusiastic about using mathematics as a tool in the world around them.

Rationale

> The amount of data available to help make decisions in business, politics, research, and everyday life is staggering: Consumer surveys guide the development and marketing of products. Polls help determine political-campaign strategies, and experiments are used to evaluate the safety and efficacy of new medical treatments. Statistics are often misused to sway public opinion on issues or to misrepresent the quality and effectiveness of commercial products. Students need to know about data analysis and related aspects of probability in order to reason statistically—skills necessary to becoming informed citizens and intelligent consumers. (NCTM, 2000, p. 48)

The *Principles and Standards for School Mathematics* (NCTM, 2000) aims to support students' ability to think and reason mathematically. Students should learn how to question the world around them, how to find answers to these questions, how to analyze data, and how to communicate their findings to others. This unit asks students to formulate questions and collect, organize, and display relevant data to answer such questions. Students learn to utilize appropriate analytical methods to derive meaning from their investigations, making inferences and predictions based on their results. By investigating questions and problems in this manner, students acquire varied modes of thinking and expressing, habits of inquiry and persistence, and confidence to question unfamiliar situations or information. Rather than a simple summary or description, students practice the art of argument and rationale.

In this unit, students also have the opportunity to participate in mathematical conversations, listening to others' explanations and justifications that allow them to explore multiple perspectives and form connections. Besides oral and written communication of ideas, students learn to represent ideas through pictures, tables, graphs, and other displays. These mathematical tools help to develop students' abilities and make them more socially aware.

NCTM Standards and Focal Points

NCTM Curriculum Standards & Focal Points	Lesson 1: Line Graphs (Part 1)—Learning About Recycling	Lesson 2: Line Graphs (Part 1)—Planning the Intervention	Lesson 3: Line Graphs (Part 1)—Implementing the Intervention	Lesson 4: Bar Graphs—Whole Lot of Sorting Going On	Lesson 5: Bar Graphs—Fair Share and How Many to Spare	Lesson 6: Bar Graphs—Displaying Shape Data	Lesson 7: Bar Graphs—Reusing and Reducing	Lesson 8: Bar Graphs—Professional Style	Lesson 9: Bar Graphs—Graphing and Analyzing Data	Lesson 10: Line Graphs (Part 2)—Learning About Line Graphs	Lesson 11: Line Graphs (Part 2)—Evaluating the Intervention	Lesson 12: Line Plots—Discovering Line Plots	Lesson 13: Line Plots—Learning About Surveys	Lesson 14: Line Plots—Designing a Survey	Lesson 15: Line Plots—Using Frequency Tables	Lesson 16: Line Plots—Reusing Our Data
DATA ANALYSIS																
1. Students design investigations to address a question and consider how data-collection methods affect the nature of the data set.	X	X		X			X					X	X	X		
2. Students collect data using observations, surveys, and experiments		X	X		X			X						X	X	
3. Students represent data by creating tables and graphs such as line plots, bar graphs, and line graphs.			X		X	X	X	X	X	X	X	X		X	X	X
4. Students recognize the differences in representing categorical and numerical data.				X					X	X	X					X
5. Students propose and justify conclusions and predictions that are based on data and design studies to further investigate the conclusions or predictions.	X	X	X			X	X		X	X	X	X		X	X	X
6. Students apply knowledge of addition, subtraction, multiplication, and division of whole numbers to construct and analyze frequency tables, bar graphs, picture graphs, and line plots and use them to solve problems.					X	X		X	X			X		X	X	X

Data Analysis: A Model-based Unit

This unit was designed with specific modifications and differentiation for high ability or gifted learners. It also was designed to be responsive to the academic diversity of the talent pool and all other students in general education classrooms. Elements of three well-known curricular models in the field of gifted and talented education were combined and utilized to develop this unit: the Differentiation of Instruction Model from Carol A. Tomlinson (2001), the Depth and Complexity Model from Sandra N. Kaplan (1998), and the Schoolwide Enrichment Model from Joseph S. Renzulli and Sally M. Reis (1997). These three research-based models support and promote qualitative differentiation of learning and the ways it is pursued, the nature and extent of student engagement, the active and investigative roles assumed by students, and the quality of student products.

The Differentiation of Instruction Model is designed to provide rich and engaging curriculum matched to the diverse interests, readiness levels, and learning profiles of individual students. The model assumes that there is no distinct, single curriculum appropriate for gifted learners, but rather that all students, including the gifted, require educational experiences suited to their individual needs (Tomlinson, 1996).

The Depth and Complexity Model emphasizes the importance of rich, deep, and complex content in appropriately serving gifted learners (Kaplan, 1998). This model emphasizes the benefit of higher level thinking skills, elaborate product development, and more advanced resources in a curriculum for the gifted, but the crux of the model's curriculum equation is the redefinition of the nature of the content. It is based on the premise that high-level content that is appropriate is synonymous with the dimensions of depth, complexity, novelty, and acceleration (Kaplan).

The Schoolwide Enrichment Model identifies a talent pool of 15 to 20% of above average ability and/or high potential students who will be served through a variety of options, including learning experiences geared toward students' interests and learning styles, curriculum compacting, and enrichment experiences (Renzulli & Reis, 1985; 1997). Over 20 years, the Schoolwide Enrichment Model research demonstrates its effectiveness with a broad range of school socioeconomic levels, program organization patterns, and gifted learners (Baum, 1985, 1988; Burns, 1987; Delcourt, 1988; Gubbins, 1982; Imbeau, 1991; Olenchak, 1988; Olenchak & Renzulli, 1989; Reis, 1981; Reis & Renzulli, 2003; Schack, 1986; Starko, 1986).

This curriculum unit reflects the commonalities of these well-known models in the field of gifted and talented education. This unit tailors essential content, processes, and products to the academic needs of students in academically diverse classrooms; emphasizes conceptual thinking, real-world disciplinary inquiry, and problem solving; assesses specific and developing learning needs of talent pool and all other students in general education classrooms; helps students acquire increasing levels of expertise; and encourages student involvement with problem solving and product development with real-world utility.

Greening Up With Graphing Pacing Chart

	Monday	Tuesday	Wednesday	Thursday	Friday
Week 1 Line Graphs (Part 1) (Lessons 1-3) Bar Graphs (Lesson 4)	**Unit** **Pretest**	**Lesson 1:** Learning About Recycling **Tiered Student Pages** Babbage Galileo & Falconer	**Lesson 2:** Planning the Intervention	**Lesson 3:** Implementing the Intervention	**Lesson 4:** Whole Lot of Sorting Going On **Tiered Student Pages** Babbage Galileo & Falconer
Week 2 Bar Graphs (Lessons 5-9)	**Lesson 5:** Fair Share and How Many to Spare **Optional Challenge**	**Lesson 6:** Displaying Shape Data **Tiered Student Pages** Babbage Galileo & Falconer	**Lesson 7:** Reusing and Reducing	**Lesson 8 (Optional):** Professional Style	**Lesson 9:** Graphing and Analyzing Data **Interest-Based Products**
Week 3 Line Graphs (Part 2) (Lessons 10 & 11) Line Plots (Lessons 12-14)	**Lesson 10:** Learning About Line Graphs **Tiered by Activity** Babbage & Galileo Falconer	**Lesson 11:** Evaluating the Intervention	**Lesson 12:** Discovering Line Plots **Tiered Student Pages** Babbage Galileo Falconer	**Lesson 13:** Learning About Surveys **Optional Challenge**	**Lesson 14:** Designing a Survey **Tiered Student Pages** Babbage Galileo Falconer
Week 4 Line Plots (Lessons 15 & 16)	**Lesson 15:** Using Frequency Tables **Tiered Student Pages** Babbage Galileo Falconer	**Lesson 16:** Reusing Our Data	**FLEX TIME – Seven flex days are built in for lessons that take more than one class period to complete. Classes may also use this time to disseminate the data they have gathered to the rest of the school.**		
Week 5 Unit Test	**FLEX TIME**				**Unit** **Posttest**

The Lesson Format

Each lesson is written in the following format to allow for easy implementation:

Planning Phase

- **Big Mathematical Ideas**
- **Lesson Objectives**
- **Materials**
- **Mathematical Language**
- **Lesson Preview**

Teaching Phase

- **Initiate**
  This is an introductory learning experience designed to engage students.
- **Investigate**
  These activities allow students to delve into the activity, exploring the concepts in interesting hands-on investigations.
- **Conclude**
  This section allows students to solidify new knowledge from the lesson, communicating what they have learned in a variety of ways.
- **Assess**
  Teachers will be given opportunities to assess, both formally and informally, student progress throughout the unit. Forms of assessment include:
  - **Student Pages**
    The Student Pages include *Student Mathematicians Think Deeply*, *Student Mathematicians Practice*, and various topic-specific pages that allow for practice of new material and encourage critical thinking. After the questions related to the lessons, many of these pages include practice problems, which are designed to gauge student progress in number and operations procedures outside of the unit. Teachers can decide whether to formally assess students on these sections of the Student Pages or to simply use them as a guide in determining which students need assistance in particular areas. All of these pages are accompanied by answer pages, which include possible student responses and possible student difficulties. Many of these pages have been tiered for different levels of readiness.
  - **Group discussion**
    This can be based on student communication on topics during the Conclude section of the lesson or questions posed on the Student Pages.
  - **Direct inquiry**
    This occurs when teachers directly ask students comprehension questions or ask students to demonstrate understanding.

The Unit Breakdown

The unit is designed to develop data analysis skills while allowing students to explore environmental themes relating to their school's practices. The unit is divided into three main segments: Line Graphs, Bar Graphs, and Line Plots. The following is a list of the topical sections throughout the unit:

- **Planning** allows students to practice as mathematicians in preparing experiments. In the first few lessons, students describe and plan an intervention that measures recycling in their school. Later in the unit, students plan a clothing, book, or crayon drive to support a cause. In this planning phase, students formulate appropriate research questions and hypotheses.
- **Collecting** data allows students to practice as mathematicians in gathering and compiling data. In several of the lessons, students are asked to record data in a table. Students also compile data, using foam shape manipulatives to introduce the idea of sorting into categories. In *Line Plots,* students administer surveys that they created. Other data collection methods are also encouraged, such as note taking and tallying.
- **Converting** data allows students to practice as mathematicians in transforming data into graphs from frequency or data tables. In the recycling intervention and in the clothing, book, or crayon drive, students are asked to convert data into graphical representations.
- **Analyzing** data trends allows students to practice as mathematicians in making inferences about representative samples. Students first analyze and infer from the results of their intervention experiment, asking themselves, "Were we successful?" and "Did it work?" Additionally, they are able to analyze existing data, such as temperatures across the United States.
- **Reflecting** on the challenges and successes of the research process allows students to practice as mathematicians in self-evaluation and modification. As students reflect on their experiments, they are also asked to suggest improvements in design and process. It is important for students to realize that research is an ongoing process and that success is not always guaranteed.

Student Mathematician Journal

The *Student Mathematician Journal (SMJ*) allows students to experience thinking and working like young mathematicians. The *SMJ* consists of all the Student Pages to promote reflection and practice with "big mathematical ideas." The activities included in the Student Pages are designed to challenge the talents and abilities of all students in academically diverse classrooms.

Throughout the manual, teachers are guided in how to use the Student Pages. The teachers' manual includes a duplicate set of Student Pages. You will note an inset indicating the corresponding page numbers for the *Student Mathematician Journal.*

Periodically, we have included Check Up Pages that serve as review and practice.

Graphic Communication

Students learn that sometimes a picture is worth a thousand words. In this unit, students move from verbally communicating their results to communicating them via writing to communicating them via picture, graph, or plot. Students learn to represent tabular data in the form of graphs and plots for others to get a snapshot of their results. They act as researchers, who have many ways to present the data they collect. They are asked questions such as, “How would your graph change if you had different categories?” Students learn that line plots can be used to give a general idea of the trends in survey data and generate summary information. Students begin to understand how representing data graphically makes communication more parsimonious, and they see this as a process that evolves from oral to written to graphic communication.

Differentiating Instruction

This unit has been developed using proven curricular models. Essentially, this unit calls for the teacher to respond to student needs. The teacher’s response is guided by general principles of differentiation, such as respectful tasks, on-going assessment and adjustment, flexible grouping, clarity of learning goals, and appropriate challenge. This unit presents both differentiated content that will engage students and differentiated products that will challenge them. Most lessons are differentiated according to learners’ readiness and interests. The content presents greater complexity and is in greater depth for students who are more familiar with the concepts. Products ask students to develop expertise, complete more open-ended tasks, and work increasingly independently.

Lessons 1, 4, 6, 10, 12, 14, and 15 contain tiered Student Pages, or differentiation by content, activity, or product. In addition, some of the optional student pages in Lessons 5, 8, and 14 are differentiated. In these lessons, the questions vary from group to group, increasing in complexity, depth, and abstraction as the groups increase in expertise of that particular concept or topic. In Lesson 1, students are presented with different tasks according to their readiness. The Babbage Group works on tasks that relate directly to the interpretation of an existing graph. The Galileo & Falconer Student Page contains an extension that asks students to consider what would happen to the graph if it continued for twice as long. The question lends itself to multiple responses but requires justification. Thus, the tasks become more complex and ask for greater depth of understanding.

Student interests are also a key feature of quality differentiation. In Lesson 9, students choose how they will present the overall results of their drive. Students are encouraged to know and focus on their individual strengths when choosing this medium of communication. They are also encouraged to use technology in creating their graphs to provide a more professional-looking product.

Two lessons contain optional challenges. These are intended to provide additional depth and complexity to the existing lessons either for individual students or for the whole class. This is in accordance with the National Math Panel's recommendation that mathematically gifted students should be encouraged and supported in learning at the accelerated rates of which they are capable (National Mathematics Advisory Panel, 2008).

Implementing the Pretest and Posttest

The pretest and posttest for this unit are the same document with the same questions—in this manual they are simply referred to as the Unit Test. This test measures students' abilities to interpret and infer information and to create the types of graphs that they will encounter throughout the unit—line graphs, bar graphs, and line plots. The Unit Test should be utilized as a pretest to determine student readiness for the investigations of the unit. We will collect pretest and posttest data from each student for analysis purposes. Students' performance on the pretest should be used in the flexible grouping of students during tiered investigations. For such investigations, a chart is provided in the unit that demonstrates the recommended grouping of students according to how they performed on a related task on the pretest. The posttest will measure students' acquisition of graphic communication skills (see below)—their ability to create and interpret graphic representations of data. Pretest and posttest scores will also be collected by the research team to assess the impact of the unit content.

After the Unit Test in this manual is an Answer Key and Unit Test Rubric that includes the focus of the question, the number of points per question, and an expected student response. The Unit Test Scoring Guide follows the Rubric. The Guide categorizes student scores according to type of graph—line graph, bar graph, and line plot—for facilitation of grouping within that section of the unit. Teachers are asked to use their judgment based on student performance within that section to classify students. This Guide is intended to aid teachers in the flexible grouping of students throughout the unit.

Lessons Tiered by Mathematician Name

Throughout the unit are tiered activities or tiered groups, with different levels of challenge. We used the names of famous mathematicians who made significant contributions to the field. The selected groups and their descriptions are below, followed by brief mathematician biographies. You should share these biographies with the students or have the students research the mathematicians. For additional information about and activities related to these famous mathematicians, refer to the Resources section of your CD-Rom.

- **Babbage**—Least challenging of the 3 levels. Designed for students with little to no prior knowledge of the topic.
- **Galileo**—Medium level of challenge. Designed for students with some understanding of the topic.

- **Falconer**—Most challenging. Designed for students who already grasp the topic and are ready for more challenging applications with less scaffolding.

Famous Mathematicians

Charles Babbage (pronounced BAB-uhj) 1791 - 1871
Charles Babbage was born in London, England, in 1791. His father was a banker who owned a big estate. Charles was taught at home by many tutors. When Charles was 8, he became very ill with a fever and was forced to rest for many months. Charles said that during this rest he had much time to think about math and he came up with many ideas. When Charles was better, he went to an academy in a nearby town. His favorite part of the academy was the library. Charles then went to the local university to study mathematics. At the university, he had many friends who later became famous mathematicians, too. After graduating, Charles married a woman named Georgiana, and they had 8 children. Charles became a professor at his university. Charles created the first machine that did calculations and printouts. This machine weighed 15 tons and was 8 feet high. Because of this invention, Charles is known as the “Father of Computing.” He also invented the type of light bulb that is used in lighthouses and parts of trains and railroad tracks. Charles studied symbols and was able to figure out many secret codes. He won awards for his work in mathematics and engineering.

Galileo Galilei (pronounced gal-ih-LAY-oh gal-ih-LAY) 1564 - 1642
Galileo Galilei was born in 1564 in Pisa, Italy, the same place as Leonardo Fibonacci. He was a mathematician, a scientist, and a philosopher. Galileo’s father was a musician but wanted his son to be a doctor. Galileo started studying medicine, but he changed his mind and decided to study mathematics at the university. Galileo’s first idea about the science of mathematics happened when he was watching a lamp swing back and forth in church. Because of this idea, Galileo came up with many theories and inventions, like the pendulum clock. He came up with many other ideas in mathematics and science. Many people call Galileo the “Father of Science” and the “Father of Astronomy.” He created the telescope and was able to look at the moon, observe supernovas, and discover sunspots and Jupiter’s moons. Galileo also invented the water pump. Galileo worked for one of the most powerful families in Italy, the Medicis. Galileo was one of the first people to argue that the Sun was the center of the universe. This made him very unpopular to some people. He went to jail because of these beliefs, even though his theories were correct. Galileo stood up for what he believed in. His ideas, theories, and inventions survive today and inspire many mathematicians and scientists.

Etta Falconer (pronounced FAL-kuhn-urh) 1933 - 2002

Etta Falconer was an African-American who was born in 1933 in Tupelo, Mississippi. Etta had a great passion for mathematics and science. Her father was a doctor and her mother was a musician. Etta had an older sister. Etta went to the University of Wisconsin in the 1950s. Here she met many people from many different races. She had friends from Africa, Thailand, and India. Etta became only the tenth African-American woman to become a doctor of mathematics. Etta became a mathematics professor who lived and taught all over the country—in Mississippi, Tennessee, Georgia, Virginia, Illinois, and California. She created the NASA Women in Science Program in 1987. Etta was also a mentor, a person who helped many African-American girls become successful mathematicians and scientists. Etta’s best mathematics skill, like Hypatia, was to create easy definitions to explain hard mathematical ideas. Etta won many awards for her work with girls and her work with mathematics.

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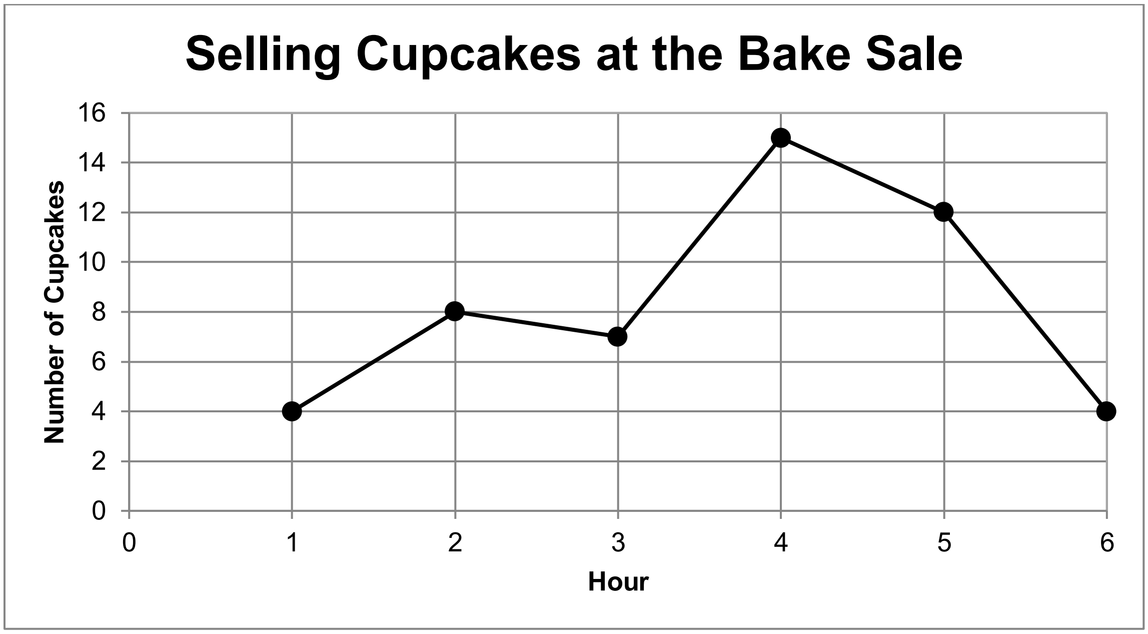
Name: ___________________________________ Date: __________________

Unit Test: Great Graphing

1. The fifth grade had a bake sale to raise money for their party. The line graph below shows how many cupcakes they sold each hour.



   a. During what hour did they sell the most cupcakes? __________________

   b. Between what two hours did the number of cupcakes sold increase the most? ___________________

   c. How many cupcakes did they sell in total at the end of the 6 hours? Explain your thinking.

____

____

____

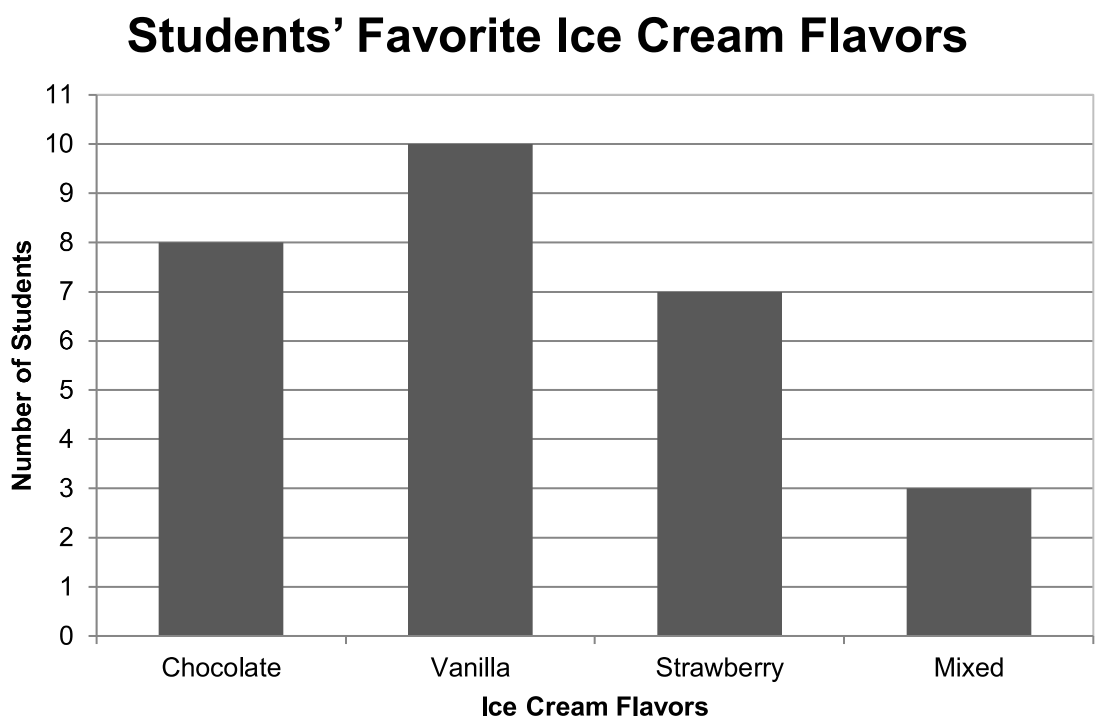
____

d. A local soccer team has kept track of the number of goals scored each game. The table shows the data for the first six games.

- Make a line graph that shows the number of goals per game.
- Give your graph a title and labels.

Game	Number of Goals
1	1
2	0
3	3
4	2
5	3
6	5

2. Students were asked about their favorite ice cream flavors. The results were made into the following bar graph.



a. What is the most popular ice cream flavor in the class? _______________

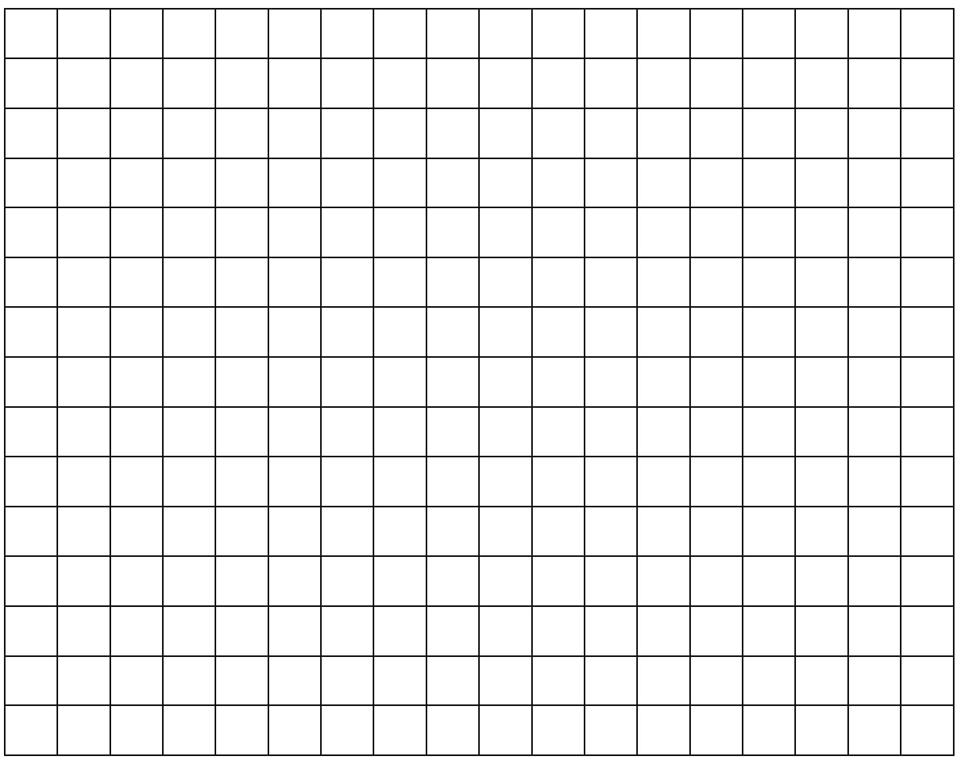
b. How many students were asked this survey question in total? __________

Collecting Toys

c. Students collected toys for a toy drive and made a table of the data.

Type of Toy	Number Collected
Games	15
Dolls	20
Trucks	30
Bikes	10
Sports	7

- Make a bar graph. Use a scale that will let all of your bars fit on the grid.
- Give the graph a title and label it.



d. Explain how you chose a scale for your graph.

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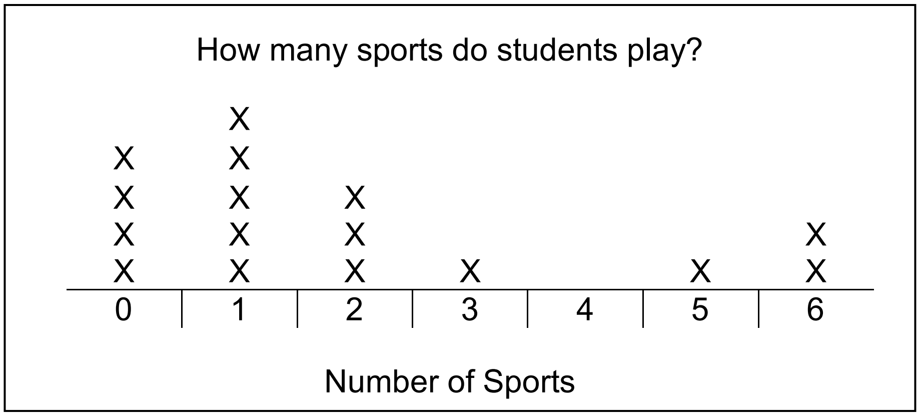
3. Mr. Reid's class asked 16 students how many different sports they play. They made a line plot.



a. How many students said they play 2 sports?

____

b. What are the mode and median of the data?

Mode: ______________ Median:____________

c. Mrs. Ryan's class asked 10 students how many people live in each of their houses. Here are the data from their survey:

4 5 2 4 7 3 5 4 5 7

Use the data to make a line plot below:

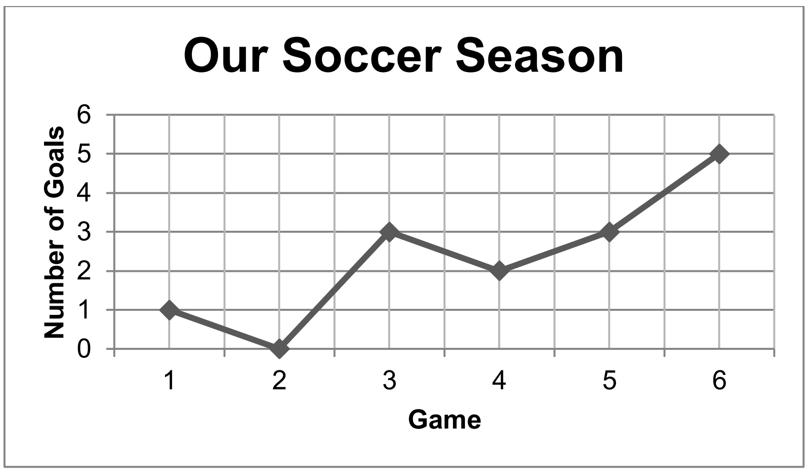
Unit Test: Great Graphing
ANSWER KEY

1a. *They sold the most cupcakes during hour 4.*

1b. *The cupcake sales increased the most between the hours of 3 and 4.*

1c. *They sold 50 cupcakes at the end of 6 hours. The sum of the number of cupcakes from each hour was totaled (4 + 8 + 7 + 15 + 12 + 4) to get 50.*

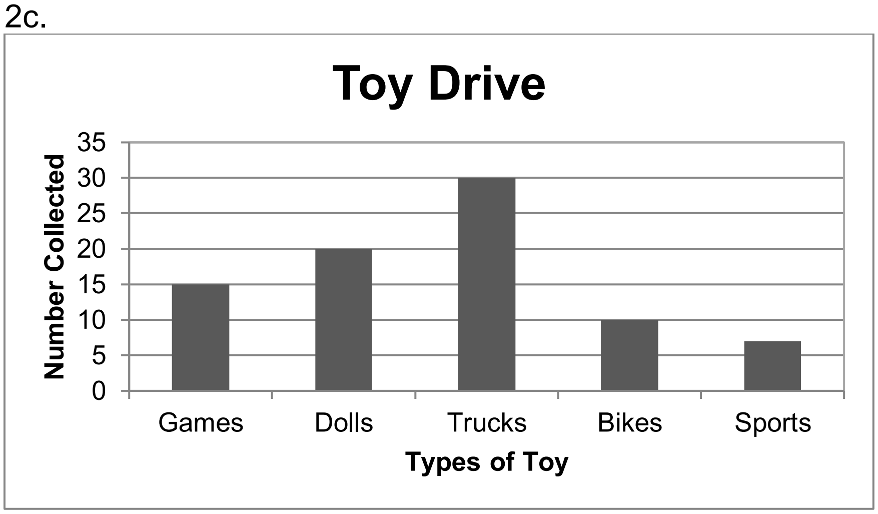
1d.



2a. *The most popular ice cream flavor in the class is vanilla.*

2b. *8+10+7+3 = 28; This is the sum of the total for each bar on the graph.*

2c.



2d. *Scale increments of 2, 5, or 10 are good choices. The grid provided has 15 boxes. Given a maximum data value of 30, each box should be counted as 2 or more to fit the data to the grid.*

3a. *Three students said they play two sports.*

3b. Mode: *1* Median: *1*

3c.

		X	X		
		X	X		X
X	X	X	X		X
2	3	4	5	6	7

Number of People in Students' Homes

Unit Test Scoring

Unit Test Rubric: Greening Up With Graphing   Question 1: Line Graphs

Problem	Mathematical Focus: Points	Mathematical Focus: Expected Student Response
1a **(max 1)** *Focus: Interpreting a line graph.*	**1**	States that they sold the most cupcakes during hour 4.
1a **(max 1)** *Focus: Interpreting a line graph.*	**0**	Answer is incorrect or student skipped question.
1b **(max 1)** *Focus: Interpreting a line graph.*	**1**	States that sales increased the most between hours of 3 and 4.
1b **(max 1)** *Focus: Interpreting a line graph.*	**0**	Answer is incorrect or student skipped question.
1c **(max 2)** *Focus: Connecting a line graph to number and operation.*	**2**	States that 50 cupcakes were sold and provides an explanation. One example might be, "I added the number from each hour, 4+8+7+15+12+4, to find the total number of cupcakes."
1c **(max 2)** *Focus: Connecting a line graph to number and operation.*	**1**	States that the answer is 50 cupcakes without explaining *OR* provides a good explanation with an answer other than 50.
1c **(max 2)** *Focus: Connecting a line graph to number and operation.*	**0**	Answer is incorrect or student skipped question.
1d **(max 2)** *Focus: Constructing a line graph.*	**2**	Draws a line graph with all of the following: labels on axes, a title, an accurate line representing the data.
1d **(max 2)** *Focus: Constructing a line graph.*	**1**	Draws a line graph that is missing labels but has an accurate line *OR* has all labels but minor errors representing the data.
1d **(max 2)** *Focus: Constructing a line graph.*	**0**	Draws a graph with major errors, makes the wrong type of graph, *OR* does not answer the question.

TOTAL POINTS: 6

Unit Test Rubric: Greening Up With Graphing Question 2: Bar Graphs

Problem	Mathematical Focus: Points	Mathematical Focus: Expected Student Response
2a (max 1) *Focus: Interpreting a bar graph.*	**1**	States that the most popular ice cream flavor is vanilla.
2a (max 1) *Focus: Interpreting a bar graph.*	**0**	Answer is incorrect or student skipped question.
2b (max 1) *Focus: Interpreting a bar graph.*	**1**	States that 28 students were asked this survey question.
2b (max 1) *Focus: Interpreting a bar graph.*	**0**	Answer is incorrect or student skipped question.
2c (max 2) *Focus: Constructing a bar graph.*	**2**	Draws a bar graph with all of the following: labels on both axes, a title, an appropriate scale, accurate bars representing the numbers.
2c (max 2) *Focus: Constructing a bar graph.*	**1**	Draws a graph that is missing title or labels *OR* uses an inappropriate scale for the space provided *OR* has errors in the bar heights.
2c (max 2) *Focus: Constructing a bar graph.*	**0**	Draws a graph with major errors, makes the wrong type of graph, *OR* does not answer the question.
2d (max 1) *Focus: Understanding how an appropriate scale is selected.*	**1**	Provides an accurate explanation of how he/she decided on a scale. For example, the student writes, “I counted the number of boxes on the grid and there were only 15, so I counted by 2s to get to 30.”
2d (max 1) *Focus: Understanding how an appropriate scale is selected.*	**0**	Explanation is mathematically inaccurate or student skipped question.

TOTAL POINTS: 5

Unit Test Rubric: Greening Up With Graphing   Question 3: Line Plots

Problem	Mathematical Focus: Points	Mathematical Focus: Expected Student Response
3a **(max 1)** *Focus: Interpreting a line plot.*	**1**	States that 3 students play 2 sports.
3a **(max 1)** *Focus: Interpreting a line plot.*	**0**	Answer is incorrect or student skipped question.
3b **(max 1)** *Focus: Interpreting a line plot.*	**1/2 point each answer**	States that the mode is 1. States that the median is 1.
3b **(max 1)** *Focus: Interpreting a line plot.*	**0**	Both mode and median are incorrect.
3c **(max 2)** *Focus: Constructing a line plot.*	**2**	Draws a line plot that is numbered across the bottom, has the correct number of X's (or other symbol) above each number, and has a title and/or label.
3c **(max 2)** *Focus: Constructing a line plot.*	**1**	Draws a line plot that contains one or more minor errors (i.e., the wrong number of X's above one number).
3c **(max 2)** *Focus: Constructing a line plot.*	**0**	Draws a line plot with major errors, makes the wrong type of graph, *OR* does not answer the question.

TOTAL POINTS: 4

Greening Up With Graphing
Unit Test Scoring Guide Directions

Note that the unit test may be used both as a pretest to assess students' prior knowledge and as a posttest to assess growth.

Directions:

1. Record **students' names** on the Unit Test Scoring Guide.

2. Use the Unit Test Rubric **to score student responses**.

3. **Record students' scores** for each question under "Scores."

4. After recording all students' scores, use the results and your judgment to classify students into low (L), medium (M), and high (H) groups.

5. Remember that a student's score may indicate that his or her **placement may change** from one curricular topic to another. For example, a student may be in a high group for line graphs, but in a medium group for line plots.

6. Assign students to groups in lessons calling for differentiation as follows:
   - Babbage (L)
   - Galileo (M)
   - Falconer (H)

Teacher ______________________________ School ______________________________

City ____________________________ State _______ Date _______________________

Please indicate the average length of time (in minutes) students needed to complete the test: ☐ 30 or less ☐ 40 ☐ 50 ☐ 60

☐ Pretest ☐ Posttest

Greening Up With Graphing
Unit Test Scoring Guide

Last Name, First Name	**Q1 (6 pts)** Line Graphs				**Q2 (5 pts)** Bar Graphs				**Q3 (4 pts)** Line Plots			
	score	L	M	H	score	L	M	H	score	L	M	H
1												
2												
3												
4												
5												
6												
7												
8												
9												
10												
11												
12												
13												
14												
15												

Teacher ______________________________ School ______________________________

City ____________________________ State _______ Date ________________________

Please indicate the average length of time (in minutes) students needed to complete the test: ☐ 30 or less ☐ 40 ☐ 50 ☐ 60

☐ Pretest ☐ Posttest

Greening Up With Graphing
Unit Test Scoring Guide

Last Name, First Name	**Q1 (6 pts)** Line Graphs				**Q2 (5 pts)** Bar Graphs				**Q3 (4 pts)** Line Plots			
	score	L	M	H	score	L	M	H	score	L	M	H
16												
17												
18												
19												
20												
21												
22												
23												
24												
25												
26												
27												
28												
29												
30												

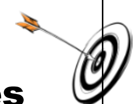
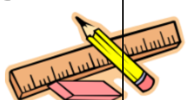
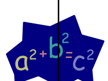
LINE GRAPHS (PART 1)—

LEARNING ABOUT RECYCLING

LESSON 1

Big Mathematical Ideas

Researchers often design interventions to try to make a change in practices. The single intervention design can be accomplished by collecting data before, during, and after the intervention.

Lesson Objectives 	• Students will interpret an increasing line graph.
Materials 	• Student Page—*Recycling in the News (Babbage and Galileo & Falconer)* [SMJ pages 1 & 3] • Student Page—*For a Good Cause: Collecting and Donating* [SMJ page 7] • Student Page—*Student Mathematicians Think Deeply* [SMJ page 11]
Mathematical Language 	• **Recycle:** To create new products from waste materials. • **Intervention:** The process of changing something to determine if you can affect the outcome. • **Experiment:** A test, trial, or procedure to discover something new or test an idea. • **Reuse:** To use again, sometimes for a different purpose.

Lesson Preview

Students learn about recycling, interventions, and experiments.

Initiate

1. **Brainstorm recyclable items**
Ask students what it means to recycle. Prompt students to think about what they recycle at home. Once students demonstrate familiarity with the concept, display the definition in the classroom. Then discuss items at

school that are or could be recycled. Write student responses on the board. Do not attempt to judge student responses at this stage, but simply record all items.

Investigate

2. **Predicting and interpreting a graph's meaning**

Unit test item #1 can be used to determine students' tasks for this part of the lesson. Students scoring in the high and middle ranges should be given the more challenging Galileo & Falconer version of the task. Students with lower scores should try the Babbage version of the task. NOTE: The article is the same for both groups.

Separate students into groups: Babbage and Galileo & Falconer. Base your grouping for this lesson on your knowledge of the students and how they scored on the pretest, as follows:

	Babbage Group	**Galileo & Falconer Group**
How student scored on #1 on the pretest	Scored 0-3 points on item	Scored 3-6 points on item
Lesson Student Pages for each group	Recycling in the News—Babbage	Recycling in the News —Galileo & Falconer

Ask students to look at the graph on the *Recycling in the News* Student Pages **[SMJ pages 1 & 3]**. Without reading the article, ask them to describe what is happening in the graph. Tell students they will read an article based on this graph. Ask them to predict what the article is going to discuss. Read *Recycling in the News* with students, which describes a school recycling program. Ask students to respond to the discussion questions based on the article. (See Appendices A and B for information on math written communication and talk moves.)

Sample Responses:

1. (Babbage version only) Why do you think that some people do not recycle?
   Suggested answers may include that people don't know what to recycle or how recycling works in their neighborhood. People may not think about their trash because they are thinking about other things.
2. (On both versions, #2 on Babbage and #1 on Galileo & Falconer) What did you learn from the article that the graph did not tell you?
   Answers may include the types of materials the students are recycling and the idea of using colored bins.

3. (#3 on Babbage and #2 Galileo and Falconer) What did you learn from the graph that the article did not tell you?
   Answers may include specific numbers, such as on the 20th day there were 20 pounds of waste recycled. Answers may also include exactly how much the recycling increased over 50 days. It started at around 8 pounds and on the 50th day, it was over 40 pounds. It would be interesting to have a conversation with students to discuss why both the text and the graph are important to understanding the story.
4. (On both versions, #4 on Babbage and #3 on Galileo & Falconer) What can be done to encourage people in your school to recycle more?
   Brainstorm such ideas as putting out bins, placing signs to encourage recycling, making announcements, and writing letters.
5. The Galileo & Falconer version (#4) asks students to extend the graph for 100 days instead of 50. Several responses are possible. Encourage students from the Galileo & Falconer groups to share and draw different responses. Ask students from all groups to agree or disagree with the given responses.
   - *Some students may believe the graph will continue to increase as more people become aware of the recycling program.*
   - *Some may expect the graph to level off because everybody is recycling as much as they can already.*
   - *Some students may expect a decrease because people forget and stop recycling.*

Conclude

3. Explain the intervention

Before explaining the intervention to the students, consider two options for completing this project in the Look Ahead section. Explain to students that in the next lesson they will be designing an experiment to try to increase recycling at their school. Encourage students to think about choosing one item that they would like to try to measure in their experiment.

Assess

4. Increasing and decreasing graphs

The *For a Good Cause: Collecting and Donating* Student Page **[SMJ page 7]** can be used to assess students' understanding of increasing and decreasing line graphs. The *Student Mathematicians Think Deeply* Student Page **[SMJ page 11]** provides additional challenge for students demonstrating mastery of concepts.

5. Possible student problems

Some students struggle to think of meaningful titles for the graphs. It is important to reinforce the idea that there is not just one right title for a graph. The purpose of a graph's title is to describe the graph and to give the readers an idea of what kind of information they can get from the graph. There are many correct answers. In some science classes, teachers may mandate that students create a title using this format: "y-axis vs. x-axis." This format, while perhaps too rigid for all graphs and purposes, may help students to think about what the purpose of each axis is. For example, in this practice, students may think about the number of toys and weeks as important parts of the graph that should be in the title. They also may want to think about who or what is being referred to in the graph.

Look Ahead

6. Planning authentic data collections

Engaging the students in meaningful data collecting activities allows them to understand why graphs are helpful and that information they are sharing is real and purposeful. In both the line graph section and bar graph section, students are asked to collect data. After both sections, students should understand the following: graphs represent real information, the question one wants to answer dictates what type of graph is used, and data can be represented in multiple ways to answer different questions.

There are two separate collection projects presented in this manual. Here in the line graph section, there is a recycling intervention, which asks students to create an intervention to increase the amount of recycling at their school (i.e., recycling cans, cardboard, or paper). In the bar graph section (see Lesson 7), students are asked to design a collection that reduces waste by reusing (i.e., a used clothes drive or a book exchange). If time is an issue, one of these projects can be used for both the line and bar graphs. If only one project is being implemented, however, it is important to understand how it can be used for both types of graphs.

For example, if students want to have a book drive, they can track the number of books that are donated on a daily basis, and they can implement an intervention to see if the number of books increases when they hang up posters. This information would best translate into a line graph because the goal of line graphs is to track data over time. The guiding question would be, "Does the number of books donated increase after the intervention is implemented?"

The same drive could also be used to create a bar graph. Bar graphs are created to show how many are in each category. Students could categorize the books in a number of ways: genre, types of series, copyright decade, or even reading level. The guiding question is, “What type of books are people donating?”

The line plot project is significantly different. It is a survey project, but if the class was interested in finding out how many books individual students in their class brought, a line plot would be the best type of graph to use. This could also be done anonymously by having the students record at the end of the drive how many books they brought in on a post-it, and then collecting all the post-its on the board.

Each graph that is covered in this unit answers a different question, but the data collection projects do not necessarily have to be different. Even if time is not an issue and both projects can be completed, it would be beneficial to have students explore the same data collection projects by creating each of the different graphs.

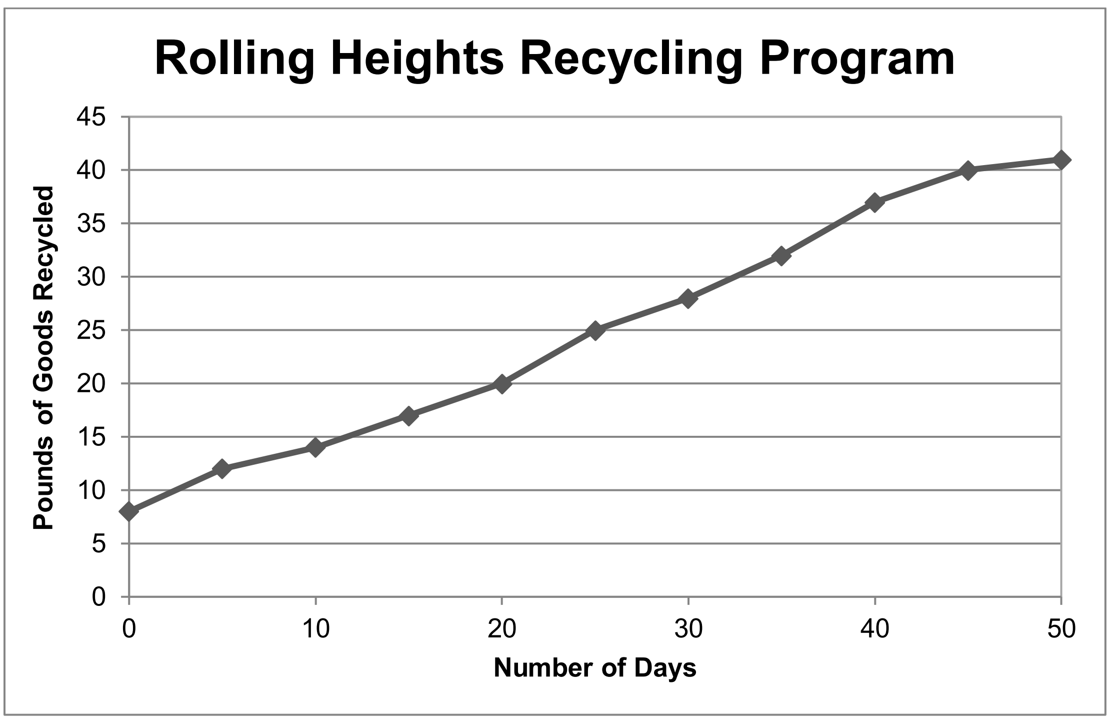
It is important to note that if the reducing by reusing project is chosen, some of the student pages may need to have the word “recycle” changed to “reuse.” Throughout the rest of the manual, the authors assume that you have time for separate projects, but teachers should feel free to exercise discretion in deciding on the projects that will best meet the needs of their classes.

Recycling in the News

Answering the Call for Recycling—The Cactus Chronicle

"Recycling is the right thing to do, and it isn't that hard," Tina Lee explained. Tina is a fourth grade student at Rolling Heights Elementary School in Southern California. She was so excited when her school began its recycling program because she knew it would help save the environment.

Students and staff thought it would be best to start in the classrooms. The school's goal was to recycle paper, aluminum, and plastic. At first, they were not sure how to keep the three items separate. Then, Tina thought of using colors to indicate which item went in which bin. Tina's friends helped by creating signs to tell other students which recyclable item went in which bin. A graph showing the change in recycling over 50 days is shown below.



By the end of the program most of the school's trash was being recycled, which meant the school was throwing out only a small amount of its waste. These students, teachers, and staff made a big difference in their school by starting the recycling program!

SMJ page 1

Discussion Questions

Babbage

1. Why do you think that some people do not recycle?

2. What did you learn from the article that the graph did not tell you?

3. What did you learn from the graph that the article did not tell you?

4. What can be done to encourage people in your school to recycle more?

SMJ page 2

Galileo & Falconer

Recycling in the News

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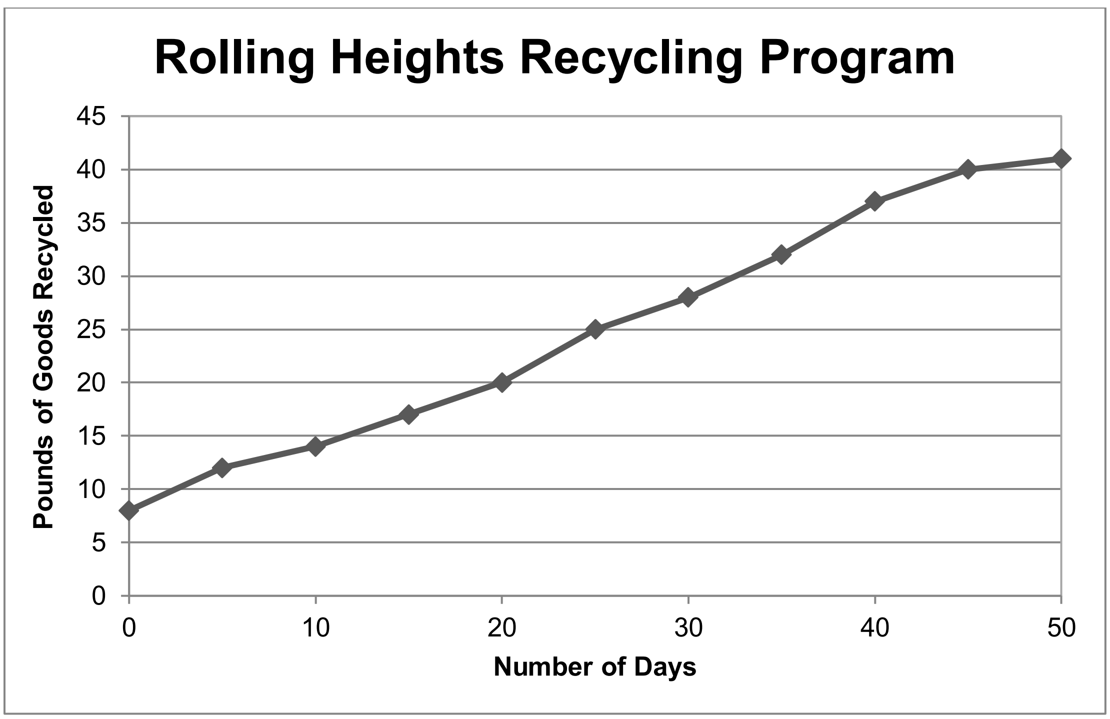


By the end of the program most of the school’s trash was being recycled, which meant the school was throwing out only a small amount of its waste. These students, teachers, and staff made a big difference in their school by starting the recycling program!

SMJ page 3

Discussion Questions

Galileo & Falconer

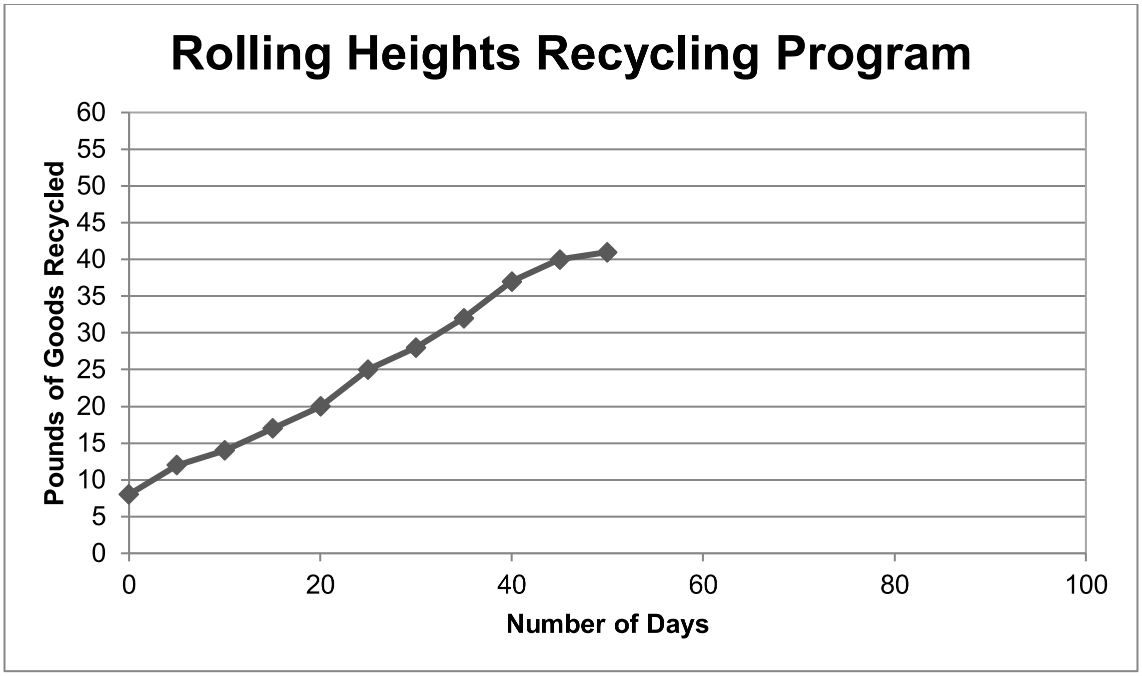
1. What did you learn from the article that the graph did not tell you?

2. What did you learn from the graph that the article did not tell you?

3. What can be done to encourage people in your school to recycle more?

SMJ page 4

4. What do you think the graph would look like if the recycling program goes on for 100 days? Draw it below.



Explain your thinking.

____

____

____

SMJ page 5

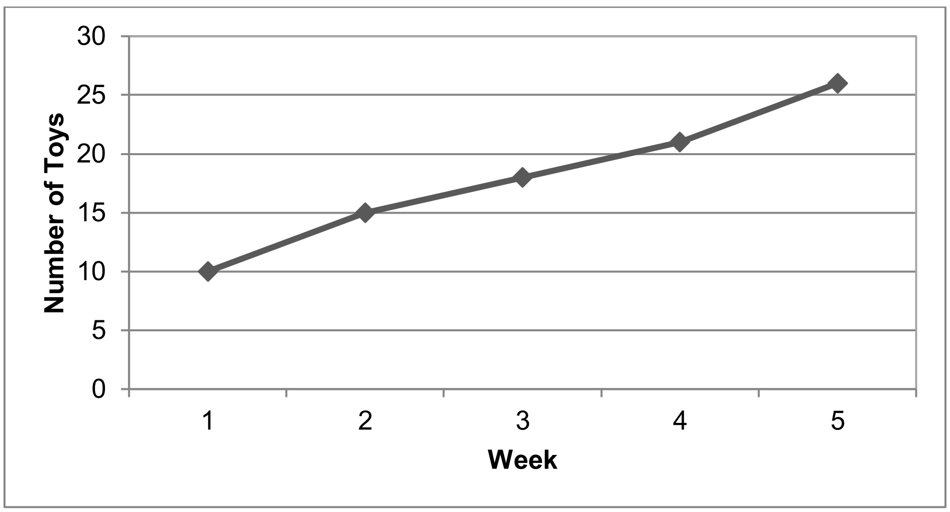
Name: ___________________________________ Date: __________________

For a Good Cause: Collecting and Donating

Read the two stories below and decide which graph goes with each.

1. Lydia has 28 toys. She decides that there are many that she no longer plays with and gives some to a different charity each week. After 5 weeks, she only has 8 toys left.

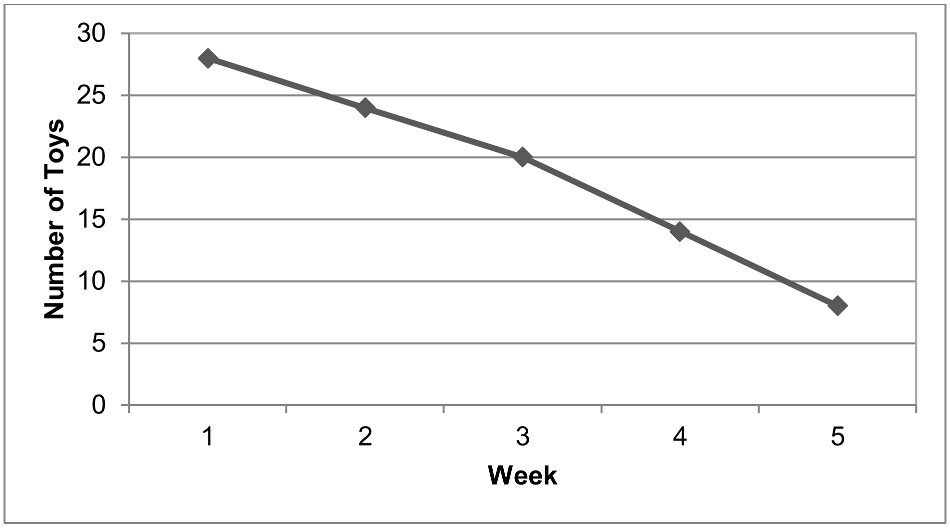
2. Antonio has 10 toys. He collects more toys from his friends each week. After 5 weeks, he has 26 toys.



Which story can be modeled by this graph? ______________________________

What would a good title for this graph be? ________________________________

SMJ page 7



Which story can be modeled by this graph? ______________________________

What would a good title for this graph be? ________________________________

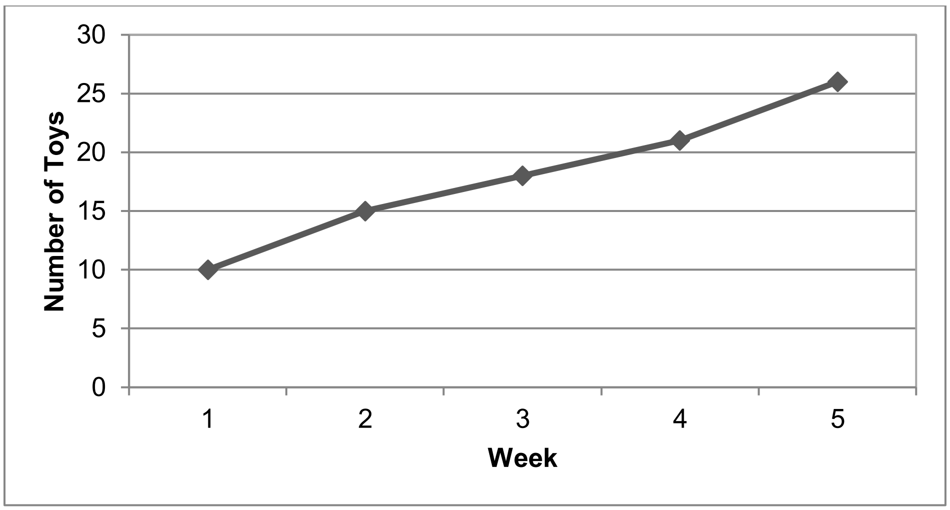
SMJ page 9

For a Good Cause: Collecting and Donating
ANSWER KEY

Read the two stories below and decide which graph goes with each.

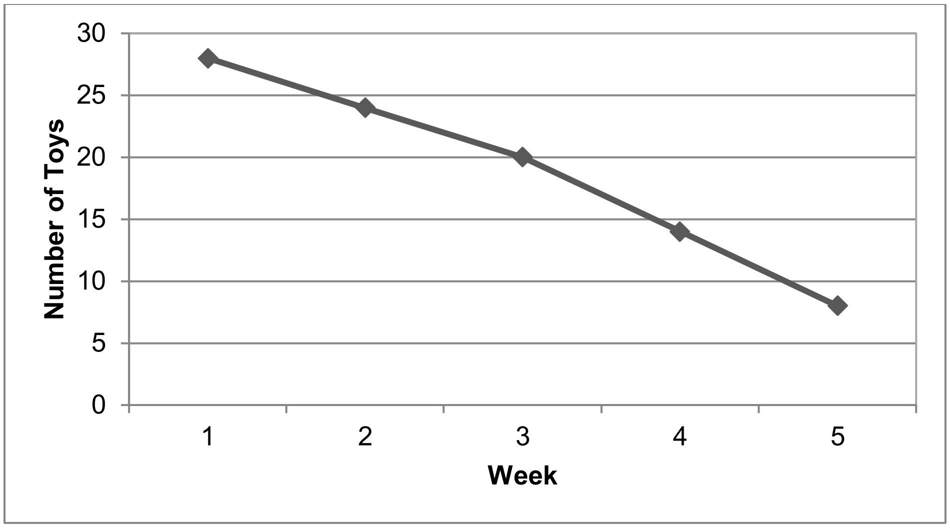
1. Lydia has 28 toys. She decides that there are many that she no longer plays with and gives some to a different charity each week. After 5 weeks, she only has 8 toys left.

2. Antonio has 10 toys. He collects more toys from his friends each week. After 5 weeks, he has 26 toys.



Which story can be modeled by this graph? *The first graph matches with story 2.*

What would a good title for this graph be? *Antonio's Toy Collection (answers may vary).*



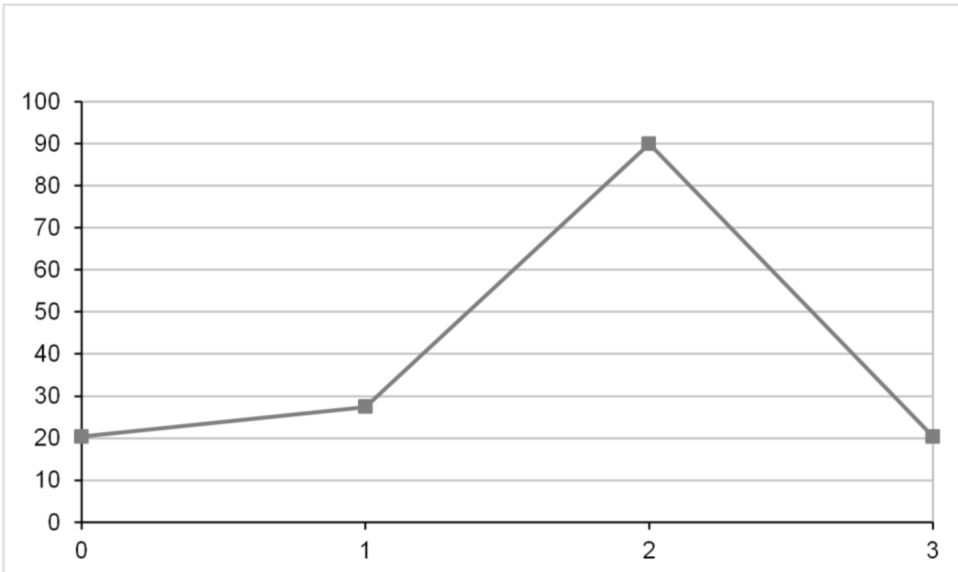
Which story can be modeled by this graph? *The second graph matches with story 1.*

What would a good title for this graph be? *Lydia's Toy Collection (answers may vary).*

Name: ______________________________ Date: ________________

Student Mathematicians Think Deeply

Make up a story that could be modeled by the graph below.



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What is a possible title for your graph? ______________________________

SMJ page 11

Student Mathematicians Think Deeply
ANSWER KEY

Sample Story:

Jerry was collecting worms for his worm farm. He started with 20 that he found in the yard. After another day of collecting, Jerry had 28 worms. He asked a bunch of his friends to help one day. After this day, Jerry had 90 worms. His mom looked pretty mad. She said, "Jerry, let some of those worms go!" Jerry released many and was back to only 20 worms.

NOTE: The number '28' may vary slightly if students choose to use numbers in their stories.

If students do not use exact numbers in their stories, look for stories to indicate a slow increase followed by a sharp increase and then a sharp decrease.

What is a possible title for your graph? *Sample Title: Jerry's Worm Collection*

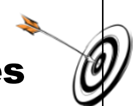
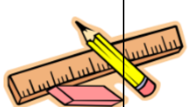
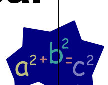
LINE GRAPHS (PART 1)—

PLANNING THE INTERVENTION

LESSON 2

Big Mathematical Ideas

Researchers must plan experiments containing interventions carefully. They should decide what it is they are measuring and how they will measure it before they collect data. They then create a question, a hypothesis, and a plan for the intervention before they start their experiment.

Lesson Objectives 	• Students will identify a recycled item that they will measure. • Students will decide how to measure the item. • Students will identify a question for their experiment. • Students will identify a hypothesis for their experiment. • Students will design an intervention for their experiment.
Materials 	• Student Page—*Recycling Rules! (Part 1)* [SMJ page 13] • Student Page—*State Capitals (Pre-Intervention)* [SMJ page 15] • Student Page—*State Capitals (Post-Intervention)* [SMJ page 17] • Student Page—*The Pirate Mart* (Optional) [SMJ page 19] • State Capitals List—[SMJ Appendix page 211]
Mathematical Language 	• **Question:** What you want to know in an experiment. • **Intervention:** The process of changing something to determine if you can affect the outcome. • **Hypothesis:** What you think will happen in an experiment, based on facts and your ideas.

Lesson Preview

Students discuss and decide on a recyclable item they want to measure. Next, they decide how to measure it and complete the *Recycling Rules! (Part 1)* Student Page **[SMJ page 13]** with the question and hypothesis for their experiment.

Initiate

1. Select an intervention idea

Begin by explaining the goal of the lesson. The goal is to increase recycling in our school by using an intervention. Define an **intervention** as an action taken to improve a situation. Refer back to the *Recycling in the News* **[SMJ page 1]**, and ask students to describe what intervention was used. Clarify by asking what students did to improve recycling at their school.

Remind students of the list they created of the items within the school that can be recycled. Create a table on the board with three columns. The first column should be titled, “Recyclable Items.” The second column should be titled, “Possible Interventions.” The third column should be titled “Ways of Measuring.” A sample table is below:

Recyclable Items	Possible Interventions	Ways of Measuring
Cans Paper Plastic	Posters Colored Bins Announcements	Weighing Counting Measuring

The next stage is to evaluate which item, intervention, and measurement technique should be used for the class’s project. Ask students the following questions:

- Which item from the list could we get people to recycle more?
- What kinds of things should we think about when deciding on an item?
  Answers may include how easy it would be to measure or collect data on the item, how important that item is to recycle, and how many of the items the school throws away. From this discussion, the class should choose the item for the class intervention.
- What is the best intervention for getting people to recycle that item? Why?
- How will we know if the intervention is working?
  Answers should include ideas such as measuring before, during, and after the intervention. Measuring before the experiment is important because it allows students to see how much the school was recycling before the intervention.
- What do you think is the best way to measure the change in recycling?

Answers will vary, but for paper, students might count or weigh it. This response depends on the item selected and the school's resources.

Investigate

2.

Form a research question

Direct students to the *Recycling Rules! (Part 1)* Student Page **[SMJ page 13]**. Explain to students that all good research begins with a question. Read the question on top of the sheet with students:

Can we increase how much our school recycles ___________

by ____?

Instruct students to write in the first blank the item that they have chosen to measure. Tell students that now it is time to plan an intervention.

- ✓ Check to ensure that students remember that an intervention is doing something to make a positive change.

Tell students that the intervention will be carried out in one or two rooms in the school. This is the teacher's choice and certainly may be expanded. However, one or two classrooms (or the office), if appropriate, would certainly be enough to collect data. The place that the intervention will take place is referred to as *the location.*

Instruct students to complete the question on the Student Page by writing the intervention on the second blank.

An example of a completed question statement might be:
Can we increase how much our school recycles ***sheets of paper*** by ***putting up signs in the hallways?***

Read the entire question out loud with the students to promote understanding.

3.

Write a hypothesis

Explain to students that when scientists and other researchers ask a question, they want to make an "educated guess" based on facts and their ideas about the answer to the question. Tell students that this is called a **hypothesis.** Write the following definition on the board for the students:

Hypothesis—What you think will happen after the intervention based on facts and your own ideas.

Encourage students to consider what their hypotheses are for their intervention. Discuss with students whether or not they think the intervention will increase recycling.

Once students have discussed their ideas, instruct them to write their personal hypotheses on the *Recycling Rules! (Part 1)* Student Page **[SMJ page 13]**. It is especially important that they also write why they believe their hypotheses to be true after the "because." Remind students that they should base their hypotheses on reason.

An example of a completed hypothesis statement might be:

Hypothesis: *I think we* ***(X)*** *CAN (   ) CANNOT increase how our school recycles this way BECAUSE* ***people will think about recycling more if we put up signs and write letters.***

OR

Hypothesis: *I think we (   ) CAN* ***(X)*** *CANNOT increase how our school recycles this way BECAUSE* ***even if we put up signs, people won't notice them because they might be thinking of other things*****.**

4. Practice gathering data

In this section, you will ask students to name as many state capitals as they can in one minute. The goal of the task is for students to understand that a baseline is the information collected before the intervention.

Ask students to open to the *State Capitals (Pre-Intervention)* Student Page **[SMJ page 15]**. Tell students that they will have one minute to write down as many state capitals as they can. After they complete this task, tell them that the intervention for this experiment is allowing them time to look at a list of state capitals.

Ask students to describe what might happen if they are given a chance to look at a list of state capitals for one minute (this is the hypothesis). Then give them one minute to look at the list located on *US State Capitals* Student Page **[SMJ Appendix page 211]**. Ask the students to then turn back to *State Capitals (Post-Intervention)* Student Page **[SMJ page 17]** and once again record all the state capitals they know without looking at the list again.

Allow the students to self-correct both Student Pages using the list in the appendix of their journals. Ask how many students were able to name more after looking at the list.

Conclude

5. **Review and preview**

Tell students that the state capital data will be used in the next lesson to learn about baselines and interventions. Review the purpose of research questions and hypotheses.

Look Ahead

6. **Preparation**

It is important to contact individuals in the location(s) where the intervention will be conducted to make necessary arrangements. The intervention will begin in Lesson 3.

If there is already a recycling bin at the location, it is important that the teacher or another adult empty the bin at least one day before the class visit so that one day of recycling can be measured. Baseline data must be collected before the intervention begins. If there is no recycling program currently, students' three baseline data points will all be zero.

Assess

7. **Lesson conclusions and optional student practice**

Ask students to reflect upon their hypotheses for the recycling intervention. Discuss how many students believe the intervention will be successful. Ask them to provide an explanation for their thinking.

The Pirate Mart Student Page **[SMJ page 19]** can be used for additional student practice with line graphs. Teachers may choose to send it home as homework, use it as an optional activity when students finish their work, or assign it as morning work. An answer key can be found in the teacher's manual on page 71.

Name: ________________________________ Date: ________________

Recycling Rules! (Part 1)

OUR QUESTION:

Can we increase how our school recycles ________________
(item name)

by ____
(intervention chosen)

____?

MY HYPOTHESIS:

I think we (   ) CAN (   ) CANNOT increase how much our school recycles this way BECAUSE

____

____.

SMJ page 13

State Capitals (Pre-Intervention)

State	Capital	State	Capital
1. Alabama		26. Montana	
2. Alaska		27. Nebraska	
3. Arizona		28. Nevada	
4. Arkansas		29. New Hampshire	
5. California		30. New Jersey	
6. Colorado		31. New Mexico	
7. Connecticut		32. New York	
8. Delaware		33. North Carolina	
9. Florida		34. North Dakota	
10. Georgia		35. Ohio	
11. Hawaii		36. Oklahoma	
12. Idaho		37. Oregon	
13. Illinois		38. Pennsylvania	
14. Indiana		39. Rhode Island	
15. Iowa		40. South Carolina	
16. Kansas		41. South Dakota	
17. Kentucky		42. Tennessee	
18. Louisiana		43. Texas	
19. Maine		44. Utah	
20. Maryland		45. Vermont	
21. Massachusetts		46. Virginia	
22. Michigan		47. Washington	
23. Minnesota		48. West Virginia	
24. Mississippi		49. Wisconsin	
25. Missouri		50. Wyoming	

Number of state capitals I knew before the intervention: ____________________

SMJ page 15

State Capitals (Pre-Intervention)
SAMPLE STUDENT WORK

State	Capital	State	Capital
1. Alabama		26. Montana	
2. Alaska	*Juneau*	27. Nebraska	
3. Arizona		28. Nevada	
4. Arkansas		29. New Hampshire	
5. California		30. New Jersey	
6. Colorado		31. New Mexico	
7. Connecticut		32. New York	
8. Delaware	*Dover*	33. North Carolina	
9. Florida		34. North Dakota	
10. Georgia		35. Ohio	
11. Hawaii		36. Oklahoma	
12. Idaho		37. Oregon	
13. Illinois		38. Pennsylvania	*Harrisburg*
14. Indiana	*Indianapolis*	39. Rhode Island	
15. Iowa		40. South Carolina	
16. Kansas		41. South Dakota	
17. Kentucky		42. Tennessee	
18. Louisiana		43. Texas	
19. Maine		44. Utah	
20. Maryland		45. Vermont	
21. Massachusetts		46. Virginia	
22. Michigan		47. Washington	
23. Minnesota		48. West Virginia	
24. Mississippi		49. Wisconsin	
25. Missouri		50. Wyoming	

Number of state capitals I knew before the intervention: 4

State Capitals (Post-Intervention)

State	Capital	State	Capital
1. Alabama		26. Montana	
2. Alaska		27. Nebraska	
3. Arizona		28. Nevada	
4. Arkansas		29. New Hampshire	
5. California		30. New Jersey	
6. Colorado		31. New Mexico	
7. Connecticut		32. New York	
8. Delaware		33. North Carolina	
9. Florida		34. North Dakota	
10. Georgia		35. Ohio	
11. Hawaii		36. Oklahoma	
12. Idaho		37. Oregon	
13. Illinois		38. Pennsylvania	
14. Indiana		39. Rhode Island	
15. Iowa		40. South Carolina	
16. Kansas		41. South Dakota	
17. Kentucky		42. Tennessee	
18. Louisiana		43. Texas	
19. Maine		44. Utah	
20. Maryland		45. Vermont	
21. Massachusetts		46. Virginia	
22. Michigan		47. Washington	
23. Minnesota		48. West Virginia	
24. Mississippi		49. Wisconsin	
25. Missouri		50. Wyoming	

Number of state capitals I knew after the intervention: ______________________

SMJ page 17

State Capitals (Post-Intervention)
SAMPLE STUDENT WORK

State	Capital	State	Capital
1. Alabama		26. Montana	
2. Alaska	*Juneau*	27. Nebraska	
3. Arizona	*Phoenix*	28. Nevada	
4. Arkansas	*Little Rock*	29. New Hampshire	
5. California		30. New Jersey	
6. Colorado	*Denver*	31. New Mexico	
7. Connecticut	*Hartford*	32. New York	
8. Delaware	*Dover*	33. North Carolina	
9. Florida		34. North Dakota	
10. Georgia	*Atlanta*	35. Ohio	
11. Hawaii		36. Oklahoma	
12. Idaho		37. Oregon	
13. Illinois		38. Pennsylvania	*Harrisburg*
14. Indiana	*Indianapolis*	39. Rhode Island	
15. Iowa		40. South Carolina	
16. Kansas		41. South Dakota	
17. Kentucky		42. Tennessee	
18. Louisiana		43. Texas	
19. Maine		44. Utah	
20. Maryland		45. Vermont	
21. Massachusetts		46. Virginia	
22. Michigan		47. Washington	
23. Minnesota		48. West Virginia	
24. Mississippi		49. Wisconsin	
25. Missouri		50. Wyoming	

Number of state capitals I knew after the intervention: 9

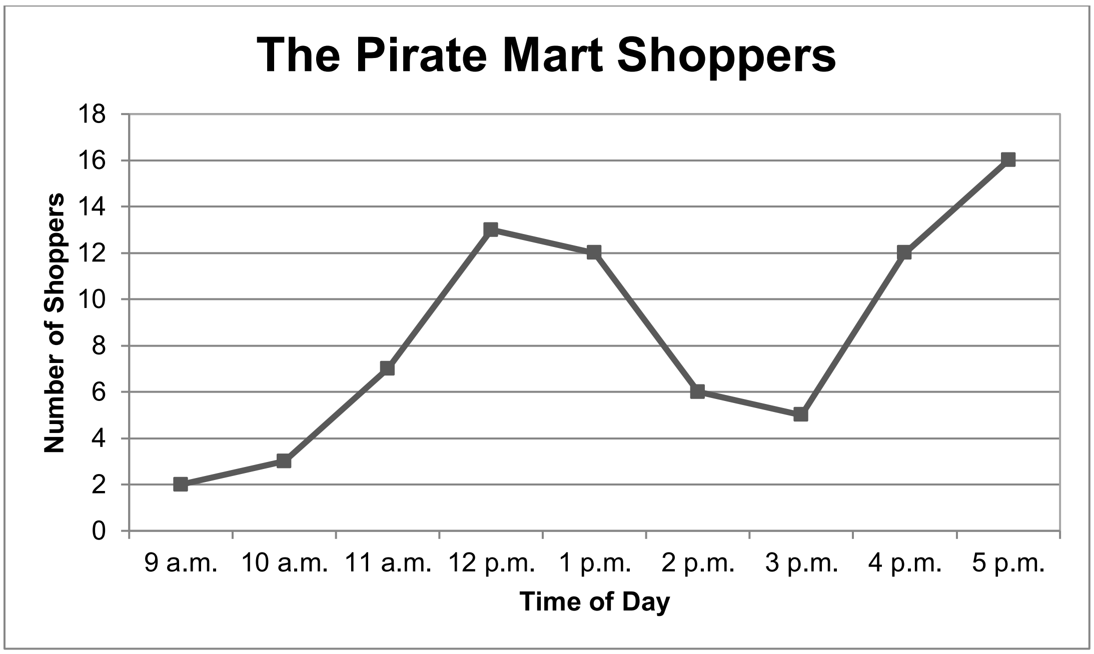
States and Their Capitals
ANSWER KEY

State	Capital		State	Capital
1. Alabama	*Montgomery*		26. Montana	*Helena*
2. Alaska	*Juneau*		27. Nebraska	*Lincoln*
3. Arizona	*Phoenix*		28. Nevada	*Carson City*
4. Arkansas	*Little Rock*		29. New Hampshire	*Concord*
5. California	*Sacramento*		30. New Jersey	*Trenton*
6. Colorado	*Denver*		31. New Mexico	*Santa Fe*
7. Connecticut	*Hartford*		32. New York	*Albany*
8. Delaware	*Dover*		33. North Carolina	*Raleigh*
9. Florida	*Tallahassee*		34. North Dakota	*Bismarck*
10. Georgia	*Atlanta*		35. Ohio	*Columbus*
11. Hawaii	*Honolulu*		36. Oklahoma	*Oklahoma City*
12. Idaho	*Boise*		37. Oregon	*Salem*
13. Illinois	*Springfield*		38. Pennsylvania	*Harrisburg*
14. Indiana	*Indianapolis*		39. Rhode Island	*Providence*
15. Iowa	*Des Moines*		40. South Carolina	*Columbia*
16. Kansas	*Topeka*		41. South Dakota	*Pierre*
17. Kentucky	*Frankfort*		42. Tennessee	*Nashville*
18. Louisiana	*Baton Rouge*		43. Texas	*Austin*
19. Maine	*Augusta*		44. Utah	*Salt Lake City*
20. Maryland	*Annapolis*		45. Vermont	*Montpelier*
21. Massachusetts	*Boston*		46. Virginia	*Richmond*
22. Michigan	*Lansing*		47. Washington	*Olympia*
23. Minnesota	*St. Paul*		48. West Virginia	*Charleston*
24. Mississippi	*Jackson*		49. Wisconsin	*Madison*
25. Missouri	*Jefferson City*		50. Wyoming	*Cheyenne*

Name: ________________________________ Date: _________________

The Pirate Mart (Optional)

Jack opened a store that sold pirate clothes, parrots, and treasure chests. He wanted to see when his customers came to the store, so he created this graph.



1. At what time are the most people shopping at Pirates Mart?

____

2. When are the fewest people shopping?

____

SMJ page 19

3. What happens around lunchtime? Why?

____

____

4. Jack is thinking about changing when the store's hours. Do you recommend opening earlier or staying open later? Why? (Use the graph to support your suggestion.)

____

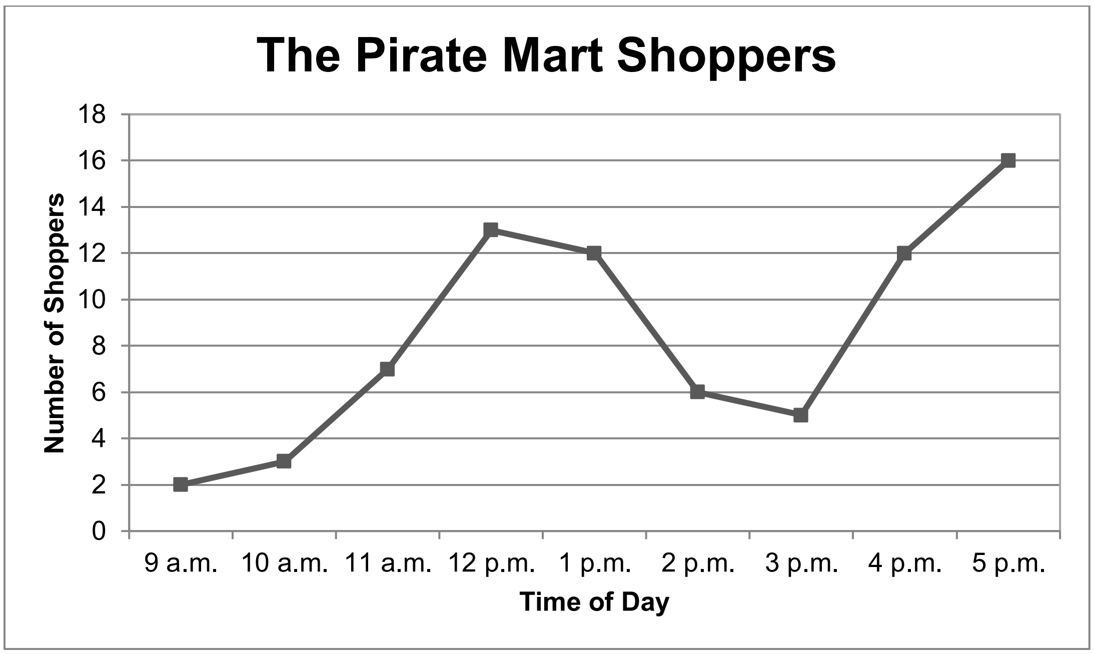
____

____

SMJ page 21

The Pirate Mart (Optional)
ANSWER KEY

Jack opened a store that sold pirate clothes, parrots, and treasure chests. He wanted to see when his customers came to the store, so he created this graph.



1. At what time are the most people shopping at Pirates Mart?

   Most people are shopping at 5 p.m.

2. When are the fewest people shopping?

   The fewest people are shopping around 9 a.m.

3. What happens around lunchtime? Why?

   Answers will vary. More people seem to visit the store. This could be because they have a lunch break, or they went out for lunch and saw the store.

4. Jack is thinking about changing when the store's hours. Do you recommend opening earlier or staying open later? Why? (Use the graph to support your suggestion.)

   Answers will vary. I would recommend staying open later because more people tend to visit the store at 5 p.m. than at 9 a.m.

LINE GRAPHS (PART 1)—

IMPLEMENTING THE INTERVENTION

LESSON 3

Big Mathematical Ideas

Real research takes time. Once the experiment has been planned, researchers need time to prepare for the intervention. Researchers collect pre-intervention data, conduct the intervention, and then collect post-intervention data. Researchers need to collect data over multiple time points to get a sense of the trends in the data.

Lesson Objectives	• Students will collect and compile data. • Students will define and understand why baselines are necessary.
Materials	• Student Page—*Graph It!* [SMJ page 23] • Student Page—*Recycling Rules! (Part 2)* [SMJ page 25] • Check Up #1 [SMJ page 27] • Data from SMJ pages 15 and 17
Mathematical Language	• **Data:** Information such as numbers that researchers gather during an experiment. • **Data Collection:** The process of collecting information and writing it down. • **Data Table:** A way for researchers to organize their data. • **Column:** The vertical, or up and down, display on a graph that represents numbers. • **Baseline:** The data collected before the intervention.

Lesson Preview

Students plan for and implement their intervention. This lesson takes one to three days depending on how the teacher decides to organize the intervention. Data collection continues for several days while other content is introduced.

Initiate

1. Analysis of the state capital data

Ask students to refer to the data they collected in the previous lesson.

- How many of you were able to increase the number of state capitals you were able to list after the intervention (studying the list for one minute)?
- How many did not increase the number of state capitals after looking at the list?
- Why do you think you were or were not able to increase the number of state capitals recalled?

Then have the students turn to the *Graph It!* Student Page **[SMJ page 23]**. Ask students to record in the table their pre- and post-intervention data for the number of states they were able to list. Then have students graph their data values as points. Model a sample graph on the board showing students how to place the data point for baseline above the word “Baseline” on their graphs. For example, if a student has a baseline of 2, his or her point should be directly above the word “Baseline” at 2. Ask them to connect their points. Analyze the line by asking:

- How can you tell by looking at your line whether or not the intervention made a difference?
    - *If the line increases or decreases, then the intervention made a difference. A horizontal line indicates no difference.*
- Tell students that a **baseline** is the data collected before an intervention. Ask students which point represents their baseline.
    - *The baseline is the number of state capitals students listed before studying the list.*
- Why is it important to collect baseline data?
    - *Baseline data allows researchers to see whether or not the intervention made a difference. For example, we needed to know how many state capitals students knew before studying the list to see if studying made a difference.*
- How can you tell whether the number increased from pre to post-intervention? Decreased? Remained the same?
    - *If the number increased, the graph will go upward. If the number decreased, the graph will go downward. If the number stayed the same, the graph will be flat (horizontal).*

Use visual examples to demonstrate what increasing, decreasing, and no change graphs look like. Provide additional challenge by looking at the steepness of different graphs. For example, a student whose number of capitals increased by three from pre- to post-intervention would have a steeper graph than a student who increased by only one.

Investigate

2. **Understanding data collection**

Write the word **data** on the board or a display. Ask students if they know what it means. If students need a hint, tell them they will be collecting data during their intervention. Guide the discussion until students understand that data is information that researchers gather during an experiment, and that it is often (but not always) numbers. Ask:

- What type of data are we gathering in our research?
  Numerical data
- What do the numbers represent?
  They represent the amount that people are recycling.
- How are we collecting our data?
  Sample response: We are going to classrooms and counting each piece of paper in the recycling bin.

Direct students to the *Recycling Rules! (Part 2)* Student Page **[SMJ page 25]**. Tell students that when researchers collect data, they often place it in a **data table.**

Review with students the word **column**, and make sure that they understand that a column is a vertical, or up and down, display that represents numbers. Review the columns on the worksheet with the students. Ask:

- How many columns are there in the data table?
  2
- What are those columns for?
  The first column shows the day we are collecting data. The second column is where we will write how many of the item we find in the recycling bin.

3. **Collecting our baseline data**

Remind students of their own intervention that they will be using to try to increase the amount of recycling in the school. Tell students where they will be collecting the data for their project and ask if anybody remembers how they will be measuring the data. Tell them they will go to the site three times to collect data *before* the intervention (e.g., hanging posters, making announcements, or reading letters). Ask if they remember the word for pre-intervention data collection (baseline). (NOTE: If the school had NO recycling program prior to the intervention, use zero for the three baseline points.)

The purpose of each location visit is to count the number of recycled items in the bin and record it in the *Number of Items* column on the Data Table (if the class has opted to use counting as its measurement).

Visit the location with students to collect the first baseline data value. Students should record the number of recycled items next to “Day 1” of the data table on their *Recycling Rules! (Part 2)* Student Page **[SMJ page 25]**.

4. Implementing the intervention

Provide in class time for groups to work on completing the materials and/or tasks required for the interventions or use language arts or science time to create posters or pamphlets to increase recycling. Students may also work on these tasks at home. The details of creating materials for and carrying out the intervention should be decided based on classroom schedule and needs. The most important thing to remember is that the intervention should begin following the baseline data collection. For example, if students have decided to write letters to read to people at their collection sites, these readings should be done *after* the baseline data is collected.

Conclude

5. Reinforce mathematical language

Reinforce key vocabulary terms by asking questions throughout the intervention process. Examples include:

- How is the intervention helping us to answer our research question?
- What hypothesis are we testing?
- Can someone report the data we have collected up to this point for students who may have been absent?
- Have we seen an increase from the baseline?

Look Ahead

6. Organizing the intervention

The first three data points will be baseline data. The next five to seven data points are to be collected during the intervention. Try to get at least eight total data points including the three baseline points.

There are many organizational methods for collecting data. One option would be to send several students each day to collect and tabulate the data before school starts. Students could take turns in this role then report the data to the rest of the class. Another option is to take the whole class to gather the data right after lunch. Decide ahead of time who will collect the data and when that will be accomplished.

Since data collection will take several days, students will explore bar graphs and the concept of reducing (the amount of waste that is produced) prior to returning to line graphs. This allows new content to be introduced while the data is being collected.

Assess

7.

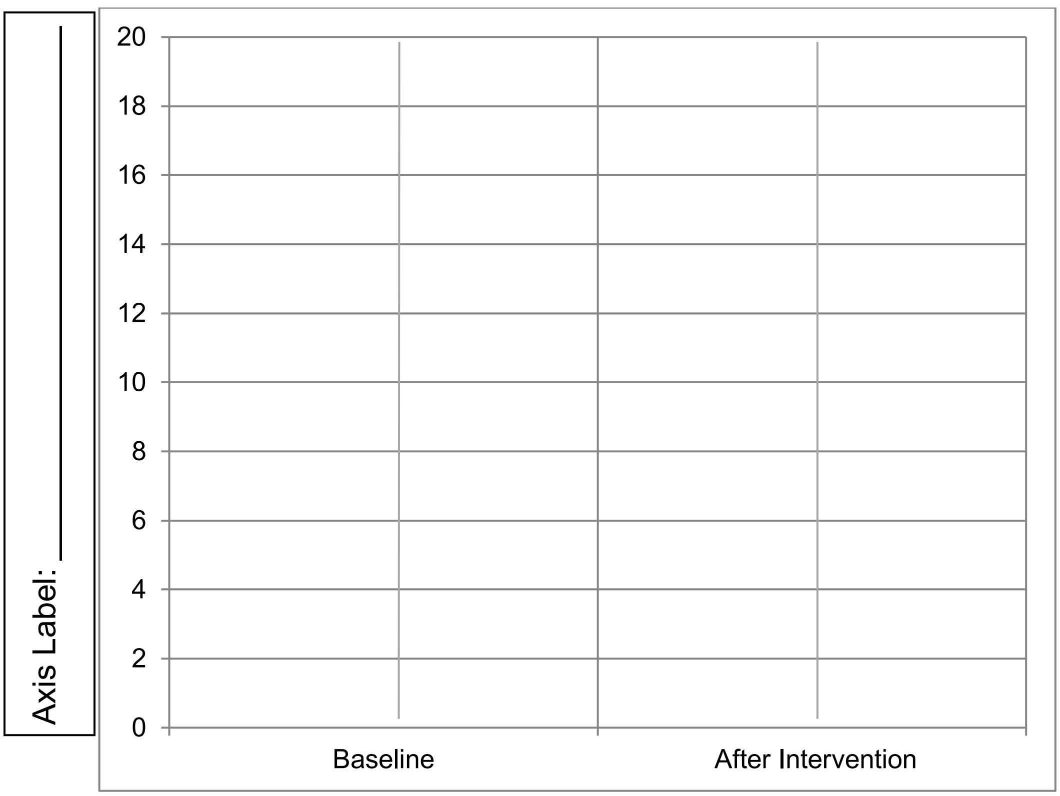
Check Up #1

Check Up #1 **[SMJ page 27]** can be used to assess students' understanding of mathematical ideas in the first three lessons. If this Check Up is done in class, students will need a coin to flip or the teacher can flip a coin to obtain data for the whole group!

Name: ______________________________ Date: _______________

Graph It!

Title: __________________________________



SMJ page 23

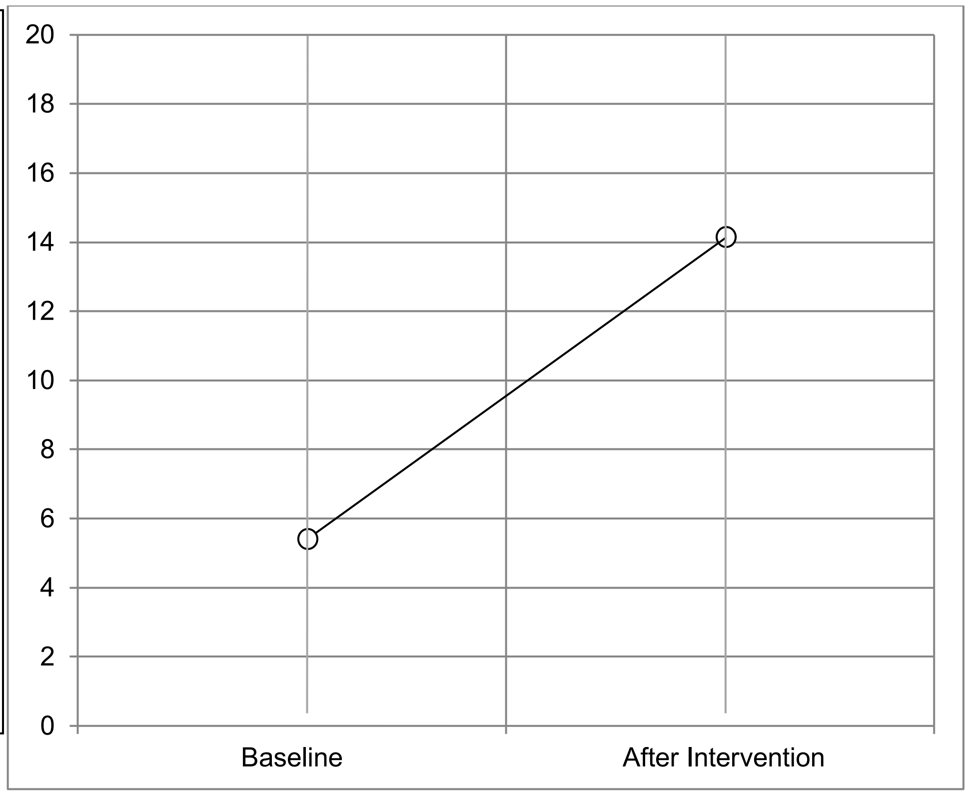
Graph It!

ANSWER KEY

(Actual numbers will vary.)

Title: *My Knowledge of State Capitals*

Axis Label: *Number of State Capitals Known*



Name: ______________________________ Date: ________________

Recycling Rules! (Part 2)

My Data Table

	Collection Total
Day 1 (Baseline)	
Day 2 (Baseline)	
Day 3 (Baseline)	
Day 4	
Day 5	
Day 6	
Day 7	
Day 8	

SMJ page 25

Recycling Rules! (Part 2)
ANSWER KEY
(Actual numbers will vary.)

My Data Table

	Collection Total
Day 1 (Baseline)	*(Students may record items with tallies or simply record the total numbers.)* *14*
Day 2 (Baseline)	*18*
Day 3 (Baseline)	*15*
Day 4	*32*
Day 5	*54*
Day 6	*39*
Day 7	*56*
Day 8	*67*

Check Up #1

Name: ___________________________________ Date: __________________

1. Jenna and Josh sold 4 cups of lemonade the first day of their lemonade stand. They wanted to sell more lemonade. What intervention could Josh and Jenna use to increase their lemonade sales?

____

____

____

____

2. What do you think will happen if Josh and Jenna use the intervention? Write your hypothesis below.

____

____

____

____

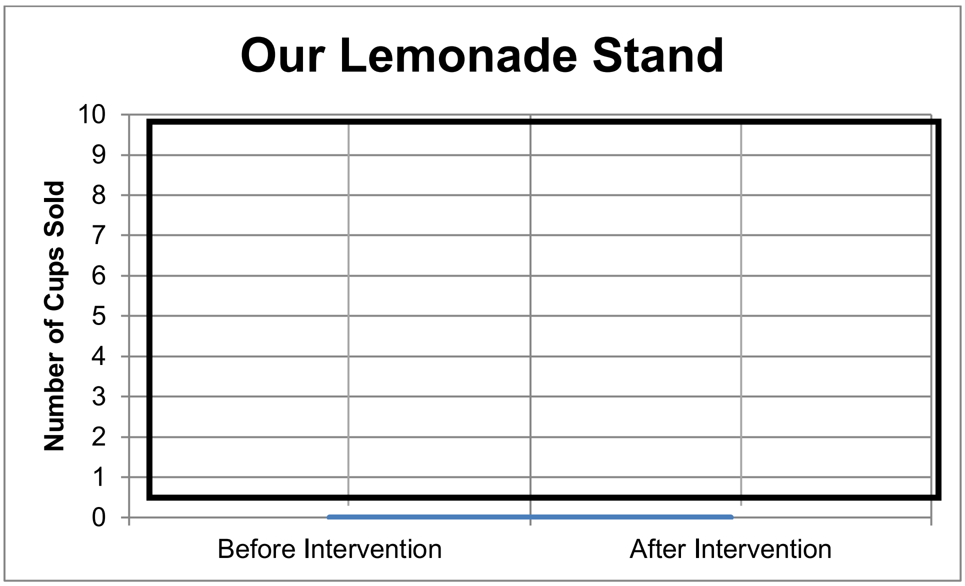
SMJ page 27

3. Flip a coin 10 times. Each time the coin lands on heads, a cup of lemonade is sold. Count the number of heads and record this number below for "Cups of Lemonade Sold After the Intervention."

   Cups of Lemonade Sold Before the Intervention: ____4____

   Cups of Lemonade Sold After the Intervention: ___________

4. Make two points on the graph to show the number of cups before and after the intervention. Connect the points with a line.



5. Did the intervention work? How do you know?

____

____

____

____

SMJ page 29

Check Up #1
ANSWER KEY

1. Jenna and Josh sold 4 cups of lemonade the first day of their lemonade stand. They wanted to sell more lemonade. What intervention could Josh and Jenna use to increase their lemonade sales?

   Answers will vary. Students could suggest that they make posters and hang them around the neighborhood or create a radio advertisement or send emails to all their friends inviting them to come to the lemonade stand.

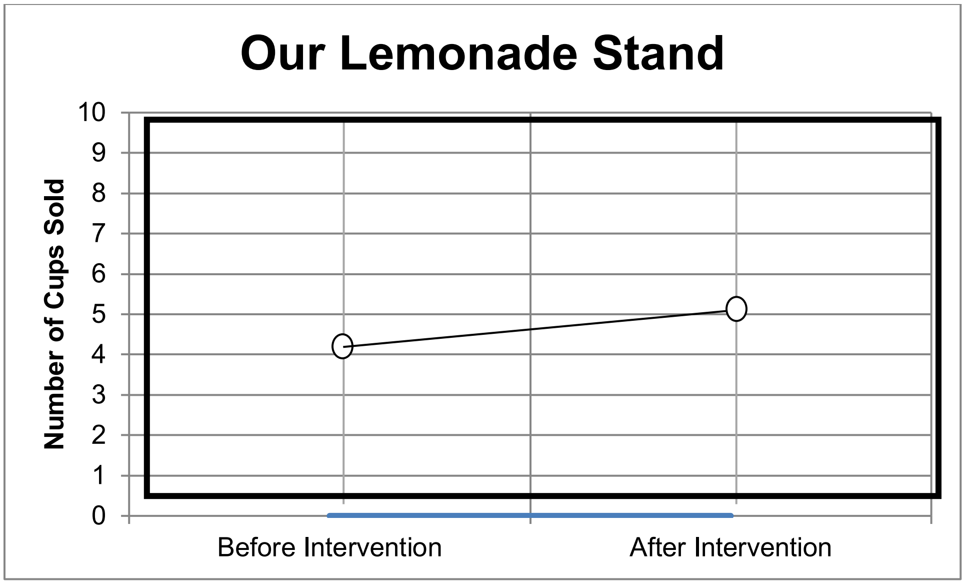
2. What do you think will happen if Josh and Jenna use the intervention? Write your hypothesis below.

   Answers will vary. Students need to make a prediction whether the intervention will work. Students may say the intervention will help people to hear about the lemonade stand and will help them to sell more lemonade.

3. Flip a coin 10 times. Each time the coin lands on heads, a cup of lemonade is sold. Count the number of heads and record this number below for "Cups of Lemonade Sold After the Intervention."

   Cups of Lemonade Sold Before the Intervention: 4
   Cups of Lemonade Sold After the Intervention: *5 (will vary)*

4. Make two points on the graph to show the number of cups before and after the intervention. Connect the points with a line.



5. Did the intervention work? How do you know?

   Answers will vary. If the number of cups sold increases, students should conclude that the intervention made a positive difference. They may also think about other variables that may have changed like the weather or the lemonade recipe. If the number of cups decreases, students should conclude the intervention may have negatively affected the sales.

LESSON 4

BAR GRAPHS—

WHOLE LOT OF SORTING GOING ON

Big Mathematical Ideas

Sorting groups of objects according to similarities or differences is an important skill that spans different disciplines. We sort animals and plants in science, people in social studies, and books in language arts. In mathematics, we often sort shapes and numbers. This skill provides the basis for future study of categorizing and displaying non-numerical data.

Lesson Objectives	• Students will be able to sort foam shapes or any other collection of items that can be sorted in multiple ways. (Ex.: blocks, buttons, or small items students bring from home)
Materials	• Student Page—*Put Me With My Four Legged Friends! (Babbage and Galileo & Falconer)* [SMJ pages 31-35 & 37-41] • Foam shapes • Scissors, glue or tape
Mathematical Language	• **Sort:** To arrange objects by something they have in common. ➢ Use an example from science to explain the idea of sorting (often called classifying). Scientists might “sort” animals as mammals or non-mammals. Which animals might be in the mammal group? The non-mammal group? Why?

Lesson Preview

Students discover different methods for sorting the same bag of shapes.

Initiate

1. Sorting by shape

Arrange students in groups of varying sizes with two, three, or four students per group. Give each group one bag of foam shapes. Ask students to sort the objects according to shape. Avoid the temptation to help students with this task. Rather, act as an observer collecting information to bring up in a later discussion. Once a group is observed completing the task correctly, call the class over to point out what that group has done well.

For groups that finish the task early, differentiate by asking them to find another way to sort the shapes (e.g., they may say “color”). Allow students to generate the new idea for sorting rather than giving them an idea. They may come up with something creative and new (e.g., sorting by round or straight edges).

Investigate

2. Sorting contest

In their groups, have students decide a method for sorting their shapes. Tell them they do not need to use every shape in the bag. Once they have finished, students gather around each group and try to guess how the group has sorted its shapes. Teachers should do their own sort while the students are working. Examples of possible ways of sorting are given below.

- Sort by number of letters in the name of the shape
- Sort into living and non-living groups

Conclude

3. Beyond shapes

Make a list of all of the different ways that students have used to sort their shapes. Discuss how the same shape can be sorted in many different ways. Relate this to the classification of an individual. You can sort students by gender, age, grade, hair color, or number of siblings. As an extension, you may want to have students sort themselves using a particular classification method.

It is important to note that sorting helps us to answer questions and the way we sort depends upon which questions we want to answer. For example, if the cafeteria worker wanted to know how many lunches to make, he would sort the students by who brought their lunch and who did not. He would not sort by eye color.

Save the data from any sorting activities done in class. These data can be used later in the unit as homework or in class work for students to practice making bar graphs.

Assess

4. How many legs

Ask students to complete the sorting activity on the *Put Me With My Four-Legged Friends!* Student Pages **[SMJ pages 31-35 & 37-41]**. Follow-up questions for this activity can be found on both Babbage and Galileo & Falconer versions. This differentiation is not tied to the Unit Pretest, so use your knowledge of the students to appropriately place them.

Name: ______________________________ Date: ________________

Put Me With My Four-Legged Friends!

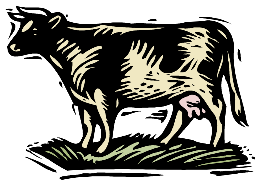
Cut out the animals on the next page. Glue or tape each animal into the table by category.

No Legs	Two Legs	Four Legs	More than Four Legs

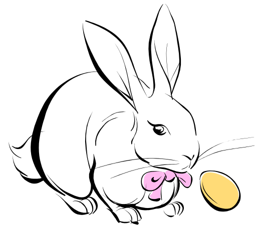
SMJ page 31

Animals to Sort

Babbage

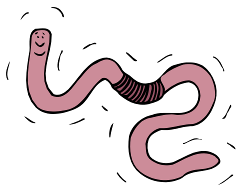


COW



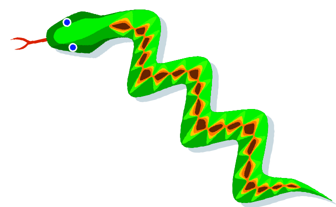
RABBIT

TIGER

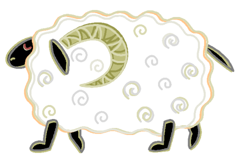


WORM

CHICKEN



SNAKE



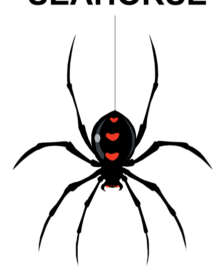
SHEEP

DONKEY

SEAHORSE

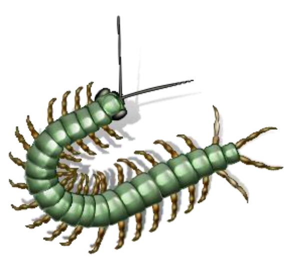
FISH

DOVE



SPIDER

OWL



CENTIPEDE

KANGAROO

SMJ page 33

Questions

Babbage

1. List a different animal that could fit in the "Four Legs" category.

____

2. Sort the same animals into the categories below. Write the name of the animal under the category that describes it.

Animals that Walk	Animals that Slither	Animals that Swim

SMJ page 35

Babbage

Put Me With My Four-Legged Friends!
ANSWER KEY

Cut out the animals on the next page. Glue or tape each animal into the table by category.

No Legs	Two Legs	Four Legs	More than Four Legs
Sea Horse *Worm* *Snake* *Fish*	*Chicken* *Dove* *Owl* *Kangaroo*	*Cow* *Rabbit* *Tiger* *Sheep* *Donkey*	*Spider* *Centipede*

Questions

1. List a different animal that could fit in the “Four Legs” category.

   Answers will vary. One possible answer may be a dog.

2. Sort the same animals into the categories below. Write the name of the animal under the category that describes it.

Animals that Walk	Animals that Slither	Animals that Swim
Cow *Rabbit* *Tiger* *Chicken* *Sheep* *Donkey* *Dove* *Spider* *Owl* *Centipede* *Kangaroo*	*Worm* *Snake*	*Sea Horse* *Fish*

Name: ___________________________________ Date: __________________

Put Me With My Four-Legged Friends!

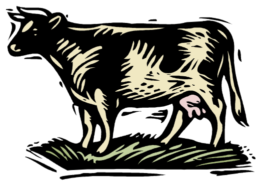
Cut out the animals on the next page. Glue or tape each animal into the table by category.

No Legs	Two Legs	Four Legs	More than Four Legs

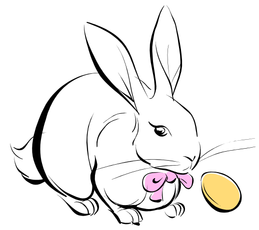
SMJ page 37

Animals to Sort

Galileo & Falconer

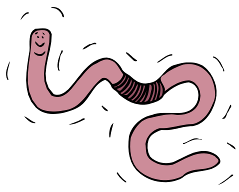


COW



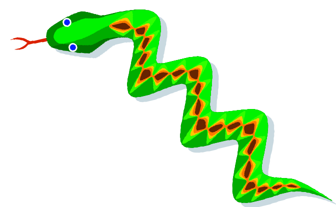
RABBIT

TIGER

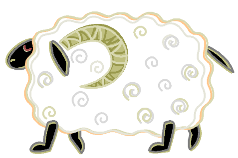


WORM

CHICKEN



SNAKE



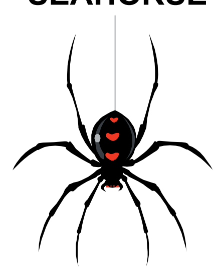
SHEEP

DONKEY

SEAHORSE

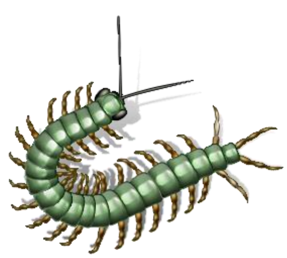
FISH

DOVE



SPIDER

OWL



CENTIPEDE

KANGAROO

SMJ page 39

Questions

Galileo & Falconer

1. List one other animal that could fit in the “Four Legs” category.

2. What is another way you could sort the animals besides by the number of legs?

   If you sort the animals the way you described, how many different categories would you need? Explain your thinking.

   In your proposed categories, where would you place a zebra?

   What about a dolphin?

3. Explain how animals might be sorted in a zoo.

SMJ page 41

Put Me With My Four-Legged Friends!
ANSWER KEY

Cut out the animals on the next page. Glue or tape each animal into the table by category.

No Legs	Two Legs	Four Legs	More than Four Legs
Sea Horse *Worm* *Snake* *Fish*	*Chicken* *Dove* *Owl* *Kangaroo*	*Cow* *Rabbit* *Tiger* *Sheep* *Donkey*	*Spider* *Centipede*

Questions

1. List one other animal that could fit in the “Four Legs” category.

   Answers will vary. One possible answer may be a dog.

2. What is another way you could sort the animals besides by the number of legs?

   Answers will vary. You could sort based on the climate they prefer, what types of food they eat, or even how they travel.

   If you sort the animals the way you described, how many different categories would you need? Explain your thinking.

   Answers will vary. For example, using the types of food the animals eat, you may use three categories: plants, animals, and a combination of both.

   In your proposed categories, where would you place a zebra?

   Answers will vary. If the students used what animals eat to sort, a zebra would be in the grass-eating category or a herbivore category.

What about a dolphin?

Answers will vary. If the students used what animals eat to sort, a dolphin would be in the fish-eating category or a carnivore category.

3. Explain how animals might be sorted in a zoo.

   Answers will vary. Animals may be sorted by continent they typically call home.

LESSON 5

BAR GRAPHS—

FAIR SHARE AND HOW MANY TO SPARE

Big Mathematical Ideas

Division is often used to break objects into equal groups. It is essential to understand the concept of remainder in division as an option for leftover objects that would be inappropriate to divide into fractional pieces.

Lesson Objectives	• Students will be able to “fair share” foam shapes. • Students will be able to explain why remainders occur.
Materials	• Student Page—*Fair Share Data* [SMJ pages 43-45] • Student Page—*Fair Sharing Frogs* [SMJ page 47] • Student Page—*Fair Share Lunches (Babbage & Galileo and Falconer)* (Optional) [SMJ pages 49 & 53] • Foam shapes
Mathematical Language	• **Fair Share:** Distributing a group of objects so that each person receives an equal number.

Lesson Preview

Students use foam shapes to learn methods for fair sharing, or distributing a group of objects equally among a number of people. Students learn that remainders occur in fair sharing when the object can no longer be distributed to each person in the group because there are not enough left to go around.

Initiate

1. **What does fair share mean?**
Arrange students in groups with three or four students per group. Give each group a bag of foam shapes and ask students to sort by shape as they did in Lesson 4. Next, ask students what they think the phrase “fair share” means. The following are some common responses:

- "Divvy up"—ask for clarification if students use this phrase; What does it mean?
- "Same amount" – nobody gets any more or any less than anybody else; equal number of each

Follow-up questions:

- What do you fair share at home?
- When you fair share cookies, are there ever any left over?
- Do you know what mathematicians call the "left over?"
  - *Remainder—what is left over after you fair share*

Investigate

2. Fair share foam shapes

Students should have their foam shapes sorted by shape. Ask students to "fair share" each group of shapes until all of the shapes have been handed out equally. Any leftover shapes will go into a "remainder pile." For example, students will fair share the pile of hearts placing any extra hearts in the remainder pile. Allow students to come up with their own methods of fair sharing. If one group has come up with a method that you would like the rest of the class to see, call the students over to watch that group's method. Remind students that there is more than one way to fair share and they do not have to use that method.

Students may try to break shapes into fractional pieces to avoid remainders. This is a great time to discuss fractions, though you may want to keep the shapes intact for future lessons! Point out that this is a great strategy for dividing things like cookies, pizza, or candy bars BUT NOT cars, puppies, books, or foam shapes.

Questions for discussion:

- What method did your group use to fair share?
- Did anyone in your group take charge?
- Did anyone in the group want to do it differently?
- How did you decide which method to use?
- Did you notice anything about the number of shapes in the remainder pile?
  - *Students may notice that the number of shapes remaining for each after the pile has been fair shared is less than the number of people in the group.*

3. All good mathematicians check their work!

Ask students to record their results in the table on the *Fair Share Data* Student Pages **[SMJ pages 43-45]**. If students have never used a data table, you may have one group demonstrate filling in one row reminding students that their data may look different due to number of shapes or group size. Tell students to verify that their data matches that of the rest of the students in their group.

Conclude

4. Stump your classmates (Optional)

There are four variables in this investigation for each shape: number of students in the group, how many shapes each member received, how many were left over, and how many total shapes the group received.

Put this table on the board:

Group Members	Shape	Each Person's #	Leftovers	Total

Then tell the students that they will choose one shape from which to create a riddle. They have to give all the information regarding that shape except for the total number of that shape their group received.

For example, one student will say, "I am going to tell you about my group's hearts. There were 3 of us in the group, and each of us got 4 hearts. There were 2 left over. How many hearts did our group have?" Record the data on the board. It may look like the following chart:

Group Members	Shape	Each Person's #	Leftovers	Total
3	Hearts	4	2	????

Then the students from the other groups will come up with the total. In this example, the total would be 14. Students should explain how they got the total (3 x 4 +2).

Then ask the groups to share any 3 out the 4 variables and see if the rest of the class can figure out what the student did not share. For example, a student might say, "I was in a group of 4 students, I got 3 spades and our team had a total of 13 spades. How many spades did we have left over?" The student can place his riddle under the last riddle.

Group Members	Shape	Each Person's #	Leftovers	Total
3	Hearts	4	2	????
4	Spades	3	????	13

(The answer would be 1 spade was left over.)

After several groups have shared, look at the pattern between the total number of group members and the remainders. Ask the students if they notice anything about these two columns. By the end of the lesson they should realize that there are never more leftovers than there are total group members. If there are more leftovers, each person could receive an additional shape.

This activity helps students to develop their understanding of division and multiplication as inverse operations without explicitly asking them to multiply or divide. Most students could create a picture or act it out to figure out the answer.

Assess

5. **Frogs and flies**

Give students the *Fair Sharing Frogs* Student Page **[SMJ page 47]** to assess their understanding of fair share.

6. **Optional student practice**

Fair Share Lunches Student Pages **[SMJ pages 49 & 53]** can be used for additional student practice with bar graphs and fair sharing. Teachers may choose to send it home as homework, use it as an optional activity when students finish their work, or assign it as morning work. This may also be used after Lesson 6 if it fits better with the teacher's schedule. Teacher instructions can be found in the teacher's manual on page 133.

Fair-Sharer: ________________________________ Date __________________

Fair Share Data

Directions: Fill in the data table by tracing each shape in the first column. Then, write how many of that shape each person has in the second column. In the third column, write how many of that shape were left over. In the fourth column, write how many of that shape your group had in all including the leftovers.
How many students are in your group (including you)? _____________________

Shape (Picture or Name)	When your group fair shared, how many of that specific shape did you get?	How many of that specific shape were **left over**?	How many of that specific shape did your group have in **total**?

SMJ page 43

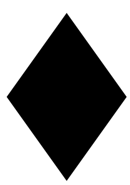
Fair Share Data (continued)

Shape (Picture or Name)	When your group fair shared, how many of that specific shape did you get?	How many of that specific shape were **left over**?	How many of that specific shape did your group have in **total**?

SMJ page 45

Fair Share Data
ANSWER KEY

Directions: Fill in the data table by tracing each shape in the first column. Then, write how many of that shape each person has in the second column. In the third column, write how many of that shape were left over. In the fourth column, write how many of that shape your group had in all including the leftovers.
How many students are in your group (including you)? ____*3*_______

Shape (Picture or Name)	When your group fair shared, how many of that specific shape did you get?	How many of that specific shape were **left over**?	How many of that specific shape did your group have in **total**?
Sample Answers *(Actual answers will vary.)* 	*4*	*2*	*14*
	*7*	*2*	*23*
	*2*	*0*	*6*
	*4*	*1*	*13*

Teacher's Note: From this sample page, you can tell that this student was in a group with 2 other students (3 total group members). You could conclude this by looking at the question at the top of the page or by taking the total shapes minus the remainders and dividing by the individual total. (For hearts, 14-2, and then 12/4 = 3.) This is helpful to think about as you get to the "Stump Your Classmates" section.

***The students' pages will allow them to continue on with the remaining 4 shapes.

Fly Catcher: ______________________________ Date: ______________________

Fair Sharing Frogs

Frogs like to be fair. Help the frogs in these riddles fair share their flies!

FAIR SHARING FROGS RIDDLE 1
On a log, there sit 3 frogs
And 18 flies in the air
How many flies should each frog catch,
If each one gets its fair share?

My Work:

Each frog gets _____ flies. There are _____ flies remaining in the air.

FAIR SHARING FROGS RIDDLE 2
Buzzing around are 15 flies
And 4 frogs playing a game,
How many flies does each frog catch,
If each one has the same?

My Work:

Each frog gets _____ flies. There are _____ flies remaining in the air.

SMJ page 47

Fair Sharing Frogs
ANSWER KEY

Frogs like to be fair. Help the frogs in these riddles fair share their flies!

FAIR SHARING FROGS RIDDLE 1
On a log, there sit 3 frogs
And 18 flies in the air
How many flies should each frog catch,
If each one gets its fair share?

My Work:

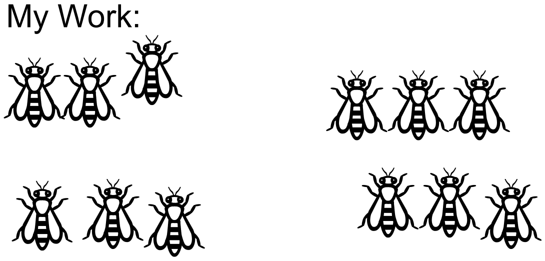
FROG 1	*FROG 2*	*FROG 3*
IIIIII	*IIIIII*	*IIIIII*

Each mark represents a fly. I gave each frog a fly, then gave each frog a fly again. I counted to 18 and finished with Frog 3. That's how I knew I was done.

Each frog gets __6__ flies. There are __0__ flies remaining in the air.

FAIR SHARING FROGS RIDDLE 2
Buzzing around are 15 flies
And 4 frogs playing a game,
How many flies does each frog catch,
If each one has the same?

My Work:



I made four piles of flies, one for each frog. I drew pictures of flies. There were 3 left but that's not enough to give each frog another one.

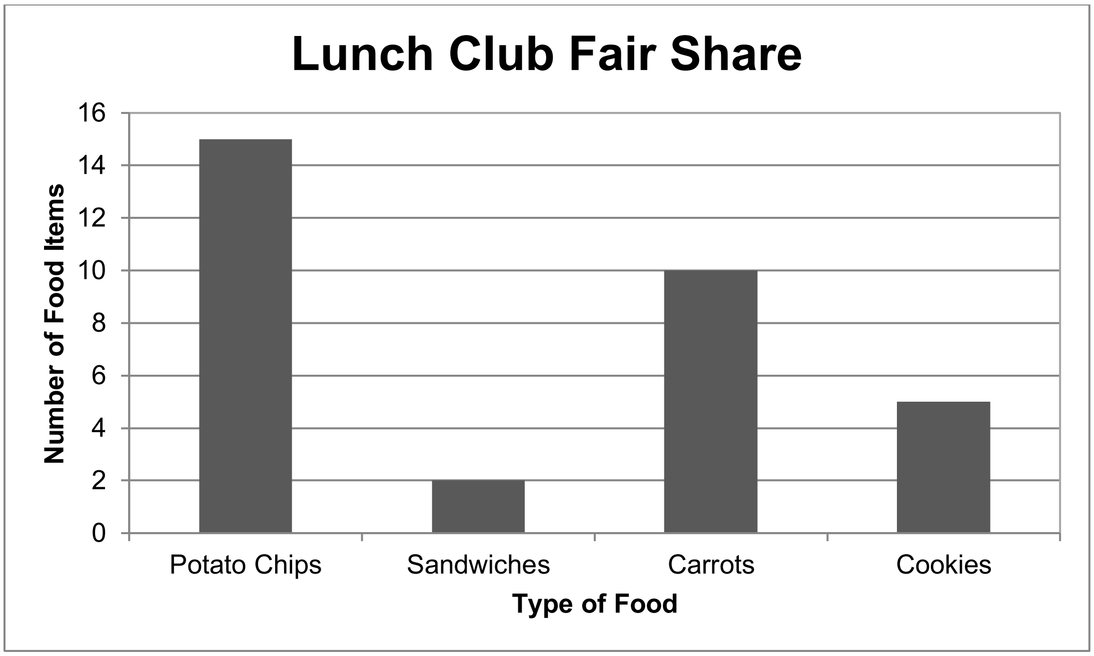
Each frog gets __3__ flies. There are __3__ flies remaining in the air.

Babbage & Galileo

Name: ______________________________ Date: _____________________

Fair Share Lunches (Optional)

Two students formed the Lunch Club to fair share their lunches. This is the graph that illustrates how many of each type of food to be shared. You may want to draw a picture to help you answer some of the questions.



1. How many total potato chips are there? __________

   If the 2 students fair shared the chips, how many would they each get?

   ____

   How many would be left? __________

2. How many total sandwiches are there? __________

   If the 2 students fair shared the sandwiches, how many would they each get?

   ____

   How many would be left? __________

SMJ page 49

3. How many total carrots are there? __________

   If the 2 students fair shared the carrots, how many would they each get?

   ____

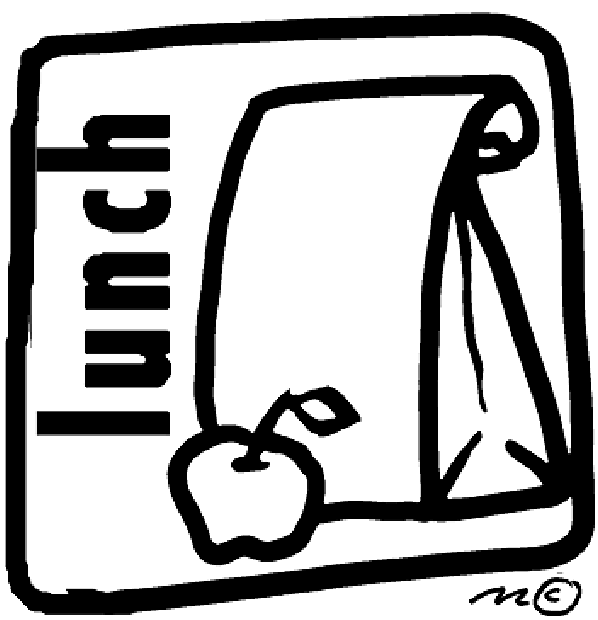
   How many would be left? __________

4. How many total cookies are there? __________

   If the 2 students fair shared the cookies, how many would they each get?

   ____

   How many would be left? __________



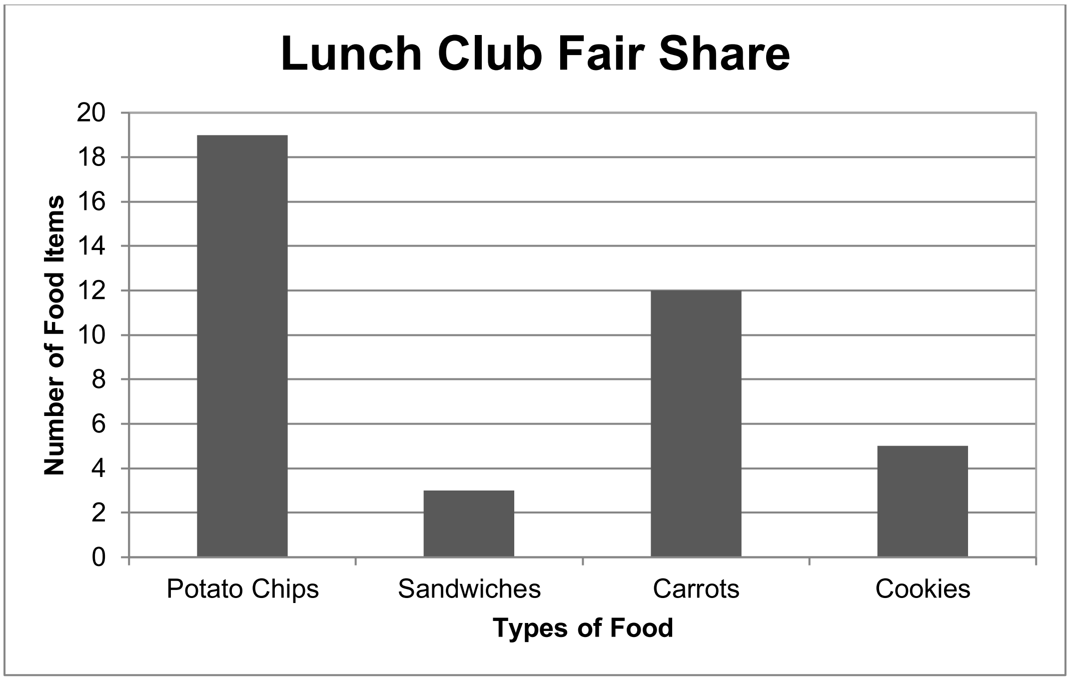
SMJ page 51

Falconer

Name: ______________________________ Date: ______________________

Fair Share Lunches (Optional)

Three students formed the Lunch Club to fair share their lunches. This is the graph that illustrates how many of each type of food is to be shared.



1. Complete the table using the data from the bar graph.

Type of Food	Total Number of Food Item	With 3 students fair sharing, how many will each get?	Number of Leftovers (Remainders)
Potato Chips			
Sandwiches			
Carrots			
Cookies			

2. What is the biggest number of leftovers possible? Why?

____

____

SMJ page 53

Fair Share Lunches (Optional)
DISCUSSION GUIDE AND ANSWER KEY

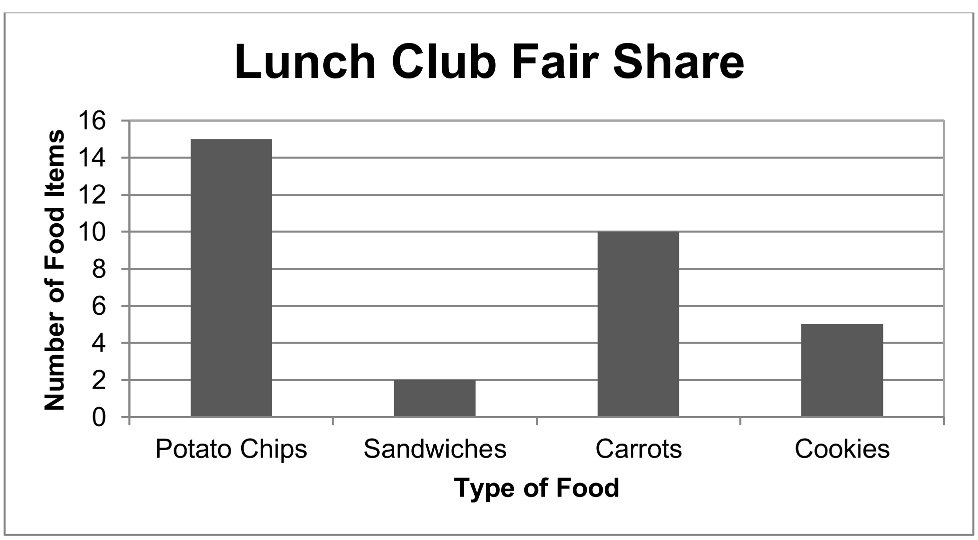
Differentiation Options: *There are 2 versions of this practice page. For the Babbage and Galileo page, the students are fair sharing between 2 students; whereas, the Falconer version requires the students to fair share among 3 students. The Babbage and Galileo version guides the student question by question, and in the Falconer version the students are expected to make a table.*

Discussion Questions: *When going over the work in class, here are a few questions/topics that foster deeper mathematical thinking.*

- *Reminders: The Falconer version has the question regarding the largest remainder possible. Help students realize that there will not be a remainder equal to or bigger than the number of people in the lunch club. After the Falconer students explain their thinking, ask the other students what is the biggest possible remainder in their lunch club.*
- *Fair Sharing: Direct the whole class to think about fair sharing between two people. When you start with an even number of items, what is left over? When you start with an odd number of items, what is left over? Is this always true for 2 people? Is it true for 3 people?*
- *Difference: Both groups have 5 total cookies. Discuss if it would be better to be in the 3 member lunch club or the 2 member lunch club.*

Babbage and Galileo

Two students formed the Lunch Club to fair share their lunches. This is the graph that illustrates how many of each type of food to be shared. You may want to draw a picture to help you answer some of the questions.



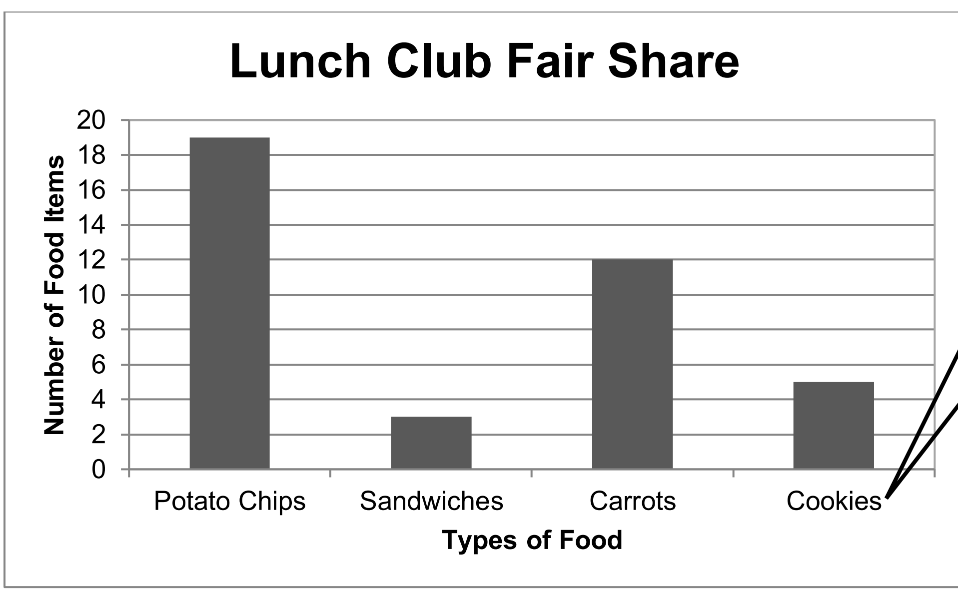
If there is an odd number, is there a remainder? What about an even number? Does it change when three people are in the group?

1. How many total potato chips are there? _15_
   If the 2 students fair shared the chips, how many would they each get? _7_
   How many would be left? _1_
2. How many total sandwiches are there? _2_
   If the 2 students fair shared the chips, how many would they each get? _1_
   How many would be left? _0_
3. How many total carrots are there? _10_
   If the 2 students fair shared the chips, how many would they each get? _5_
   How many would be left? _0_
4. How many total cookies are there? _5_
   If the 2 students fair shared the chips, how many would they each get? _2_
   How many would be left? _1_

Ask the students about the strategies they used to get their answers. Did they draw pictures? Act it out?

Falconer

Three students formed the Lunch Club to fair share their lunches. This is the graph that illustrates how many of each type of food is to be shared.



Both groups shared 5 cookies. Which Lunch Club members were luckier?

1. Complete the table using the data from the bar graph.

Type of Food	Total Number of Food Item	With 3 students fair sharing, how many will each get?	Number of Leftovers (Remainders)
Potato Chips	*19*	*6*	*1*
Sandwiches	*3*	*1*	*0*
Carrots	*12*	*4*	*0*
Cookies	*5*	*1*	*2*

2. What is the biggest number of leftovers possible? Why?

The biggest number of leftovers possible is 2 because if there are more than 2 than everyone in the group of 3 can have at least 1 more fairly.

Have the Falconer group explain, then ask the other students what is the biggest number of leftovers possible in a group of two people. What about 4 people?

LESSON 6

BAR GRAPHS—

DISPLAYING SHAPE DATA

Big Mathematical Ideas

Bar graphs are ideal for displaying data because they are fairly easy to create and interpret. They introduce students to mathematical concepts such as axes, scale, and equal spacing. In addition, a bar graph can be used in a media report or article to provide a summative representation of what is in the text.

Lesson Objectives	• Students will be able to construct a bar graph and interpret data based on information in a table.
Materials	• Student Page—*Write About It! (Babbage)* [SMJ page 55] • Student Page—*Reaching Our Goal! (Galileo & Falconer)* [SMJ page 57] • Student Page—*A Bar Graph Debate* [SMJ page 59] • Student Page—*Giants Love Jellybeans!* [SMJ page 61] • Student Page—*Museum Madness* (Optional) (Appendix C) [SMJ page 207] • Data from SMJ pages 43 & 45 • Graph paper • Rulers • Poster board
Mathematical Language	• **Scale:** A series of marks along the axes to determine unit lengths; a scale could vary by 1 unit, 2 units, 10 units, 1/2 units, and so forth. • **Bar Graph:** A graph that uses bars to display quantities of categorical data.

Lesson Preview

Students represent their data from Lesson 5 by creating a bar graph.

Initiate

1. Interpreting bar graphs

Two versions of the initiation task are available. You can use students' performance on Unit Test item #2 to help decide which task students are given.

	Babbage Group	**Galileo & Falconer Group**
How student scored on #3 on the pretest	Scored 0-2 points on item	Scored 2-5 points on item
Student Pages for Lesson	*Reaching our Goal*—Babbage	*Reaching our Goal*—Galileo & Falconer

The Babbage *Write About It!* Student Page **[SMJ page 55]** asks students to write a brief article. The Galileo & Falconer *Reaching Our Goal!* Student Page **[SMJ page 57]** asks students to create a poster. Both tasks involve interpreting bar graphs, but the complexity is slightly greater in the *Reaching Our Goal!* task.

A second grouping option is to pair students by interest and have each pair create a poster and an article to gain greater understanding of multiple representations of the information in the graph.

Below are some ideas for working with groups or individuals on each of the different products:

- For students working on the newspaper article, scaffold using prompts like the following:
  - What might students in this classroom be doing?
  - Why might they be doing this?
  - What is a good title for this article?
  - Can you report how many of each item was collected?
  - In which category might they want to increase donations?
- For students working on the poster, provide prompts like the following:
  - What should you title the poster?
  - What might you add to get more people to look at your poster?
  - How many rows and columns will your table need? How do you know?
  - How can you figure out how many more of each item is needed? Does anybody have a different way of figuring this out?

2. A bar graph debate (Optional)

Some mathematics textbooks argue that bars should never touch in a bar graph, while others indicate that it is fine for the bars to touch. Allow students to weigh in on the issue by assigning the *A Bar Graph Debate* Student Page **[SMJ page 59]**.

Investigate

3. Graphing our foam friends

Give students a sheet of graph paper and tell them they will create a bar graph of the data from the Lesson 5 *Fair Share Data* Student Pages **[SMJ pages 43 & 45]**. If students have never created a bar graph before, ask them to examine the first graph from the initiation, *Classroom 102's Winter Collection*. Ask students:

- What do you think you will do first?
  Create axes using a ruler. Create even spaces between bars.

For first-time graphers, discuss what you mean when you refer to the horizontal axis or the vertical axis. Tell students in some graphs, these are referred to as the x- and y-axes. Model how to create vertical and horizontal axes. Be sure students' horizontal axes are far enough from the bottom of the paper that they have room to write the name of each shape.

Recommend that students make "hash marks" on the horizontal axis to show where their bars will go. One strategy for spacing is shown in *Figure 1* (using bars one unit in width and two spaces in between bars). Explain that having equal spacing and same-width bars makes graphs more visually appealing.

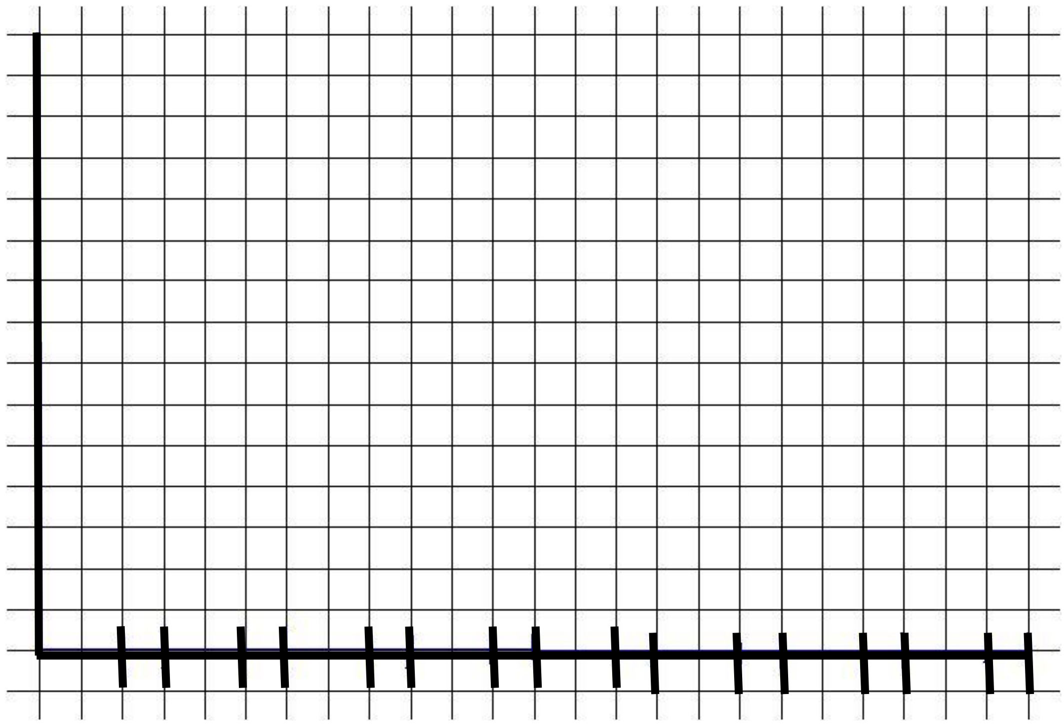


Figure 1

- Once students have set up their spacing, ask them to label their horizontal axis with the different shape names. Model on the board or chart paper. Students may need to abbreviate or draw the shapes if they do not have room for the entire name.
- Ask students how high the numbers on the vertical axis should go and what they represent.
  The vertical axis maximum numeric label must be greater than or equal to the highest number in students' tables. They represent the number of each shape.
- Draw one bar on the board or chart paper and discuss what the bar represents. Many students will be able to complete the rest of the graph on their own. They should refer back to **SMJ pages 43 & 45** for the data. Circulate as students complete the task to check for even spacing of bars and numbers. Help students who have difficulty using rulers. Check for axes labels and titles on students' papers.

Conclude

4. A classroom comparison

Use the following questions to guide a discussion of students' graphs:

- Does your graph look the same as the other students' in your group? Should it? Why or why not?
  Students' graphs should look the same if they were told how to space and label their graphs. If one student in a group chose a

scale that had increments greater than one on the vertical axis, his/her graph should look different than the rest of the group.

- Students who worked in a group of three, how might your graph differ from a group with four students?
  It is probable that the group of three would have higher bars on their graphs than students who worked in groups of four because they were able to distribute more of each shape to each person. Groups of three may also differ from other groups of three because the number of shapes is not consistent in each bag.

Assess

5.

Giants and jellybeans

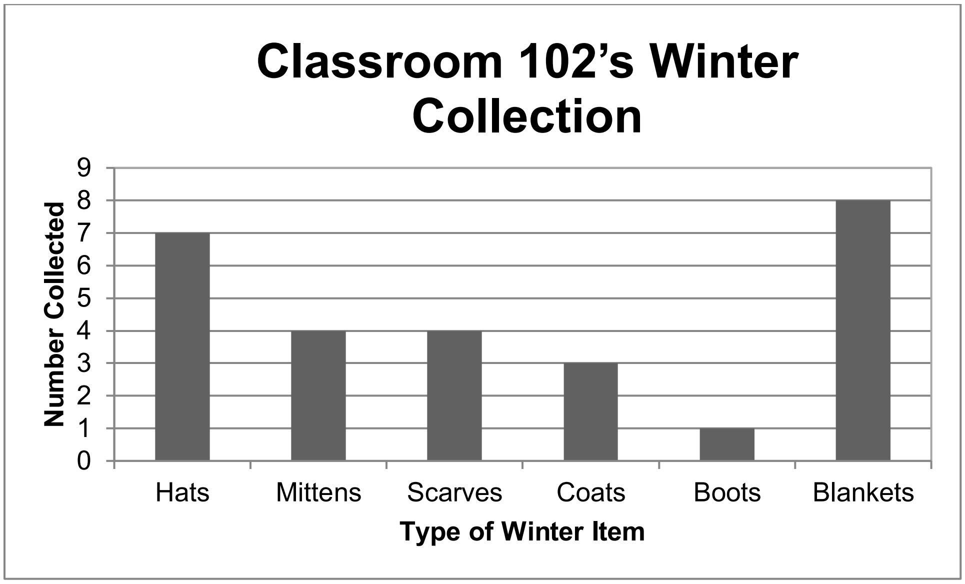
Assess students using the *Giants Love Jellybeans!* Student Page **[SMJ page 61]**.

Additional pictograph challenge

In the Appendix, there is an additional activity, *Museum Madness* Student Page **[SMJ page 207]**, which examines both pictographs and bar graphs. It can be used as an extension for this lesson with students who need an extra challenge.

Bar Graph Analyst: ___________________________________

Write About It!



Write a newspaper article that could go with the bar graph above.

____

____

____

____

____

____

____

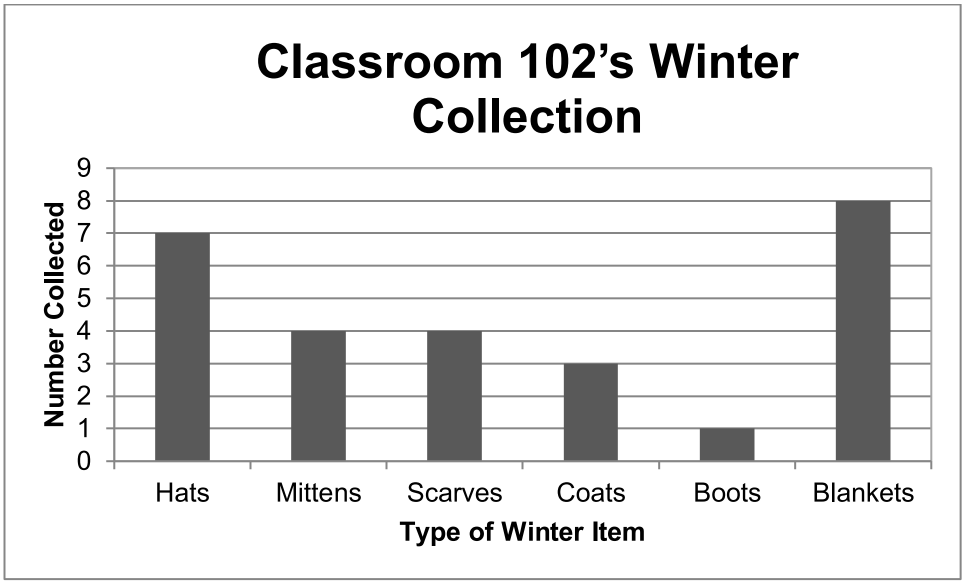
____

____

SMJ page 55

Babbage

Write About It!
ANSWER KEY



Write a newspaper article that could go with the bar graph above.

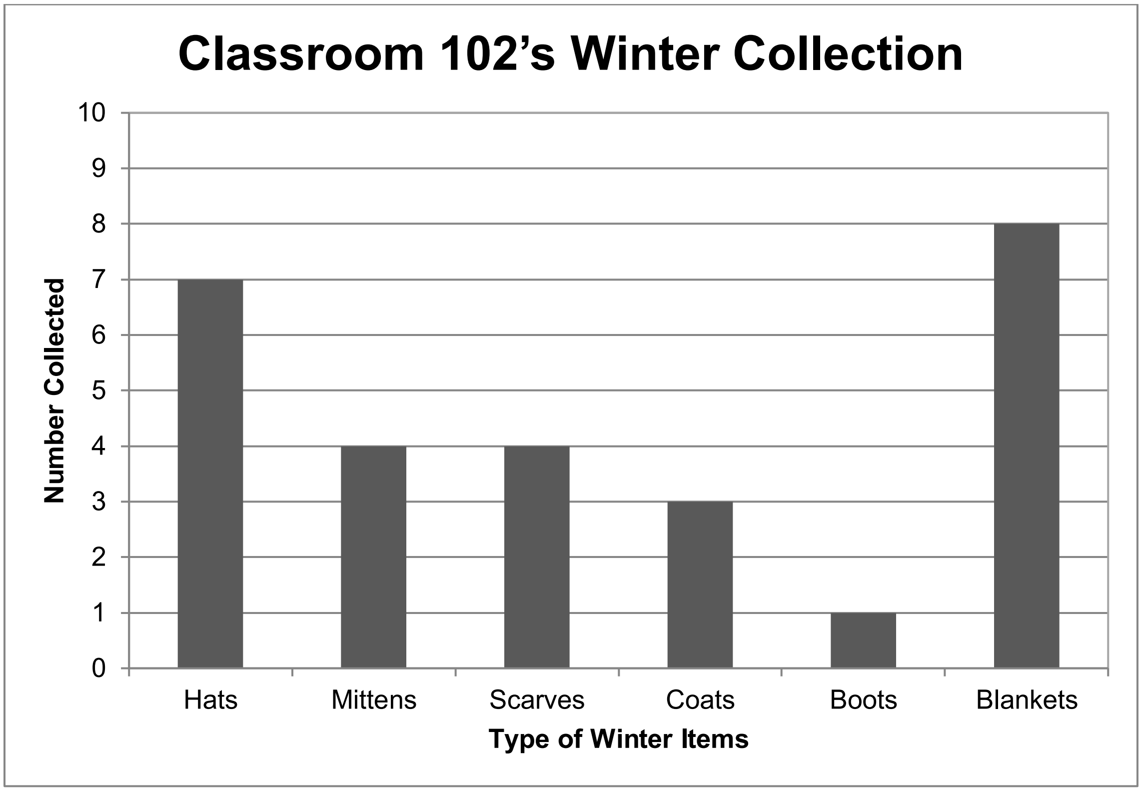
SAMPLE RESPONSE (ANSWERS WILL VARY):

Keeping Kids Warm in Winter
A local school decided that they were going to help keep kids warm this winter. Classroom 102 students collected winter items such as hats, mittens, scarves, boots, blankets, and coats. They have plenty of blankets and hats but would like more boots, mittens, scarves, and coats. In fact, they collected eight blankets but only one pair of boots. Students are still collecting, so please bring your winter items to help out!

Galileo & Falconer

Bar Graph Analyst: ___________________________________

Reaching Our Goal!



Students in classroom 102 collected the items listed in the bar graph to donate to a local shelter. Their goal is to have 10 of each item before bringing the items to the shelter.

Create a poster to inform other students which items are still needed. Provide a table or graph to show how many *more* of each is needed to reach the goal.

SMJ page 57

Galileo & Falconer

Reaching Our Goal!
ANSWER KEY

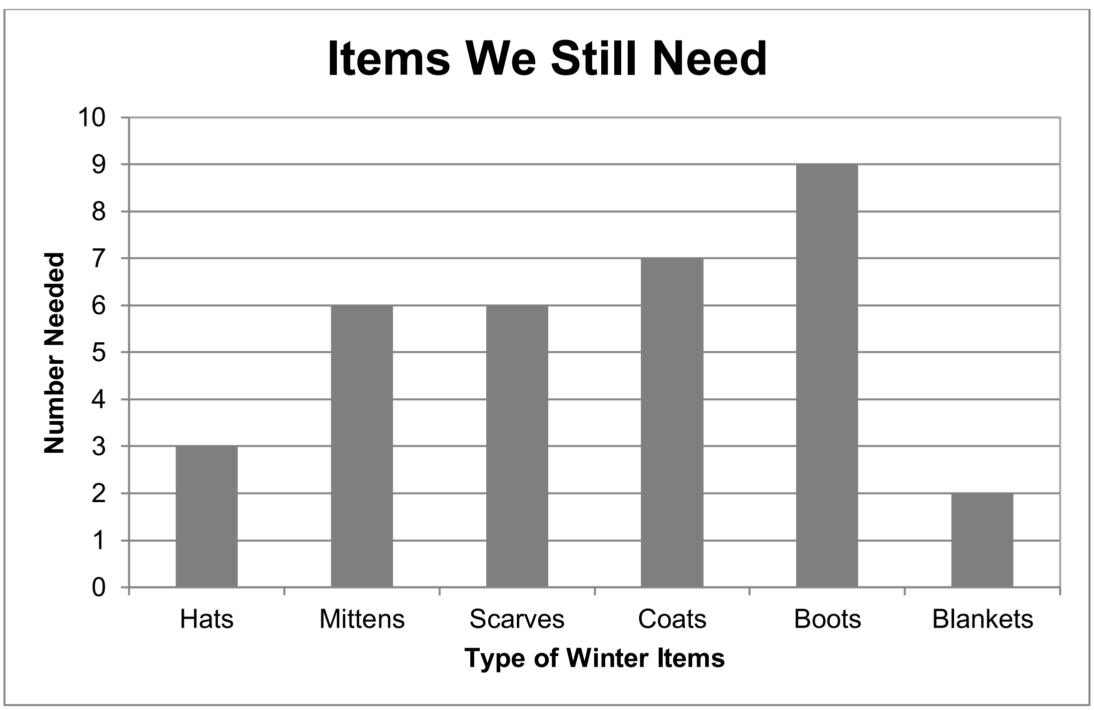
Students should create a poster asking for additional items. The poster should contain the following information in the form of a table or graph:

Sample Table

Items we still need:

Item	*How many MORE we need to reach our goal of 10*
Hats	*3*
Mittens	*6*
Scarves	*6*
Coats	*7*
Boots	*9*
Blankets	*2*

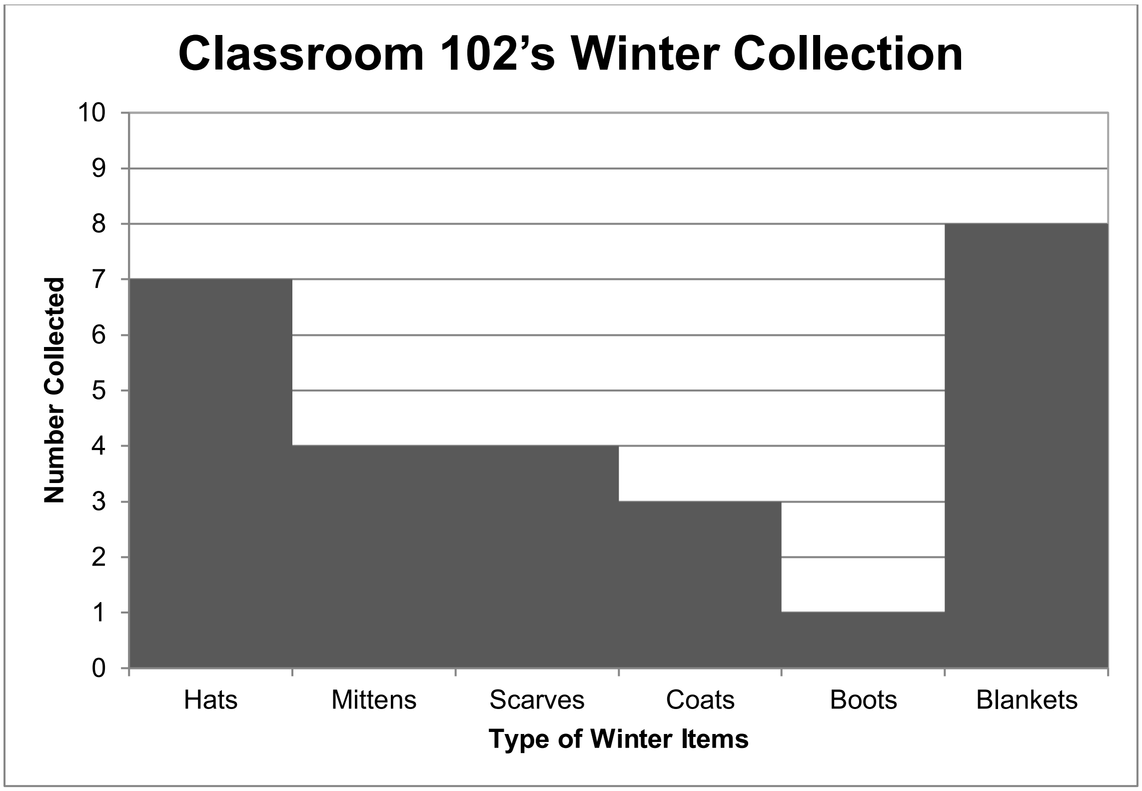
Sample Graph



Name: ___________________________________ Date: __________________

A Bar Graph Debate

Some math experts say that it is okay to have no spaces between the bars in a bar graph. Other people think that the bars in a bar graph should always have spaces in between. Below is a picture of the same graph, Classroom 102's Winter Collection, with no spaces.



Which of the two graphs do you like best (the one with the bars touching or not touching)? Explain why.

____

____

____

____

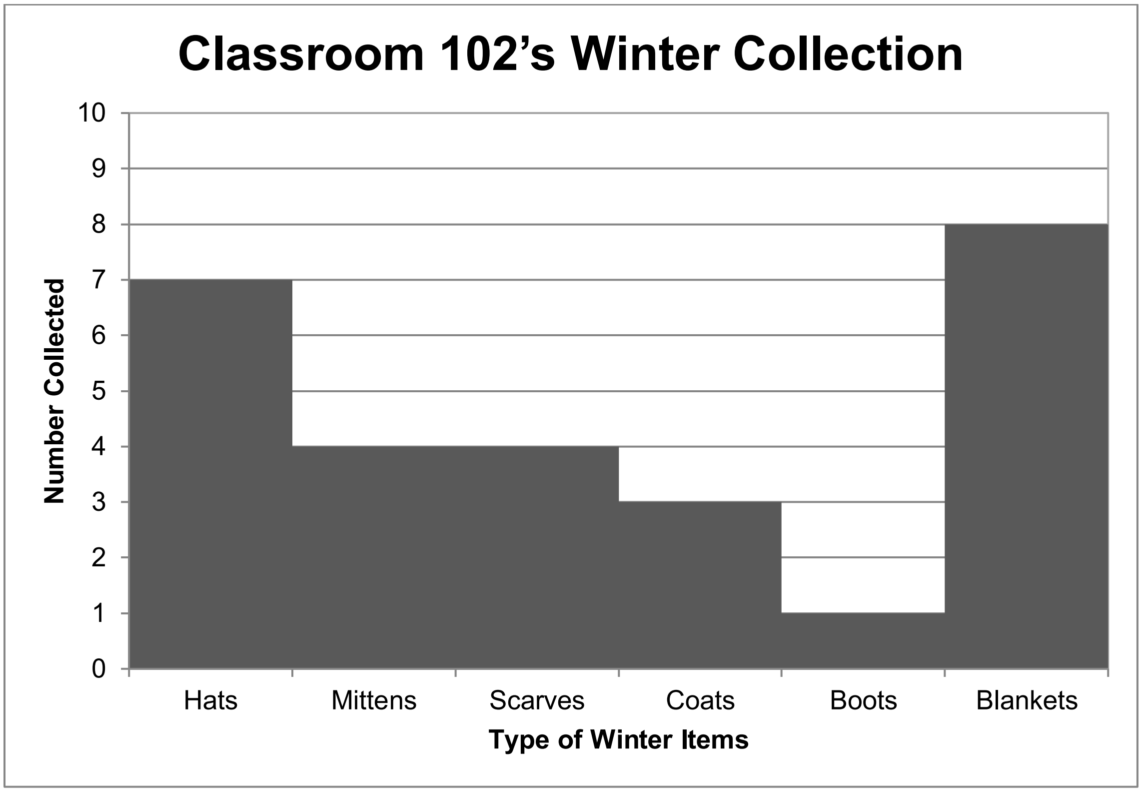
____

____

SMJ page 59

A Bar Graph Debate
ANSWER KEY

Some math experts say that it is okay to have no spaces between the bars in a bar graph. Other people think that the bars in a bar graph should always have spaces in between. Below is a picture of the same graph, Classroom 102's Winter Collection, with no spaces.



Which of the two graphs do you like best (the one with the bars touching or not touching)? Explain why.

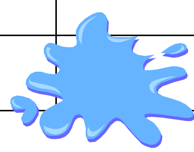
Sample Response (Answers will vary): I really like the bars that are not touching. When they are touching each other it is hard to tell where one ends. It doesn't look as neat when they are all together like that. If those math experts ask me, I will tell them to make bar graphs where the bars don't touch!

Jellybean Grapher: ___________________________________

Giants Love Jellybeans!

Jackie the Joyful Giant just loves jellybeans. One day, she decides to sort her pile of 39 jellybeans by color. She records her information in a table.

COLOR	NUMBER OF JELLYBEANS
RED	9
GREEN	12
PURPLE	8
ORANGE	



Jackie is so excited about making a bar graph of the data that she spills a jar of ink on the paper and can no longer see how many orange jellybeans she had. She is very upset because she already ate the jellybeans!!

1. Help Jackie figure out how many orange jellybeans she had. Show your work so she can tell how you got your answer.

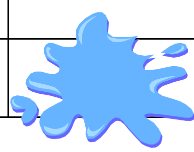
2. On a piece of graph paper, make a graph of the jellybean data.

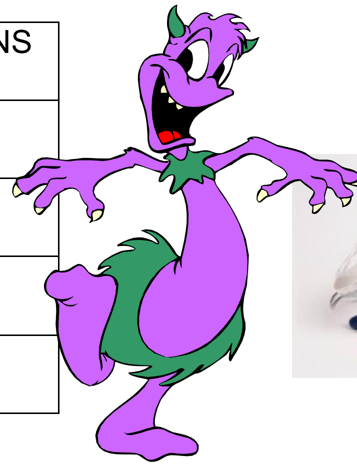
SMJ page 61

Giants Love Jellybeans!
ANSWER KEY

Jackie the Joyful Giant just loves jellybeans. One day, she decides to sort her pile of 39 jellybeans by color. She records her information in a table.

COLOR	NUMBER OF JELLYBEANS
RED	9
GREEN	12
PURPLE	8
ORANGE	



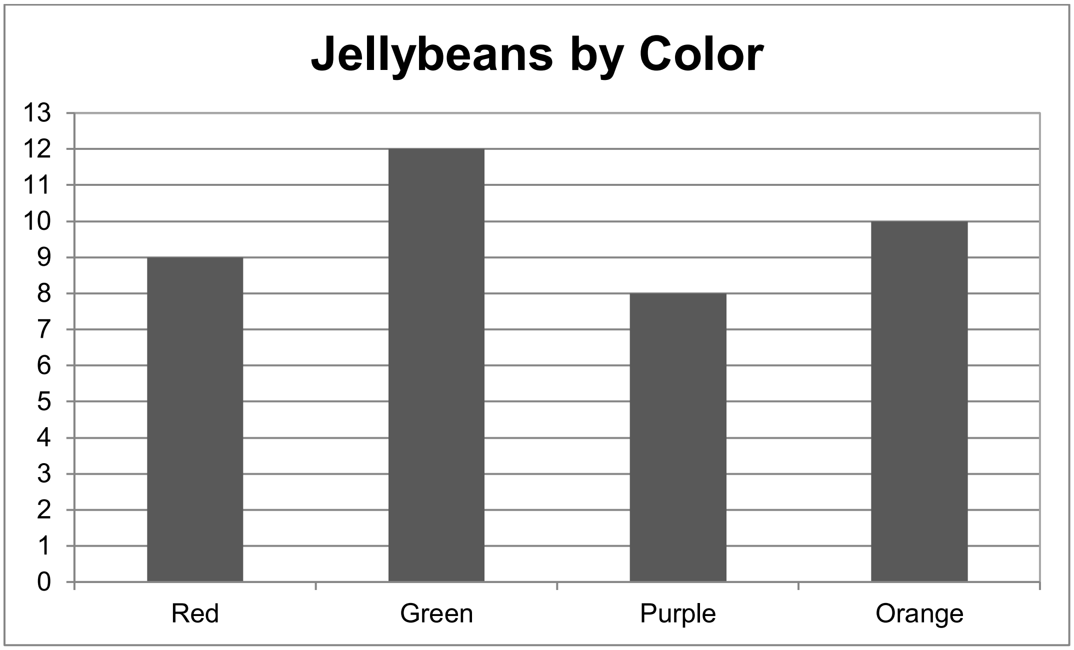


Jackie is so excited about making a bar graph of the data that she spills a jar of ink on the paper and can no longer see how many orange jellybeans she had. She is very upset because she already ate the jellybeans!!

1. Help Jackie figure out how many orange jellybeans she had. Show your work so she can tell how you got your answer.

   The table has 9 red, 12 green, and 8 purple. That's a total of 9 + 12 + 8 = 29 jellybeans. My hundreds chart makes it easy to see how many more she needs to get to 39. She needs 10 more jellybeans, so there are 10 orange jellybeans.

2. On a piece of graph paper, make a graph of the jellybean data.



BAR GRAPHS—

REUSING AND REDUCING

LESSON 7

Big Mathematical Ideas

Students learn a powerful message when social responsibility and mathematics are taught in an interdisciplinary way. Similar to the recycling intervention that students used to study how data changes over time, clothing drives provide an interactive way for students to categorize and graph real data.

Lesson Objectives	• Students will conduct a clothing, book, or crayon drive to generate categorical data.
Materials	• Student Page—*Keeping Track: Data on* __________ [SMJ page 63] • Student Page—*Keeping Track: Data on* __________ *(Alternative Version)* [SMJ page 65] • Student Page—*Our Goals* [SMJ page 67] • Student Page—*Crawly Creatures* (Optional) [SMJ page 69] • Student Page—*Student Mathematicians Practice* (Optional) [SMJ page 71]
Mathematical Language	• **Recycle:** To create new products from waste materials. • **Reduce:** To decrease the amount of waste that you produce. • **Reuse:** To use again; sometimes for a different purpose. • **Categorical Data:** Data that is placed in groups/categories (ex., types of food, eye color, book genre). • **Category:** A set of things grouped together because they share a common trait.

Lesson Preview

Students consider the connections between reducing, recycling, and reusing. They plan a “reducing by reusing” drive by collecting clothing, books, or crayons.

Initiate

1. Introduce reduce

Unlike recycle, the term **reduce** may not be familiar to students. First, review the definition for **recycle**. Second, ask:

- What do you already know about reducing?
- What does it mean if your parents tell you that you need to reduce the amount of time you watch TV?
- Would you want me to reduce the amount of homework I give you each week?

Ask students to discuss the following question with a partner before discussing as a class:

- Joey brings his lunch in a lunchbox each day. Mrs. Garrett, his teacher, brings her lunch in a paper bag. Which person do you think is practicing reducing behavior?
  Joey is practicing reducing behavior by bringing a lunchbox instead of a paper bag. This cuts back on the amount of garbage produced. Mrs. Garrett probably throws her bag away each day.

Based on the previous example, ask students to help define the term **reduce** as it relates to the greening up theme.

- This definition should address the idea that reducing means decreasing or cutting back on the amount of waste that is produced through a variety of different means.
- Students might understand reducing as *making less garbage*.

Investigate

2. Reducing by reusing

Ask students to think about their recycling intervention. Discuss the relationship between recycling and reducing. Challenge students to think of ways to reduce the amount of garbage produced that are not directly related to recycling.

*Students should discuss how recycling cuts back on or **reduces** the amount of garbage the school is producing. Actions such as using cloth instead of paper towels, regular plates instead of paper, or writing on the back of a piece of paper are all reducing behaviors that are distinct from recycling.*

Explain that another way to reduce the amount of waste produced is to reuse things we might normally throw away. Find out if students know how things around the house, school, or other places get reused to save them from being thrown away. Some questions to help prompt their thinking include the following:

- Have you ever seen a tire being used as a swing?

- Have you ever donated clothes for somebody else to use?
- Have you ever washed out a yogurt cup and used it as a drinking cup?
- Does your teacher keep a pile of scrap paper?
  All of these things reduce the amount of garbage we produce by reusing.

3. Selecting a drive

As a class, decide on a way to help reduce by reusing. Some options for your class include the following:

- Set up a clothing drive either within the class or by inviting other grades to join in the effort. Students can categorize the clothing and collect data to generate a bar graph of their efforts. If multiple classes are involved, students might categorize by classroom instead of by clothing item.
- Set up a book drive. Several organizations collect gently used children's books. Students can categorize books by age appropriateness using categories such as Toddler, Elementary, Middle, High, Adult, or a similar categorization scheme. Again, categorizing by number of books donated per classroom is another option for examining and later graphing the data.
- Ask students to bring in old, broken crayons to be melted into new ones. Several websites (example: <http://frugalliving.about.com/od/frugalfun/ht/Make_Crayons.htm>) are available to provide ways to reuse old crayons. These can be categorized by color or by classroom if multiple classrooms are involved and weighed (rather than counting). This project might be carried out with the help of an art teacher.

Ensure a successful drive and data gathering experience by taking the following steps:

- Involve students in the decision of what to collect and tell them where it will be donated (if applicable).
- Find supplemental articles on the particular cause to use during language arts or social studies to make this an interdisciplinary experience.
- Decide whom to involve. Is it just your classroom, or will you set up a competition with multiple classrooms?
- Decide how to categorize data prior to data collection. Will you count or weigh items by classroom? By item type? By color? Once you have decided *what* your class will be collecting, fill in the categories with students on the *Keeping Track: Data on* ______________ Student Page **[SMJ page 63]**. Students can fill in the blank with an appropriate word to match the selected collection item.

(NOTE: Although the table provides room for 10 categories, you may have as few as two. If you have to go beyond 10, your class may want to select an alternative categorization method.)

- Collect data each day *or* perform the data collection following the drive. Decide which method works best for your class.
- The column on the Student Page for “Tally” will be filled in as you collect data. The column for “Frequency” will be filled in following the drive.
  (An alternative form of the Student Page is provided for classrooms measuring by weight **[SMJ page 65]**.)

Conclude

4. Finalize plans

Use talk moves (see Appendix C) to have students “restate” or “add on” to discuss how the drive will take place. Integrate mathematical vocabulary, asking questions like the following:

- How will we collect data during our drive?
- What categories will we be using to examine our data?
- How will we measure our data?
- What type of graph can we use to display the data once we are done?
  A bar graph can be used to show how many items were brought in from each class or each category. If students want to collect data on how much was brought in each day, a line graph could be used to show the change over time.

Look Ahead

5. Remind students each day

Remind students to continue bringing in items. Tell them they want the bar graphs they make at the end to show the success of their drive!

Assess

6. Setting and graphing class goals

Create a data set of “goals.” The *Our Goals* Student Page **[SMJ page 67]** has been provided for this purpose. It may be easier to decide on class goals rather than each student having his or her own. Ask students how many of each item they hope to get over the course of the drive. Help students to set realistic goals given the time period your class has selected (especially if weight is the method of data measurement). Students can create bar graphs to represent their goals and use them for comparison purposes at the end of the drive. Some things to remember to make this task successful:

- Graph paper has a limited number of spaces. If students are setting high goals, a scale of 2 or 5 may be necessary.
- Help students set up axes and space their bars evenly.

7. Optional student practice

Crawly Creatures Student Page **[SMJ page 69]** can be used for additional student practice with bar graphs. This worksheet requires students to critically think about clues to create labels for a bar graph. Teachers may choose to send it home as homework, use it as an optional activity when students finish their work, or assign it as morning work.

For number and operation practice, the students can work on the *Student Mathematicians Practice* Student Page **[SMJ page 71]**.

Name: ______________________________ Date: ________________

Keeping Track: Data on ___________________

Category	Tally	Frequency

SMJ page 63

Keeping Track: Data on Our Clothing Drive
ANSWER KEY
(Actual data will vary.)

Category	Tally	Frequency
Pants	𝍸 𝍸 𝍸	*15*
Shirts	𝍸 𝍸 𝍸 𝍸 𝍸	*25*
Socks	𝍸 𝍸	*10*
Dresses	𝍸	*5*
Skirts	𝍸 𝍸 𝍸 𝍸	*20*

Name: ________________________________ Date: _________________

Keeping Track: Data on ___________________
(Alternative Version)

Category	Weight

SMJ page 65

Keeping Track: Data on Crayons Collected (Alternative Version)

ANSWER KEY

(Actual data will vary.)

Category	**Weight (lbs.)**
Red	*4.7*
Orange	*3.6*
Blue	*2.9*
Yellow	*8.7*
Green	*6.5*
Purple	*2.4*

Name: ________________________________ Date: ________________

Our Goals

Category	Goal

SMJ page 67

Our Goals

ANSWER KEY

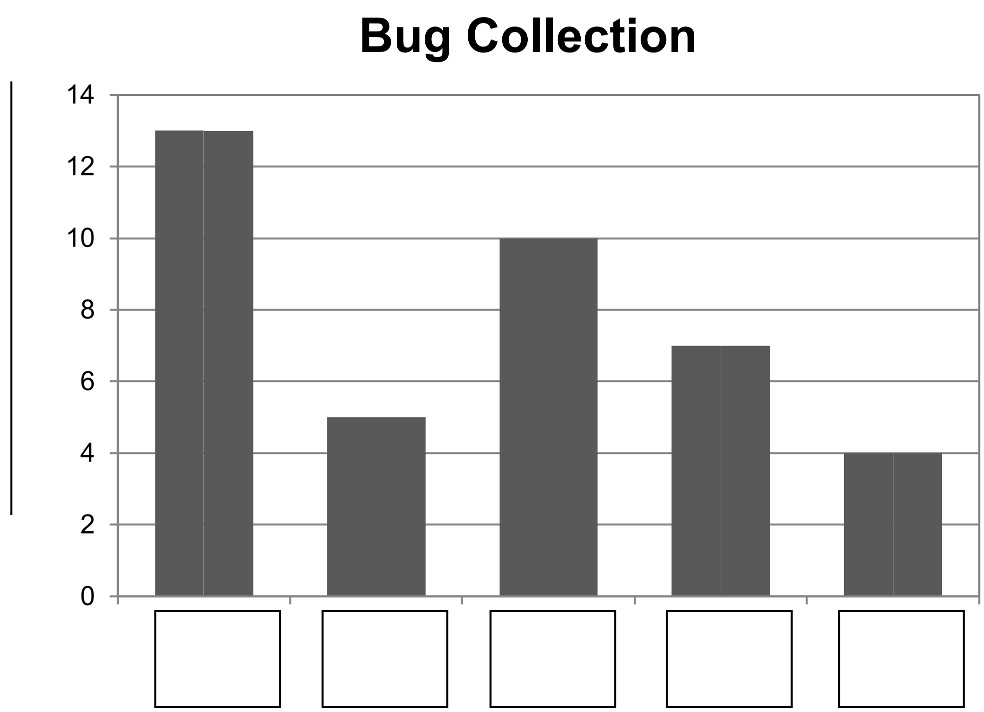
(Answers will vary.)

Category	Goal
Pants	*10*
Shirts	*20*
Skirts	*5*
Dresses	*2*
Socks	*15*

Name________________________________ Date:________________________

Crawly Creatures (Optional)

Read this carefully: Mr. Vito's class collected bugs to keep in their classroom. They collected ladybugs, flies, ants, caterpillars, and beetles. They made a graph of their collection.



Unfortunately, Mr. Vito forgot to label the graph. Here is what the students remember:

1. The flies were the hardest to catch.
2. They caught twice as many ladybugs as caterpillars.
3. They found more ants than any other bug.

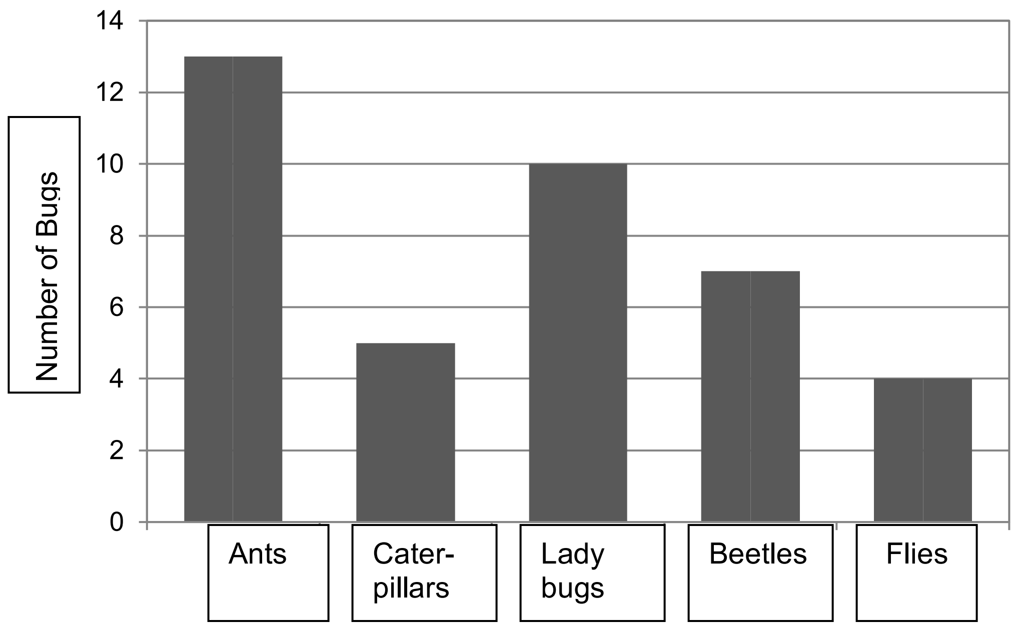
Place the bug name where it belongs on the graph and label the axes.

SMJ page 69

Crawly Creatures (Optional)
ANSWER KEY

Read this carefully: Mr. Vito's class collected bugs to keep in their classroom. They collected ladybugs, flies, ants, caterpillars, and beetles. They made a graph of their collection.

Bug Collection



Unfortunately, Mr. Vito forgot to label the graph. Here is what the students remember:

1. The flies were the hardest to catch.
2. They caught twice as many ladybugs as caterpillars.
3. They found more ants than any other bug.

Place the bug name where it belongs on the graph and label the axes.

Name: ________________________________ Date: _________________

Student Mathematicians Practice (Optional)

1. 427 - 20 =

2. 28
   <u>- 19</u>

3. 45 + 25 =

4. Who counted the most pencils?

Student	Pencils Counted
Jenny	12
Chan	28
Keisha	14
Scott	29

A. Jenny
B. Chan
C. Keisha
D. Scott

5. Which amount of money is greatest?

A. 3 quarters
B. 7 dimes
C. 16 nickels
D. 74 pennies

SMJ page 71

Student Mathematicians Practice (Optional)
ANSWER KEY

1. *407*
2. *9*
3. *70*
4. *D. Scott*
5. *C. 16 nickels*

LESSON 8

BAR GRAPHS—

PROFESSIONAL STYLE (OPTIONAL)

Big Mathematical Ideas

Presentation is everything. Technology has provided an easy way to make sharp-looking graphs to use on posters, in brochures, or in newspaper articles. Entering and displaying data are both important skills of the 21st century.

Lesson Objectives	• Students will enter data in a spreadsheet. • Students will create bar graphs using spreadsheet software. • Students will create pie graphs using spreadsheet software (extension activity).
Materials	• Student Page—*Flying Phenomena (Babbage & Galileo and Falconer)* (Optional) [SMJ pages 73 & 75] • Student Page—*Student Mathematicians Practice* (Optional) [SMJ page 77] • Computers with spreadsheet software
Mathematical Language	• **Spreadsheet:** A table used in computer programs to organize data in rows and columns.

Lesson Preview

Students use computer software programs to create professional-looking graphs.

Initiate

1. **Collect class data on a categorical variable**
   Students will need a data set to use when they create their graphs in spreadsheet software programs. Use one of the following choices to get a simple dataset. Ask the students to record the data on paper or directly into their spreadsheets with the category in column A and the number of people fitting that category in column B.

- Eye color—Brown, Green, Blue, Hazel, Other
- Hair color—Brown, Black, Blonde, Red, Other
- Socks—White, Black, Patterned, None, …

See Figure 1 for a sample of entering data into a spreadsheet.

	A	B
1	White	9
2	Black	4
3	Red	1
4	Blue	2
5	Patterned	3
6	None	3

Figure 1. Sample Spreadsheet Data Entry

Investigate

2. Creating graphs in spreadsheet software

Most spreadsheet software programs are fairly user-friendly. Use the following directions to help guide the creation of a bar graph:

- Highlight all boxes containing data. (In many programs, one box will appear to not be highlighted. This is normal.)
- Use the commands Insert → Chart → Column or Bar *OR* find the appropriate icon in the tool bar. (In spreadsheet software programs, "Column" graphs have vertical bars and "Bar" graphs have horizontal bars.) Students may use either option.
- Some variability exists in labeling options. Help students with the graph's layout, finding options for "Chart Title" and "Axis Title."
- Ask students to print their graphs (if possible) and summarize the information in the graphs. Summaries should include the following components:
    - What data was collected?
    - Which _________ was the most popular or frequent?
    - Which _________ was the least popular or infrequent?
    - How many students were included in the data for the graph? How can you use the graph to find this out?

3. Pie graphs—A different way to display

There may be an option in the software program that allows students to click on a different chart type. Students can quickly change from a bar to a pie graph and look at the same data in a different display format.

Help students to find the pie chart option. Look for an option allowing students to display the percentages. Use the following questions to help students understand the graph:

- Which ________ was the most popular or frequent according to the pie chart? How do you know? Is this the same answer as with the bar graph?
- How is the pie chart similar to the bar graph? How is it different?
    - *Students may notice that they can still tell which responses are most and least frequent with the pie graph. These do not change. The display is different and the numbers are often given as percentages rather than frequencies (though you can obtain either). The pie graph uses area to represent the information whereas the bar graph uses height (or length).*

Conclude

4. Personal preferences

Ask students to report on their personal preferences.

- Did you prefer the bar graph or the pie chart? Why?
- Do you prefer to draw the graphs by hand or use technology? Why?
- What is the advantage of creating a graph on the computer as opposed to by hand?
    - *The graphs are often neater and can be easily cut and pasted into writing, brochures, or printed to use on a poster.*

Look Ahead

5. Data from the drive

Students may want to use technology to create their final graphs from their clothing, book, or crayon drives. Begin thinking about possible products for students to showcase the graphs of the results of their drives: posters, brochures, newspaper articles.

Assess

6. Lesson conclusions

Collect student work for assessment and to display in the classroom. Alternatively, generate one more set of data using another option and ask students to try to repeat the steps for creating either a bar or pie graph. Students should also be asked to analyze their graphs either orally or in writing.

7. Optional student practice

Flying Phenomena Student Pages **[SMJ pages 73 & 75]** can be used for additional student practice with creating bar graphs. Teachers may choose to send it home as homework, use it as an optional activity when students finish their work, or assign it as morning work. Teacher instructions can be found in the teacher's manual on page 191.

If needed, the *Student Mathematicians Practice* Student Page **[SMJ page 77]** could also be used to assess number and operation concepts.

Name________________________________ Date:________________________

Flying Phenomena (Optional)

One summer camp counselor challenged all the campers to keep track of all the flying objects they saw. Your task is to create a graph of their findings. **Don't forget to label all axes and give the graph a title!**

Graph Title:

	Shooting Stars		UFOs		Hot Air Balloons		Kites		Airplanes

1. The campers reported 12 kite sightings.
2. They saw 4 fewer shooting stars than kites.
3. They observed twice as many airplanes as shooting stars.
4. Carlos and Eva were the only ones who thought they saw UFOs. Carlos thought his looked yellow, and Eva thought hers was more purple-ish.
5. The campers saw twice as many hot air balloons as UFOs.

SMJ page 73

Falconer

Name______________________________ Date:______________________

Flying Phenomena (Optional)

One summer camp counselor challenged all the campers to keep track of all the flying objects they saw. Your task is to create a graph of their findings. **Don't forget to label all axes and give the graph a title!**

Graph Title:

	Shooting Stars		UFOs		Hot Air Balloons		Kites		Airplanes

Read all the clues first and then create the graph. The campers observed twice as many airplanes as shooting stars. They saw 4 fewer shooting stars than kites. The campers reported 12 kite sightings. Carlos and Eva each observed a UFO. Carlos thought his looked yellow, and Eva thought hers was more purple-ish. For every 4 airplanes sighted, the students saw 1 hot air balloon.

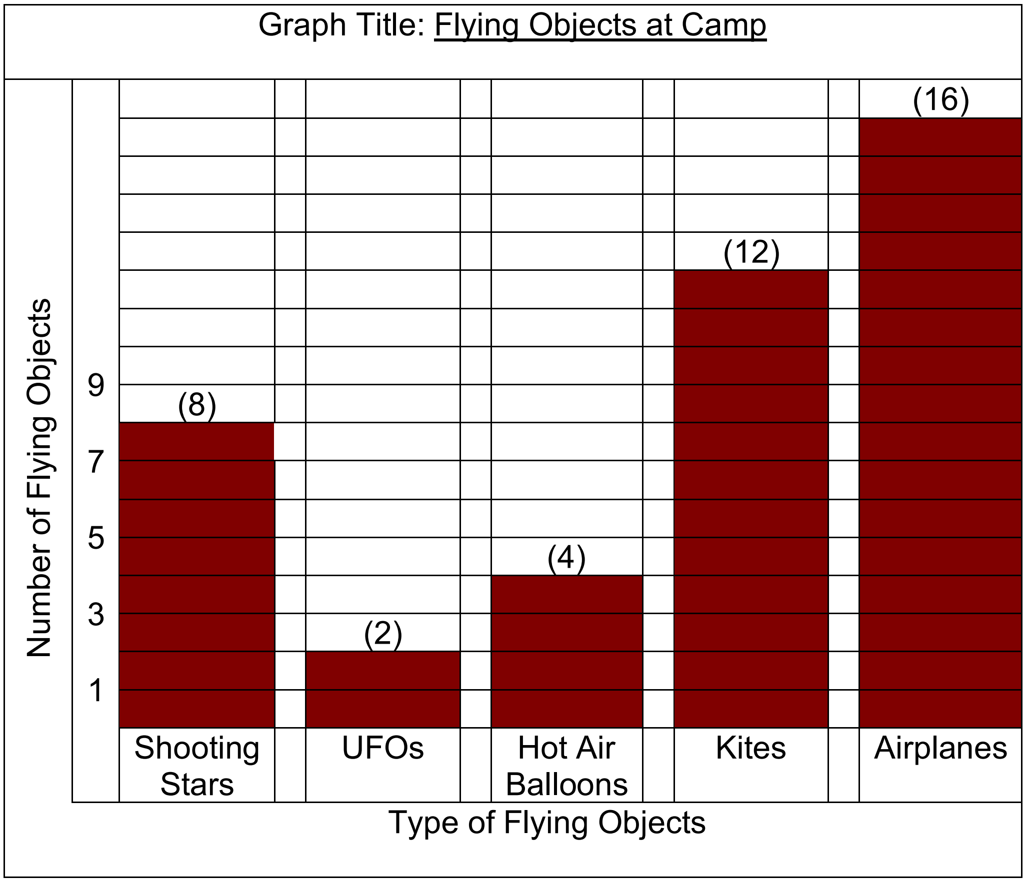
SMJ page 75

Flying Phenomena (Optional)
DISCUSSION GUIDE AND ANSWER KEY

Differentiation Options*: The Babbage/Galileo version has the clues in order that the students will need them. The Falconer version requires the students to sift through the clues to pull out the starting point. The Babbage/Galileo version requires students to understand "twice as many" and the Falconer version requires that and "for every 4 of these, there is 1 of this." Both versions, however, have the same answers.*

Discussion: *The mathematical language in this assignment is so important. Ask the students to explain what mathematical operation they used when the clue said "4 fewer shooting stars." What would the clue say to get you to add? What operation did you use for "twice as many?" Ask the Falconer group to explain the following "for every 4 airplanes sighted, there was 1 hot air balloon."*

Directions: *One summer camp counselor challenged all the campers to keep track of all the flying objects they saw. Your task is to create a graph of their findings.* ***Don't forget to label all axes and give the graph a title!***



Babbage/Galileo Clues

1. *The campers reported 12 kite sightings.*
2. *They saw 4 fewer shooting stars than kites.*
3. *They observed twice as many airplanes as shooting stars.*
4. *Carlos and Eva were the only ones who thought they saw UFOs. Carlos thought his looked yellow, and Eva thought hers was more purple-ish.*
5. *The campers saw twice as many hot air balloons as UFOs.*

Falconer Clues

The campers observed twice as many airplanes as shooting stars. They saw 4 fewer shooting stars than kites. The campers reported 12 kite sightings. Carlos and Eva each observed a UFO. Carlos thought his looked yellow, and Eva thought hers was more purple-ish. For every 4 airplanes sighted, the students saw 1 hot air balloon.

Name: ________________________________ Date: _________________

Student Mathematicians Practice (Optional)

1. 512 + 73 + 4 =

2. 625
   <u>- 207</u>

3. 4,376 - 2,062 =

4. The closest estimate of 712 + 424 is _____.

   A. 11
   B. 110
   C. 1,100
   D. 11,000

5. The closest estimate of 67¢ - 39¢ is _____.

   A. 10¢
   B. 20¢
   C. 30¢
   D. 40¢

6. Three children brought in pictures of trees. Bob brought 6, Michelle brought 12, and Kelly brought the rest. To find out how many pictures Kelly brought, what else do you need to know?

   A. The total number of children in Kelly's group
   B. The total number of trees in the pictures
   C. The total number of children who brought pictures
   D. The total number of pictures brought

SMJ page 77

Student Mathematicians Practice (Optional)
ANSWER KEY

1. *589*
2. *418*
3. *2,314*
4. *C. 1,100*
5. *C. 30¢*
6. *D. The total number of pictures brought*

BAR GRAPHS—

GRAPHING AND ANALYZING DATA

LESSON 9

(NOTE: This lesson should be done only if the data collection for the class drive is completed. Otherwise, do Lessons 10 & 11, then come back to this one.)

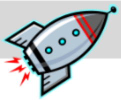
Big Mathematical Ideas

Being able to analyze a bar graph and to understand its nuances are important steps in becoming a good mathematician. To properly form conclusions about the data set, you must look at the categories, the scale, and the height of the bars.

Lesson Objectives	• Students will create bar graphs of the data from their book, clothing, or crayon drives. • Students will select an appropriate scale to display data. • Students will compare the data to the goals they had set.
Materials	• Student Page—*Create Your Own Adventure* Guide (Optional) [SMJ page 79] • Student Page—*Create Your Own Adventure* (Optional) [SMJ page 81] • Check Up #2 [SMJ page 83] • Graph paper • Rulers
Mathematical Language	• **Vertical Axis (*y*-axis):** The line on a graph that runs up and down. • **Horizontal Axis (*x*-axis):** The line on a graph that runs left to right.

Lesson Preview

Students create and compare bar graphs.



Initiate

1. Tallies into frequencies (This section does not apply to classrooms measuring their donations by weight.)

Ask students to turn to their data on the *Keeping Track: Data on* ________ Student Page from Lesson 7 **[SMJ page 63]**. If the number of items in each of the categories has not been tallied, it is time to get a final count. Ask students to use the tally marks to determine the frequency of the item for each category.

Explain that tallies are often used to count data because a mark can be made for each piece of data. They are easy to count because they are grouped in fives. If students have studied multiplication already, the following connection can be highlighted:

~~||||~~ ~~||||~~ ~~||||~~ ~~||||~~ II **=** (5 x 4) + 2 **=** (4 x 5) + 2 **=** 22

The numerical representation of the tally marks is the frequency, or the number of each item collected within a particular category.

Investigate

2. Graphing data by hand

Give students graph paper and rulers. Help students decide how to set up a graph to display their data. Some things to consider as a class as you review the group data include the following:

- How many categories? (This information can help you decide how wide to make your bars. For example, if there are only two or three categories, you might want to make each bar 3 units wide to use more paper. If there are 10 categories, then each bar might only be one unit wide.) Use the “hash mark” strategy to help students set up their horizontal axes with even spaced and equal bar widths.
- What is the maximum data value? Do we have room on our paper to count that high? If not, what scale can we use to make sure our bars fit? (Help students understand that scale in this case refers to the count on the vertical, or *y*-axis. Instead of counting each box as one on the vertical axis, they may have to count by 2s or 5s to fit all the data. Ask students to discuss their responses before sharing with the class. Acknowledge that there is more than one acceptable answer and allow students to debate which scale works best with the class’s data.)
- Once students have set up their graphs, ask them to create their bars and use colors to represent different categories. Students should include title, axes labels, and key (optional).

Optional Extension: Follow-up by asking students to create their bar graphs using the computer spreadsheet software program.

3. **Sharing our results**

There are several ways for students to analyze and share their graphs. If students have elected to give their items to charity, a newspaper article competition might be appropriate for disseminating the results. Consider making this an interdisciplinary assignment by moving some portions to another content area. A few options for analyzing the data and creating a product are below:

- A brochure on “How to Lend a Helping Hand”
    - Ask students to include a smaller version of the graph in a brochure (easier to do if the graph is electronic). Tell them to explain the results of their drive and give advice to others on how to help out.
- A poster showing others “How We Helped”
    - Students can create a poster with a large graph showing their results. The poster should include a description that includes what information can be gathered from looking at the graph.
- A newspaper article sharing the experience and results
    - This can be either a competition among students to submit an article on the results to the local paper or the class can take some ideas from different students’ work to create a class version. Again, the article should be accompanied by a graph and an explanation of what the graph says about the success of the drive.

Conclude

4. **Assess student understanding of bar graphs**

Give students a different scale to use in creating their bar graphs. Ask them to predict what would happen if they drew the bar graph using the new scale. Students can actually create the bar graph in class or at home to see if their predictions are correct. (Give students a scale that is twice as big to see the bar graphs cut by half in size!)

Look Ahead

5. **Changing graphical representations**

In the next several lessons, students will either return to line graphs or move on to line plots (depending on the intervention and drive selected).

Assess

6. **Lesson conclusions**

Collect students’ bar graphs and other products. Assess graphs and related products then display work throughout the classroom and school.

7. Optional student practice

Create Your Own Adventure Student Pages **[SMJ pages 79 & 81]** can be used for additional student practice with creating bar graphs. Teachers may choose to send it home as homework, use it as an optional activity when students finish their work, or assign it as morning work. Teacher instructions can be found in the teacher's manual on page 207.

Use Check Up #2 **[SMJ page 83]** to assess students' knowledge of bar graphs.

Name: ________________________________ Date: ________________

Create Your Own Adventure Guide (Optional)

You get to write your own worksheet! Your job is to give clues to help one of your classmates create the graph you planned.

1. Pick a topic. (You may want to use your favorite sport or activity. You could even use your favorite foods or toys.)

 ____

 ____

2. Think of 5 examples or pieces of your topic. (If you choose baseball, you could use bats, balls, cleats, mitts, caps.)

 ____

 ____

3. Record how many of each item you will use in your clues. Keep your numbers below 20. (You could use 5 bats, 4 balls, 6 cleats, 8 mitts, and 2 caps.) THIS IS YOUR ANSWER KEY!

 ____

 ____

4. Create your clues. Start by giving the actual number of one item. Then give the next item based on the first item. Keep going until you have five clues. (There were 5 bats. They had one less ball than bat…)

 ____

 ____

 ____

5. Record your story and clues on the worksheet.

SMJ page 79

I created this worksheet:________________________

I completed this worksheet:______________________

Create Your Own Adventure (Optional)

Story:____

____

Graph

Graph Title:

Clues

1. ____
2. ____
3. ____
4. ____
5. ____

SMJ page 81

Create Your Own Adventure (Optional)
DISCUSSION GUIDE AND ANSWER KEY

Implementation Directions: *If you are using this as homework, you may want to go over the worksheet as a class before the students take it home. The idea is to have the students create their own graph puzzle. Lessons 7 and 8 had homework that gave the students clues to help them label or create the graphs. In this homework, they will create their own clues. Then they will bring their clues to class and exchange with a partner in the same group. They will need a few minutes to work through each other's clues to create a graph and a few minutes for the creators' to check over the work.*

Differentiation Option: *Even though there is not a designated worksheet for the different groups, the students should exchange worksheets within their typical group. Galileo students should exchange with other Galileo students. This will hopefully allow students to work within their challenge level.*

If you want to make it extra challenging for some students, tell them they must use at least one clue that incorporates multiplication or division.

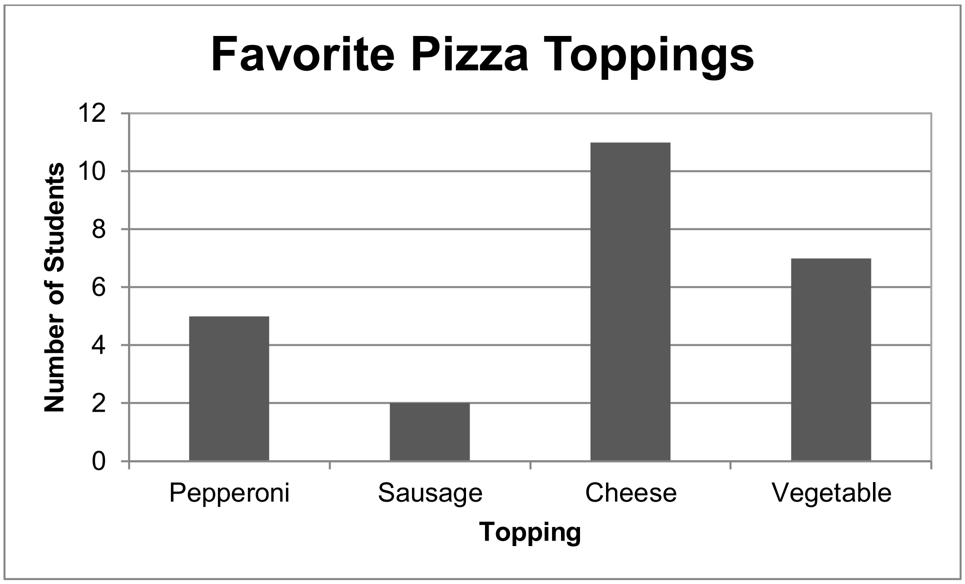
Discussion Questions*: What clues were particularly challenging? Did anyone think he/she gave a clear clue, but then when someone tried to solve it, he or she had problems? What words did you use to describe addition? Subtraction? Multiplication?*

Answers will vary for every student.

Name: ____________________________________ Date: ___________________

Check Up #2

Students were asked the question, "What is your favorite pizza topping?" They were asked to choose from pepperoni, sausage, cheese, or vegetable. A graph of the data is below.



1. What pizza topping is most popular in this class? Explain your thinking.

____

____

____

2. What is the least popular pizza topping? Explain your thinking.

____

____

____

SMJ page 83

3. Do more people in this class like pepperoni pizza or vegetable pizza? Explain your thinking.

____

____

____

____

____

4. How would the bar graph change if pepperoni and sausage were combined into a MEAT category?

____

____

____

____

____

5. Use the information from the bar graph to complete the table.

Pizza Topping	**Number of People (Frequency)**
Pepperoni	
Sausage	
Cheese	
Vegetable	

How many students in total answered the survey question? _________

SMJ page 85

6. 724
- 402

7. 312 + 34 + 5 =

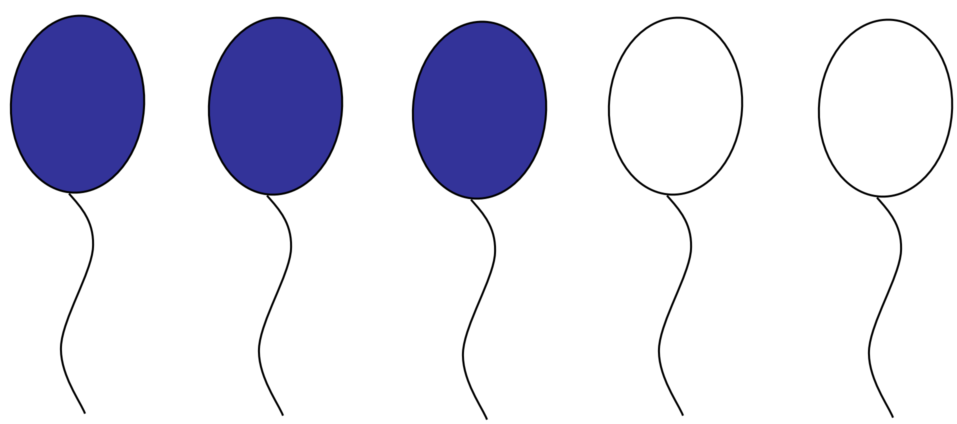
8. 417 - 10 =

9. 857
+ 36

10. 3,050 + 249 =

11. 60
-16

12. What fraction of balloons is shaded?



A. 1/2
B. 1/3
C. 2/5
D. 3/5

SMJ page 87

Check Up #2
ANSWER KEY

1. What pizza topping is most popular in this class? Explain your thinking.

   Cheese is the most popular pizza topping because it has the highest bar.

2. What is the least popular pizza topping? Explain your thinking.

   The least popular pizza topping is sausage. Only two people voted for this topping.

3. Do more people in this class like pepperoni pizza or vegetable pizza? Explain your thinking.

   Seven people like vegetable and only five like pepperoni. Vegetable is more popular.

4. How would the bar graph change if pepperoni and sausage were combined into a MEAT category?

   There would only be three bars and the MEAT category would go up to 7.

5. Use the information from the bar graph to complete the table.

Pizza Topping	**Number of People (Frequency)**
Pepperoni	*5*
Sausage	*2*
Cheese	*11*
Vegetable	*7*

How many students in total answered the survey question? *25*

6. *322*

7. *351*

8. *407*

9. *893*

10. *3,299*

11. *44*

12. What fraction of balloons is shaded?

    D. 3/5

LESSON 10

LINE GRAPHS (PART 2)—

LEARNING ABOUT LINE GRAPHS

Big Mathematical Ideas

Researchers have many ways of presenting data they collect. One way is by using a **line graph**, which plots changes in data over time. Researchers look at patterns in the lines to determine whether the intervention has worked or not.

Lesson Objectives	• Students will use a data table to complete a line graph.
Materials	• Student Page—*Navigating Number Lines* [SMJ pages 89] • Student Page—*Weather Data Across the United States (Babbage & Galileo and Falconer)* [SMJ pages 91-95 & 97-101] • Student Page—*Student Mathematicians Think Deeply* [SMJ page 103] • Student Page—*Bake Sale (Babbage, Galileo, and Falconer)* (Optional) [SMJ pages 105, 109, & 113] • Student Page—*Student Mathematicians Practice* (Optional) [SMJ page 119] • Rulers • Colored pencils or crayons
Mathematical Language	• **Line Graph:** A graph that shows how data changes over time.

Lesson Preview

Students continue collecting recycling data and entering it into their data tables. They learn about line graphs and practice constructing a line graph.

Initiate

1.

Pre-assess the concept of betweenness

Ask students to turn to the *Navigating Number Lines* Student Page **[SMJ page 89]**. Tell students that in the first question they will make a point for each number listed on the number line. Show students an example by drawing the number line on the board or chart paper and asking where you would put the number 32.

Students should understand that the number 32 is between 30 and 40. They should also be able to explain that it is closer to 30 than it is to 40.

Ask students to complete the points for the numbers in question 1. Informally assess students' levels of understanding by asking students to describe where they put the point for each number.

The second question asks students to plot points on a grid. Ask students to use the temperature data in question 2 to make points on the graph. Circulate the room and help students as needed. Students can connect their points to look at increasing and/or decreasing trends. Again, informally assess students' levels of understanding of the task by asking them to describe the location of each point.

A sample conversation might look like this:

Teacher:	Could somebody describe where they put the point for the temperature at 4 p.m.?
Delia:	I found 4 p.m. on the bottom and I moved my finger up until I got almost to the top.
Teacher:	Why did you go almost to the top?
Delia:	Well, I saw that the top line was seventy. At four o'clock it's 69° which is almost 70. So, I made my point right below the line for 70.
Teacher:	Can somebody rephrase what Delia said?

Students who demonstrated sophisticated understanding of number placement in these two tasks should be given the *Weather Data Across the United States* Student Pages (Falconer) **[SMJ pages 97-101]** for the next task. Students who require additional support in this area can work on the *Weather Data Across the United States* Student Pages (Babbage & Galileo) **[SMJ pages 91-95]**.

Investigate

2.

Weather data across the United States

Tell students that today they are going to create **line graphs**. Ask students to look at the graph from the first *Navigating the Number Line* Student Page **[SMJ page 89]**. If they have not already connected the points, ask them to connect them now. Tell students that a line graph allows us to look at trends over time, or see how something changes. Use the following questions to guide a discussion:

- What happens to the temperature from 8 a.m. to noon? *It increases or goes up.*
- How can you tell by looking at the graph whether the temperature is going up or down? *The graph increases when the temperature is going up and decreases when the temperature is going down. (Provide a visual or kinesthetic representation; for example, ask students to move a hand as though they are drawing a graph where the temperature is going up.)*

Determine whether students will continue with the *Weather Data Across the United States* Student Pages (Babbage & Galileo) **[SMJ pages 91-95]** and (Falconer) **[SMJ pages 97-101]**. Tell all students they will be graphing temperature data for different places in the United States. Remind students that they are making a point on the graph for each temperature (not bar graphs). Before allowing students to begin the task, the following concepts should be reviewed or introduced:

- Temperatures in the United States are typically measured and written in degrees Fahrenheit. Give an example, such as, 35^{o}F is read "thirty-five degrees Fahrenheit."
- Explain that average daily temperature by month means it may be 58^{o}F one part of the month and 62^{o}F another part of the month. Ask students what they think the average for that month might be. You can also tell them that when a month has an average of 60^{o}F, it might be 58^{o}F one day, 59^{o}F another day, and 63^{o}F another day. 60^{o}F is a fair representation of all of the data for the whole month.

Give each student or group of students a set of colored pencils or crayons. You might suggest that students draw each graph in a regular pencil first then trace in color in case they make errors. If students require additional scaffolding to begin their graphs, help them get the first few points either individually, in small groups, or as a whole class. Since both mathematician groups are graphing the data for Phoenix, AZ, demonstrating the first three points for this graph would be helpful. Remind students to switch colors when they graph each new location.

Conclude

3. Making generalizations

Although students worked on different graphs and questions, the same general trends should emerge. Use the following questions to guide a discussion of these trends:

- When do the temperatures typically increase? Why?

The temperatures are typically increasing from February until July (this may vary slightly depending on the graph) where the graph is increasing. This is because the U.S. tends to be hotter in the summer months.

- Explain how you can tell by looking at the graph that the temperature is decreasing.
  You can tell that the average temperature is decreasing because the line on the graph is going downward also.

4. **Geography connection (Optional)**
Allow students to look up the average daily temperatures by month for a place that has opposite trends (for example, trends for places south of the equator). Students can prepare reports (including graphs) to discuss why places on opposite sides of the equator differ in their temperature trends throughout the year. The website <http://www.weatherbase.com> is an excellent resource for this type of activity.

Assess

5. **Lesson conclusions**
The *Student Mathematicians Think Deeply* Student Page **[SMJ page 103]** can be used to assess understanding of trends in line graphs.

6. **Optional student practice**
Bake Sale Student Pages **[SMJ pages 105, 109, & 113]** can be used for additional student practice with creating line graphs. Teachers may choose to send it home as homework, use it as an optional activity when students finish their work, or assign it as morning work. This activity is differentiated and further teacher instructions can be found in the teacher's manual on page 259.

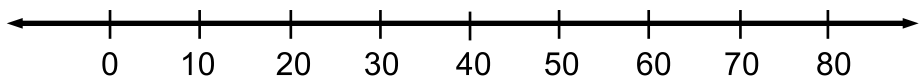
Also there is a separate number and operation practice page (*Student Mathematicians Practice* Student Page **[SMJ page 119]**) that can be used if needed.

Name: ___________________________________ Date: __________________

Navigating Number Lines

1. Make a point on the number line below for each of the following numbers:

   15, 48, 35, 60, 76, 3



Explain how you decided where to put the point for the number 76.

____

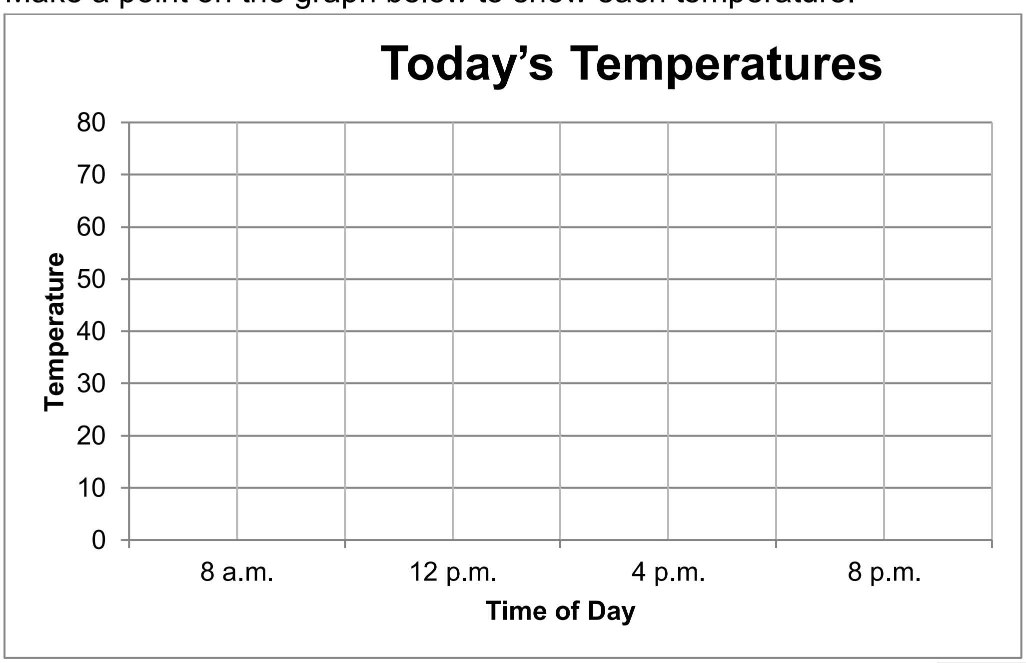
____

____

2. Shelly kept track of the temperature at different times of the day. She recorded her data in a table.

8 a.m.	12 p.m.	4 p.m.	8 p.m.
56°F	65°F	69°F	61°F

Make a point on the graph below to show each temperature.

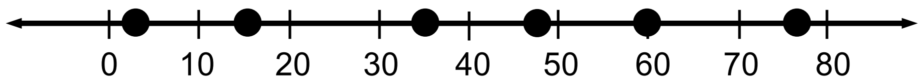


SMJ page 89

Navigating Number Lines
ANSWER KEY

1. Make a point on the number line below for each of the following numbers:

   15, 48, 35, 60, 76, 3 *(The answer key uses lines.)*



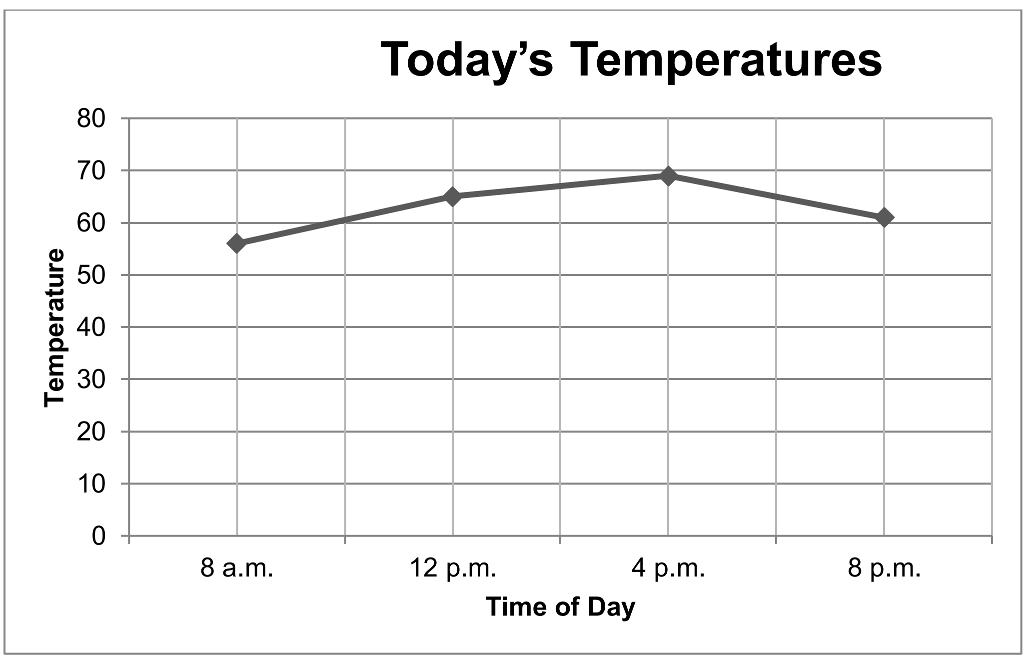
Explain how you decided where to put the point for the number 76.

Answers will vary. Students may explain how they thought the midpoint between 70 and 80 is 75 and 76 is just a little closer to 80.

2. Shelly kept track of the temperature at different times of the day. She recorded her data in a table.

8 a.m.	12 p.m.	4 p.m.	8 p.m.
56ºF	65ºF	69ºF	61ºF

Make a point on the graph below to show each temperature.



Babbage & Galileo

Name: ________________________________ Date: _________________

Weather Data Across the United States

Phoenix, AZ - Average Daily Temperature by Month (^oF)											
Jan.	**Feb.**	**Mar.**	**Apr.**	**May**	**Jun.**	**Jul.**	**Aug.**	**Sep.**	**Oct.**	**Nov.**	**Dec.**
54	**58**	**62**	**70**	**79**	**88**	**93**	**91**	**86**	**75**	**62**	**54**

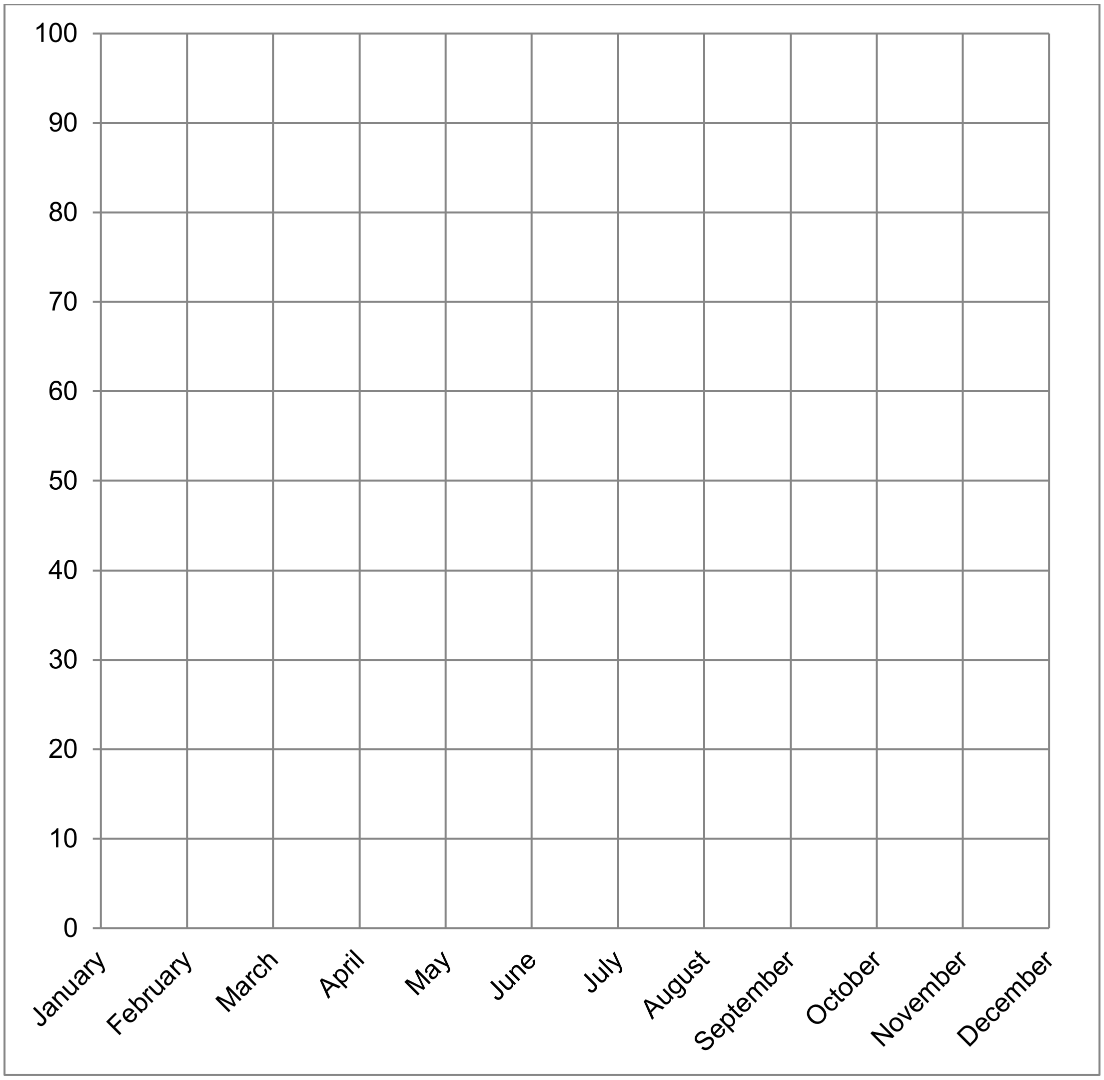
Washington, DC - Average Daily Temperature by Month (^oF)											
Jan.	**Feb.**	**Mar.**	**Apr.**	**May**	**Jun.**	**Jul.**	**Aug.**	**Sep.**	**Oct.**	**Nov.**	**Dec.**
36	**38**	**46**	**57**	**66**	**75**	**79**	**78**	**71**	**60**	**49**	**39**

Retrieved June 17, 2009 from http://www.weatherbase.com

SMJ page 91

Babbage & Galileo

Title: ____



COLOR KEY
Phoenix, AZ
Washington, DC

Directions:
1. Select two different colored pencils or crayons.
2. Graph the data for Phoenix, AZ by making a point for each month. Connect the points.
3. Graph the data for Washington, DC in a different color.
4. Complete the COLOR KEY by making a colored mark next to each place to match the color on the graph.

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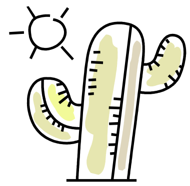
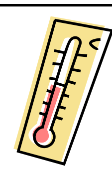
Babbage & Galileo

Questions

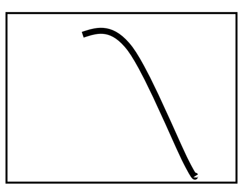



1. Which place is warmer, Phoenix, AZ or Washington, DC? _______________

   Explain how you can tell by looking at your graph.

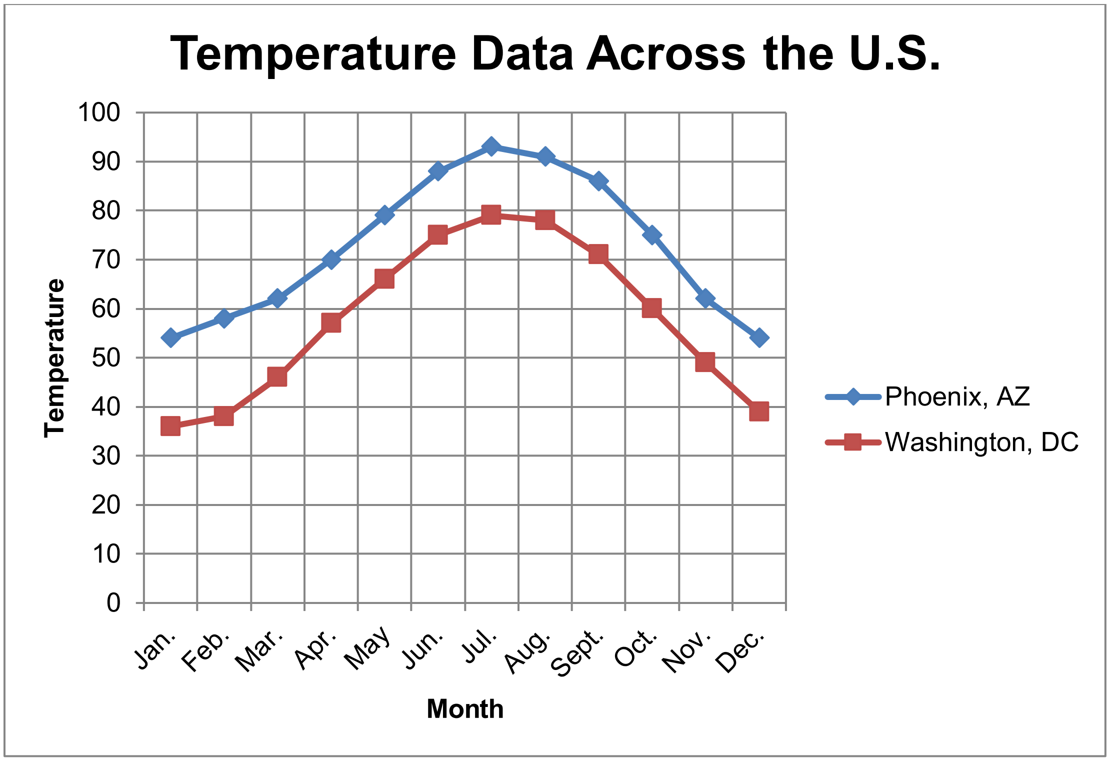


2. If you saw part of a temperature graph like the picture below, would you think it was getting WARMER or COLDER? Explain your thinking.



SMJ page 95

Weather Data Across the United States
Answer Key



Questions

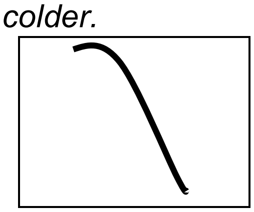
1. Which place is warmer, Phoenix, AZ or Washington, DC? *Phoenix, AZ*

   Explain how you can tell by looking at your graph.

   You can tell that Phoenix is warmer because I made that graph in orange and the Washington, DC graph in blue. The orange line is always above the blue line for the whole year.

2. If you saw part of a temperature graph like the picture below, would you think it was getting WARMER or COLDER? Explain your thinking.

   I would think it was getting colder. The graph goes downward. On my graphs, when the graph for Phoenix looked like that, it was getting colder.



Falconer

Name: ________________________________ Date: _________________

Weather Data Across the United States

Orlando, FL - Average Daily Temperature by Month (°F)											
Jan.	**Feb.**	**Mar.**	**Apr.**	**May**	**Jun.**	**Jul.**	**Aug.**	**Sep.**	**Oct.**	**Nov.**	**Dec.**
61	62	67	72	78	81	83	83	81	75	67	62

Phoenix, AZ - Average Daily Temperature by Month (°F)											
Jan.	**Feb.**	**Mar.**	**Apr.**	**May**	**Jun.**	**Jul.**	**Aug.**	**Sep.**	**Oct.**	**Nov.**	**Dec.**
54	58	62	70	79	88	93	91	86	75	62	54

Washington, DC - Average Daily Temperature by Month (°F)											
Jan.	**Feb.**	**Mar.**	**Apr.**	**May**	**Jun.**	**Jul.**	**Aug.**	**Sep.**	**Oct.**	**Nov.**	**Dec.**
36	38	46	57	66	75	79	78	71	60	49	39

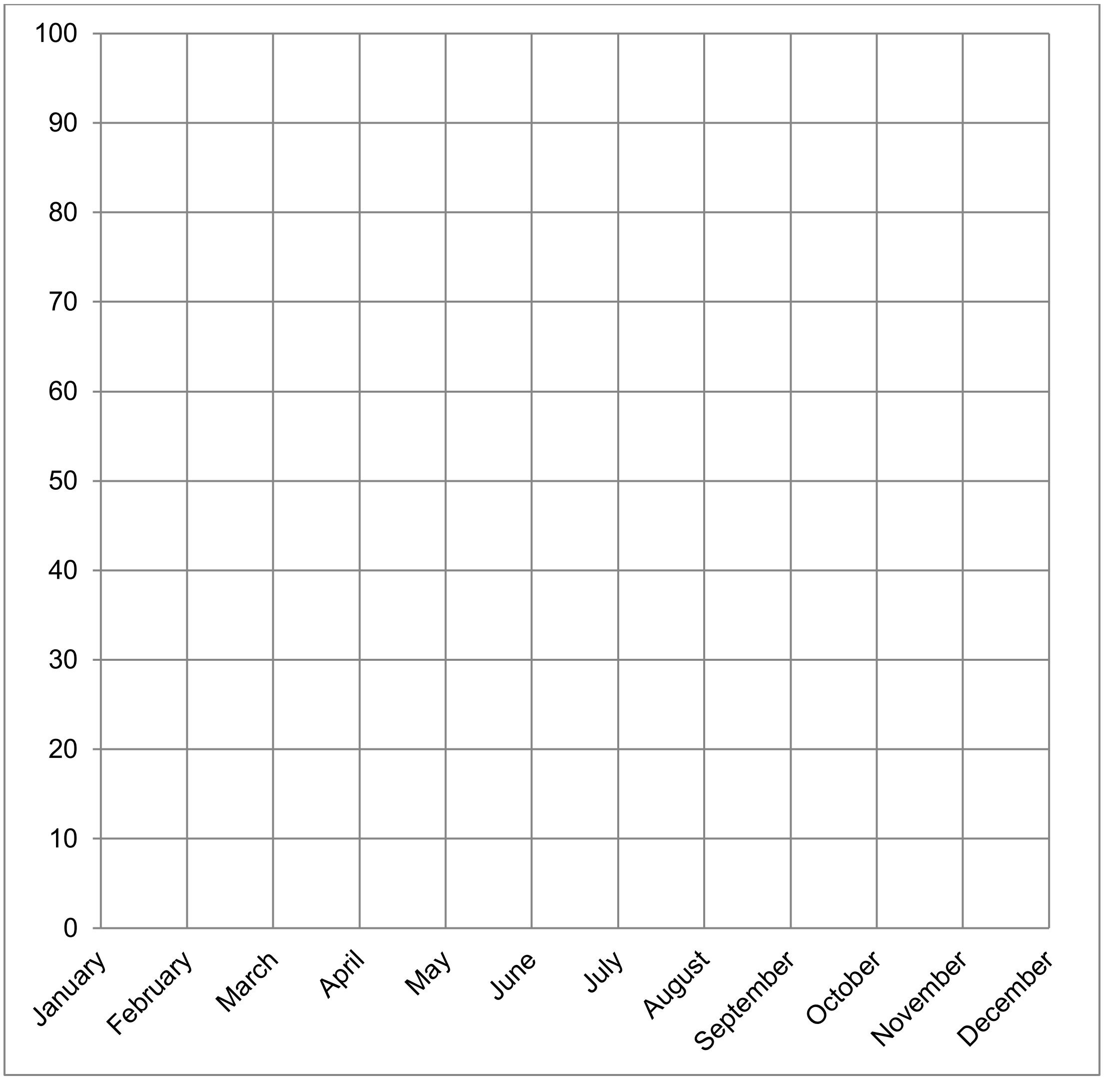
Storrs, CT - Average Daily Temperature by Month (°F)											
Jan.	**Feb.**	**Mar.**	**Apr.**	**May**	**Jun.**	**Jul.**	**Aug.**	**Sep.**	**Oct.**	**Nov.**	**Dec.**
26	26	33	44	55	64	69	68	60	50	39	30

Retrieved June 17, 2009 from http://www.weatherbase.com

SMJ page 97

Falconer

Title: ____



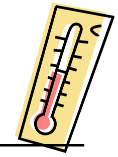
COLOR KEY
Orlando, FL
Phoenix, AZ
Washington, DC
Storrs, CT

Directions:
1. Select four different colored pencils or crayons.
2. Graph the data for Orlando, FL by making a point for each month. Connect the points.
3. Graph the data for the other three places, using a different color for each.
4. Complete the COLOR KEY by making a colored mark next to each place to match the color on the graph.

SMJ page 99

Questions

Falconer



1. Describe the graph for Storrs, CT. What does the shape tell you?

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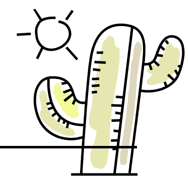
2. What is the MAXIMUM temperature for Phoenix, AZ?____________________



   How can you tell where the maximum is by looking at your graph?

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3. How can you tell by looking at a graph that it is getting colder?

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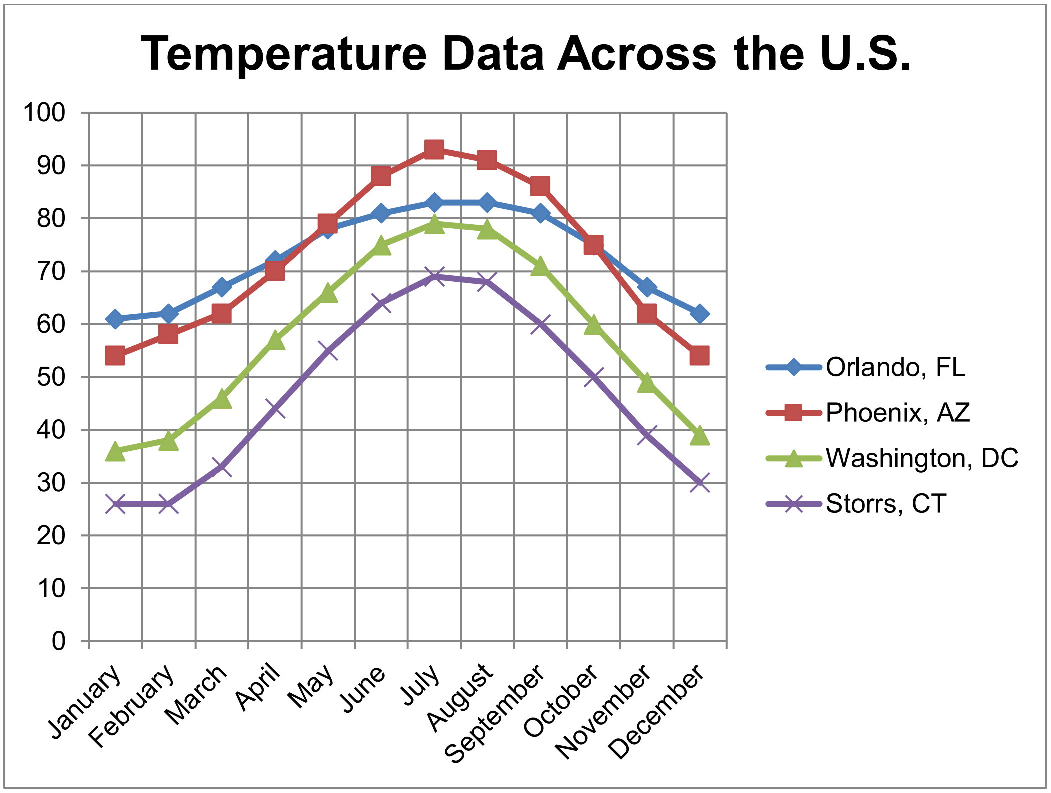
SMJ page 101

Falconer

Weather Data Across the United States
Answer Key

The Falconer Student Pages should have all four of the lines on the graph pictured.



Questions

1. Describe the graph for Storrs, CT. What does the shape tell you?

   Sample Response: The graph for Storrs, CT starts at 26ºF. It climbs upward then starts going back down again after July. The shape tells you that the temperature goes up from January until July then back down from July to December.

2. What is the MAXIMUM temperature for Phoenix, Arizona? *93ºF*

   How can you tell where the maximum is by looking at your graph?

   The maximum on the graph is the highest point. If you put your finger on the highest point and go straight down, you can find out that the highest temperature is in July.

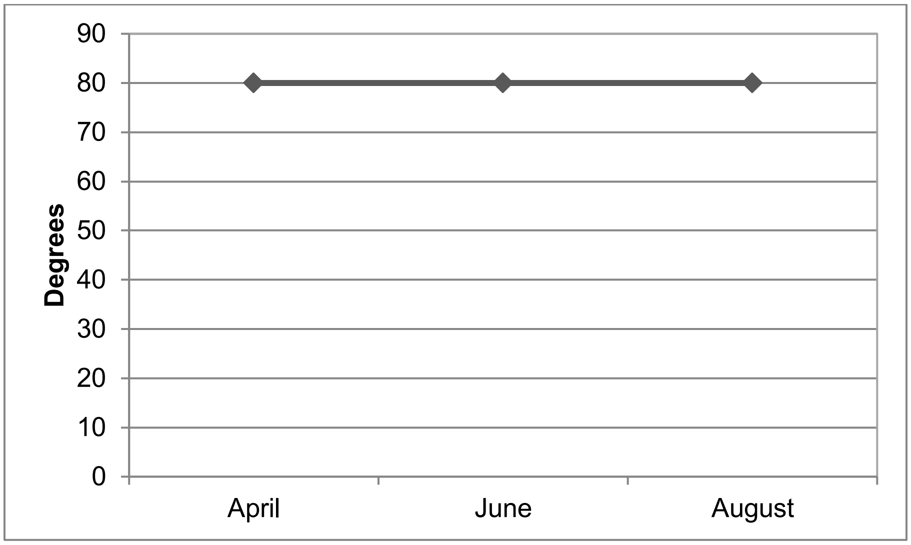
3. How can you tell by looking at a graph that it is getting colder?

   You can tell that it is getting colder because the graph decreases.

Name: ___________________________________ Date: __________________

Student Mathematicians Think Deeply

Average Monthly Temperature for Mathematicians' Island



1. Explain why the temperature graph for Mathematicians' Island is a straight, horizontal line.

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2. If the average monthly temperature on Mathematicians' Island is 75ºF in May and 85ºF in July, what would the graph look like from April to August? (You can make these two points on the graph above to help you.)

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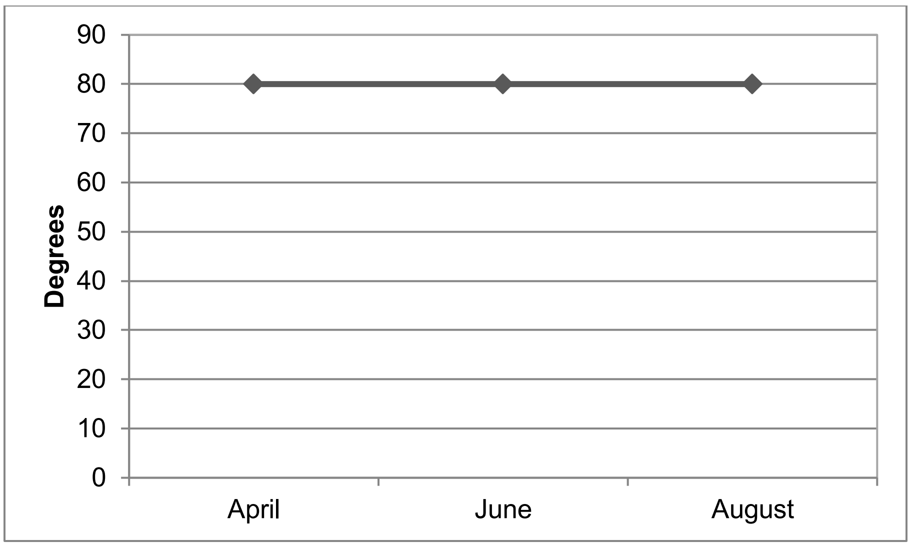
SMJ page 103

Student Mathematicians Think Deeply
ANSWER KEY

Average Monthly Temperature for Mathematicians' Island

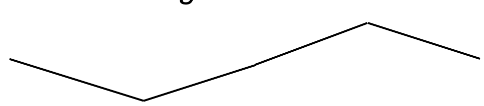


1. Explain why the temperature graph for Mathematicians' Island is a straight, horizontal line.

   The temperature graph is a straight, horizontal line because the temperature on Mathematicians' Island is 80º in April, June, and August. Since it doesn't change, the line does not go up or down.

2. If the average monthly temperature on Mathematicians' Island is 75ºF in May and 85ºF in July, what would the graph look like from April to August? (You can make these two points on the graph above to help you.)

   It would look something like this:



Babbage

Name________________________________ Date:________________________

Bake Sale (Optional)

The Cool School held a Bake Sale to raise money for their field trip.
Here is their cookie data:

Days	Number of Cookies Purchased
Monday	5
Tuesday	20
Wednesday	15
Thursday	10
Friday	25

Create a line graph with the cookie data. Don't forget a title!

Title ____

Number of Items Sold

	Monday	Tuesday	Wednesday	Thursday	Friday
	Days of the Week				

SMJ page 105

Babbage

Answer the following questions using the data from the graph. If needed, show your work under the question.

1. On what day did they sell the most cookies? ______________________

2. How many more cookies did the sell on Friday than on Tuesday?

____

3. Cookies cost $1 a piece. How much money did the students raise on Monday?

____

4. How much did they raise for the entire week? ______________________

SMJ page 107

Galileo

Name________________________________ Date:________________________

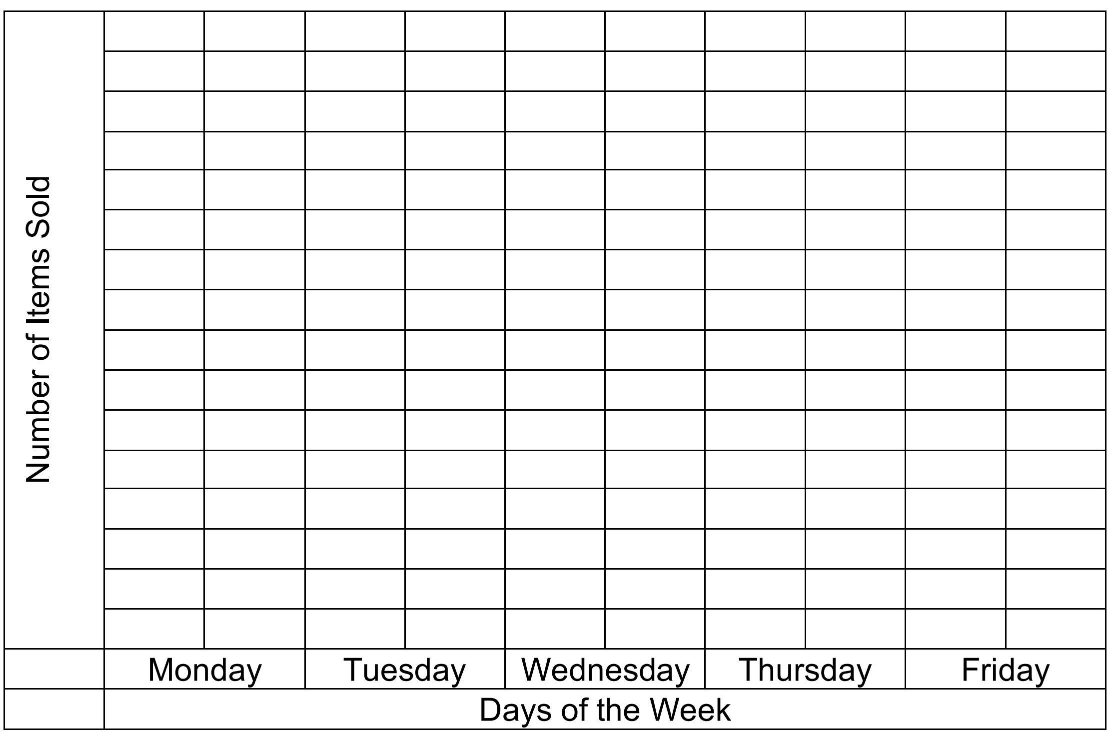
Bake Sale (Optional)

The Cool School held a Bake Sale to raise money for their field trip.
Here is their brownie data:

Days	Number of Brownies Purchased
Monday	5
Tuesday	20
Wednesday	12
Thursday	14
Friday	32

Create a line graph with the brownie data. Don't forget a title!

Title ____



SMJ page 109

Galileo

Answer the following questions using the data from the graph. If needed, show your work under the question.

1. On what day did they sell the most brownies? ______________________

2. Brownies cost $0.50 a piece, how much money did the students raise on Friday?

   ____

3. How much did they raise for the entire week? ______________________

4. How much more would they have made if they would have charged $1 for each brownie and the same amount of people purchased brownies?

   ____

SMJ page 111

Falconer

Name________________________________ Date:________________________

Bake Sale (Optional)

The Cool School held a Bake Sale to raise money for their field trip.
Here is their cupcake data:

Day	Number of Cupcakes Purchased
Monday	4
Tuesday	11
Wednesday	12
Thursday	18
Friday	5

Create a line graph with the cupcake data. Don't forget a title!

Title ____

Number of Items Sold										
	Monday		Tuesday		Wednesday		Thursday		Friday	
	Days of the Week									

SMJ page 113

Falconer

Answer the following questions using the data from the graph. If needed, show your work under the question.

1. On what day did they sell the most cupcakes? ______________________

2. Cupcakes cost $0.75 a piece, how much money did the students raise on Thursday? (Hint: Calculate how much 4 cupcakes cost and use that to help you.)

   ____

3. How much did they raise for the entire week? ______________________

4. How much more would they have made if they would have charged $1.75 for each cupcake and the same amount of people purchased cupcakes?

   ____

5. Why do you think they did not charge $1.75 for each cupcake? Should they have charged $1.75?

   ____

   ____

   ____

SMJ page 115

Bake Sale Group Questions (Optional)

1. Did the students reach their goal of raising $100 for their field trip? (Use the totals from the brownies, cookies, and cupcakes.)

2. If the students could only have a bake sale for one day, which day of the week should they choose?

3. Which baked good raised the most money? Is that also the best seller?

SMJ page 117

Bake Sale (Optional)
DISCUSSION GUIDE AND ANSWER KEY

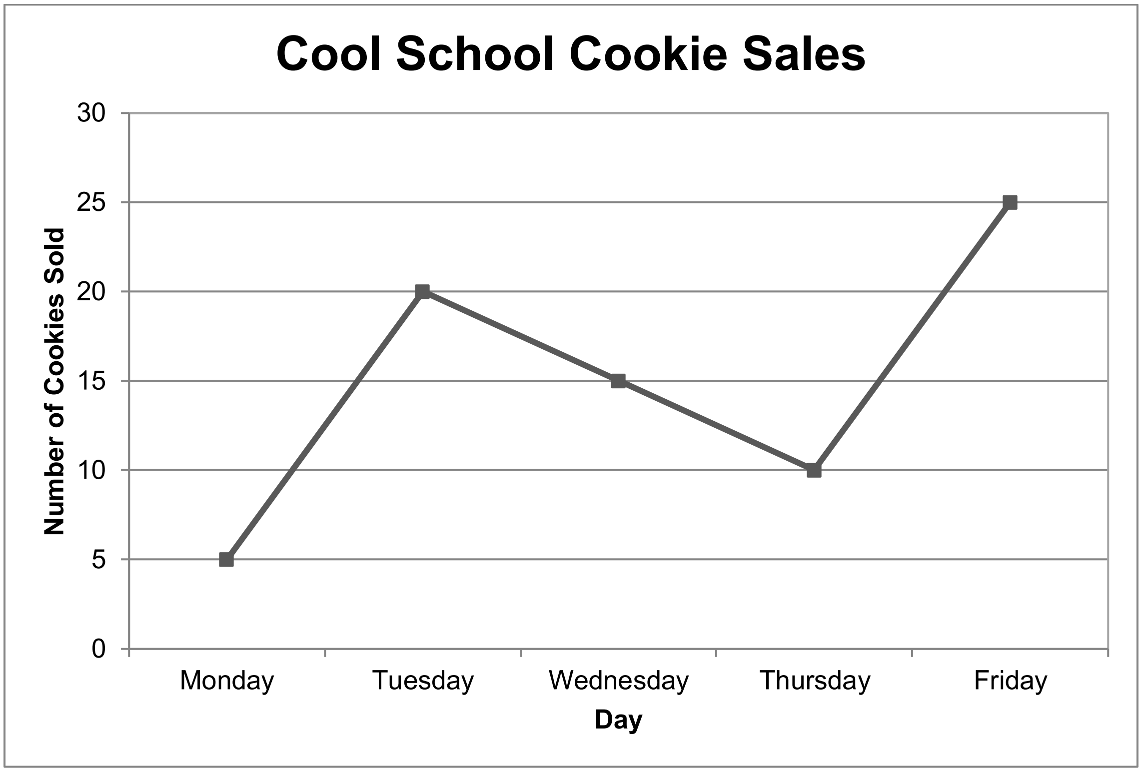
Implementation Directions: *Each student can work independently, and then offer something unique to the group. Place each student in the appropriate group when handing out the assignment. When students finish their assignment, instruct the students to get into new groups. Their new groups must have at least one person from the Babbage group, one from the Galileo group, and one from the Falconer group. The students then work together to complete group questions using the data from all three of their previous worksheets.*

Differentiation Options*: This assignment works best when equal numbers of students complete each option. The Babbage students have all multiples of 5 to graph, and then the cost of their cookies is $1.00 for easier computation. The Galileo students have a range of data to graph and they will probably need to estimate where to place the point. Their brownies cost $0.50 for a little more challenge. The Falconer group will also need to estimate on their graph and their cupcakes cost $0.75. They are also asked to think about the effect of the cupcake cost on the quantity of cupcake sales.*

Discussion Questions: *After the students have a chance to work with their mixed groups, discuss their results. Did they reach their goal? How did you figure this out? How much extra did they make? How did you work together to figure out which day they should choose? Is there another way you could have figured that out? What if the cupcakes were gourmet and cost $5.00 a piece? Would that affect which day you would have picked? Why? How did your group come to a consensus on the best seller?*

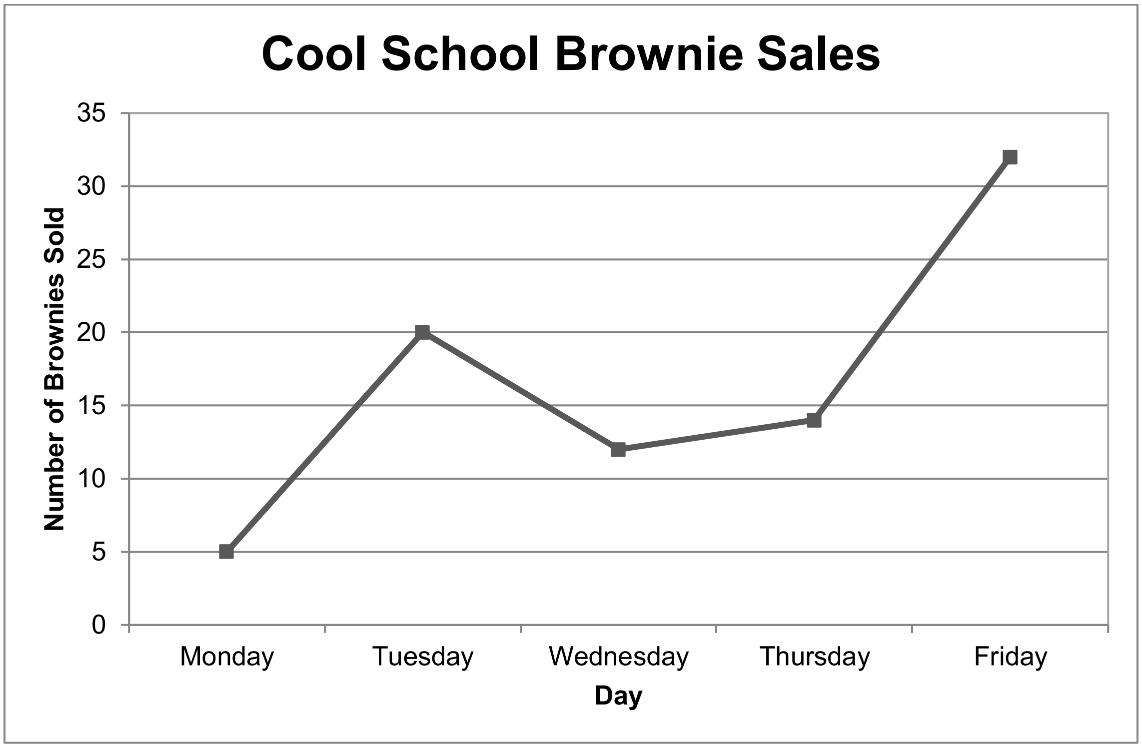
You may also want the Falconer group to share answers to question 5 and have a discussion about pricing. What would have happened if we raised the prices? What is too much to ask people to pay?

Babbage Key



1. On what day did they sell the most cookies? *Friday*
2. How many more cookies did the sell on Friday than on Tuesday? *5*
3. Cookies cost $1 a piece. How much money did the students raise on Monday? *$5*
4. How much did they raise for the entire week? *$75*

Galileo Key



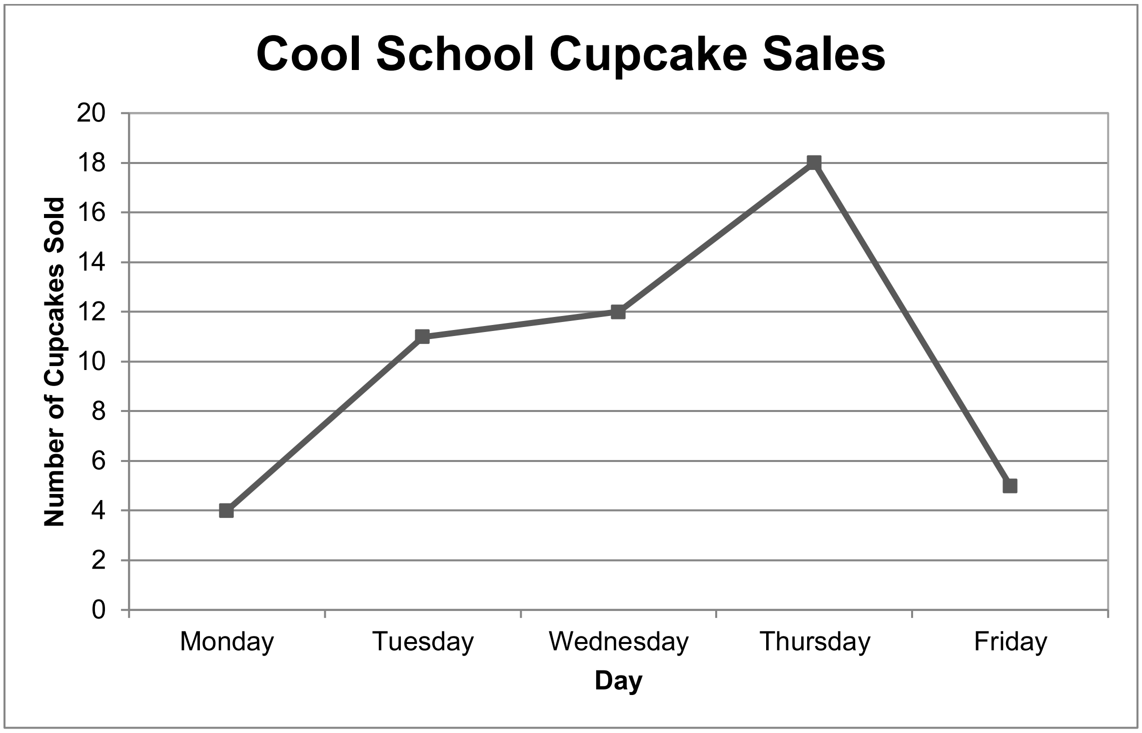
1. On what day did they sell the most brownies? *Friday*

2. Brownies cost $0.50 a piece, how much money did the students raise on Friday? *$16*

3. How much did they raise for the entire week? *$41.50*

4. How much more would they have made if they would have charged $1 for each brownie and the same amount of people purchased brownies? *$41.50 more*

Falconer Key



1. On what day did they sell the most cupcakes? *Thursday*

2. Cupcakes cost $0.75 a piece, how much money did the students raise on Thursday? (Hint: Calculate how much 4 cupcakes cost and use that to help you.) *$13.50*

3. How much did they raise for the entire week? *$37.50*

4. How much more would they have made if they would have charged $1.75 for each cupcake and the same amount of people purchased brownies? *$50 more*

5. Why do you think they did not charge $1.75 for each cupcake? Should they have charged $1.75?

   Answers will vary. This is a sample: More people may not have bought them because they cost so much money. I don't think they should have charged that much because

Bake Sale Group Questions (Optional)

1. Did the students reach their goal of raising $100 for their field trip? (Use the totals from the brownies, cookies, and cupcakes.)

   Yes. They raised $154. They raised $54 more than their goal.

2. If the students could only have a bake sale for one day, which day of the week should they choose?

   Answers will vary. Both the Babbage and Galileo group will believe Friday is the best day and Falconer will believe Thursday is the best day. They may do a calculation of how much total money is made on Friday compared to Thursday. They also may just say 2 out of 3 believe Friday so Friday is the best day.

3. Which baked good raised the most money? Is that also the best seller?

   The cookies raised the most money ($75), but the brownies were the best seller (83 sold).

Name: ________________________________ Date: _________________

Student Mathematicians Practice (Optional)

1. Which number has the digit 9 in the thousands' place?

   A. 34,396 B. 58,942

   C. 69,248 D. 95,561

2. James noticed that his magazine was missing pages numbered 138 through 156. How many of the missing pages end with a 2?

   A. 1 B. 2

   C. 4 D. 18

3. Connor's class is learning about different kinds of trees. So far, they have studied 4 out of 15 different kinds. How many more kinds do they still have to study?

   A. 1 B. 10

   C. 11 D. 19

4. Macy read that the largest full-grown tree is about 43 feet tall and the smallest is about 4 feet tall. To find out how many feet taller the largest tree is than the smallest, Macy could do which of the following?

   A. multiply 43 by 4 B. divide 43 by 4

   C. add 4 and 43 D. subtract 4 from 43

5. 4,794 - 32 =

6. 512 + 74 + 3 =

7. $\begin{array}{r} 90 \\ \underline{-\ 18} \end{array}$

SMJ page 119

Student Mathematicians Practice (Optional)
ANSWER KEY

1. *C. 69,248*
2. *B. 2*
3. *C. 11*
4. *D. subtract 4 from 43*
5. *4,762*
6. *589*
7. *72*

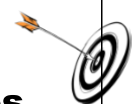
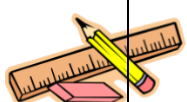
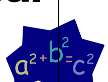
LINE GRAPHS (PART 2)—

EVALUATING THE INTERVENTION

LESSON 11

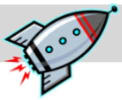
Big Mathematical Ideas

Researchers often collect data in a data table. They then organize and dis the data in a graph, looking for patterns.

Lesson Objectives 	• Students will use a data table to complete their recycling line graph.
Materials 	• Student Page—*Recycling Rules! Part 1* and *Part 2* [SMJ pages 13 & 25] • Student Page—*Did It Work?* [SMJ pages 121] • Student Page— *Student Mathematicians Think Back: Analyzing Our Graph* [SMJ page 123] • Student Page—*Understanding Line Graphs* [SMJ page 125] • Student Page—*A Gnome's Wish* (Optional) [SMJ page 127] • Student Page—*Exciting Exercise* (Optional) [SMJ page 129] • Student Page—*Student Mathematicians Practice* (Optional) [SMJ page 133] • Rulers—1 per student
Mathematical Language 	• **Conclusion:** A supported answer to a question in an experiment

Lesson Preview

Students collect their final recycling data and enter it into their Data Table. They plot the data as a line graph.



Initiate

1. Visual analysis of data

Students should now have collected and recorded recycling data for 5 to 8 days. Direct students to the *Recycling Rules! Part 1* and *Part 2* Student Pages **[SMJ pages 13 & 25]**.

Ask:

- Do you think we were successful in increasing recycling at our location? Why or why not?
  Answers will vary. Discuss with students whether the numbers in the data table went up, down, or varied each day.

Help students understand what the graph of the data will look like prior to plotting. If the data starts low and increases, ask them to put their hand low and move with the data. For example, you might say, “Our data starts down here at 12 items then it increases (*move hand upward to the right to show an increase)* then it goes down just a bit (*move hand downward and to the right*).” This activity will give students a sense of what their graphs are supposed to look like.

Investigate

2. Setting up a graph

Tell students that they will be plotting the data from the data table as a line graph. The data they have collected should be on the *Recycling Rules Part 2* Student Page **[SMJ page 25]** from Lesson 3. Direct students to the *Did It Work?* Student Page **[SMJ page 121]**. The horizontal axis has been pre-labeled on the Student Page. Help students create a line graph of the data they have collected using the following questions and steps:

- Which part of the graph has been labeled for us? What does this represent? (Tell students that mathematicians sometimes call this the *x*-axis.)
  The horizontal axis (across the bottom) has been labeled with the days. Each day represents a day of data collection similar to the information in the table.
- What information goes on the other axis (that mathematicians often call the *y*-axis)? (Point to the vertical axis for students to see and have students label this axis “Number of Items” or another label that makes sense with the selected intervention.)
  The number of items recycled goes on this axis.
- How high should we make the numbers on the vertical axis? What should we count by to get there? (Discuss the maximum data value with students.)
  Answers may vary considerably. If there are less than 10 items each day, then counting by 1s is appropriate. If there are close to 100 items on any given day, then students may need to count by 10s. This depends on the maximum data value.

- **Plot the coordinates:** Direct students to find “1” on the horizontal axis and put an index finger on the word. Holding their finger in place, direct students to create a data point to show the number of items recycled on the first day using the method they learned in previous lessons.

Ask:

- Where did you put your point?
  Above 1 on the x*-axis and in line with the number of items recycled on the* y*-axis.*
- What does the point show?
  It shows the number of items recycled the first day of data collection.

Instruct students to plot the number of items recycled for the remaining days. Monitor students for accuracy, and help any students requiring assistance. This task varies in difficulty depending on the actual data collected.

- **Connect the points:** Direct students to place their rulers (if necessary) to line up between the first point on Day 1 and the second point on Day 2. Next, instruct them to draw a straight line connecting the two points. Monitor students to see if they are drawing accurate lines, then instruct them to connect the remaining points. After checking student graphs for accuracy, instruct them to go over the lines with a colored pencil or crayon. Ask students to highlight the baseline (Days 1, 2, & 3) in one color and the remaining trend line in another color. This will help students later with the analysis.
- **Write a title:** Tell students that they need to think of a title that will describe the data.

3. Analyze the line graph

Discuss the results of the intervention with the students, using their graphs as a guideline.

Ask:

- Does the baseline (first three days) differ from the rest of the graph?
  Answers will vary. If the intervention is successful, the points on the line graph will be higher after day 3 (even if they do not consistently go up).
- Did the amount of recycling go up or down?
  Answers will vary. Students should look at the direction of the line on the line graph to help them answer this question.
- Why do you think we got the results we did?
  Answers will vary.

- Did the intervention work the way we thought it would? Why or why not?
  Answers will vary. Reasons for increased recycling might include good intervention, students at the location were motivated and excited, or others. Reasons recycling may not have increased may have been that the intervention did not work the way students thought it would, the intervention needs to be conducted longer, or others.
- Was our experiment conducted for a long enough time? What would happen if we allowed it to go on for longer?
  If the experiment was allowed to go on for more days, we could collect more data. More data might give us a better idea of whether our intervention had a lasting effect on the amount recycled.

Conclude

4. Write the conclusion

Ask students to summarize the results of their intervention. It is important to give students a chance to share their results. If students have worked with another class or the entire school, allow them to develop a creative way to share the results (e.g., in the morning announcements or displaying the graph of the results on posters to hang around the school). If the results were not as the students may have hoped, ask them to write a letter to next year's class explaining what they would change and why.

Other options for sharing results include creating a newspaper article or developing a video storyboard demonstrating how an intervention can lead to a positive change. Students may also choose different methods for dissemination based on their personal preferences. Explain that the goal of this wrap-up project is to share the results with an audience to encourage others to use similar interventions.

Assess

5. Lesson conclusions

Allow students to complete the *Student Mathematicians Think Back: Analyzing Our Graph* Student Page **[SMJ page 123]** to check for understanding of the graph and the intervention. Assess students' general understanding of line graphs using the *Understanding Line Graphs* and *A Gnome's Wish* Student Pages (Optional) **[SMJ pages 125 & 127]**. There is a section on the *Understanding Line Graphs* Student Page **[SMJ page 125]** that only addresses number and operation concepts. This section is optional.

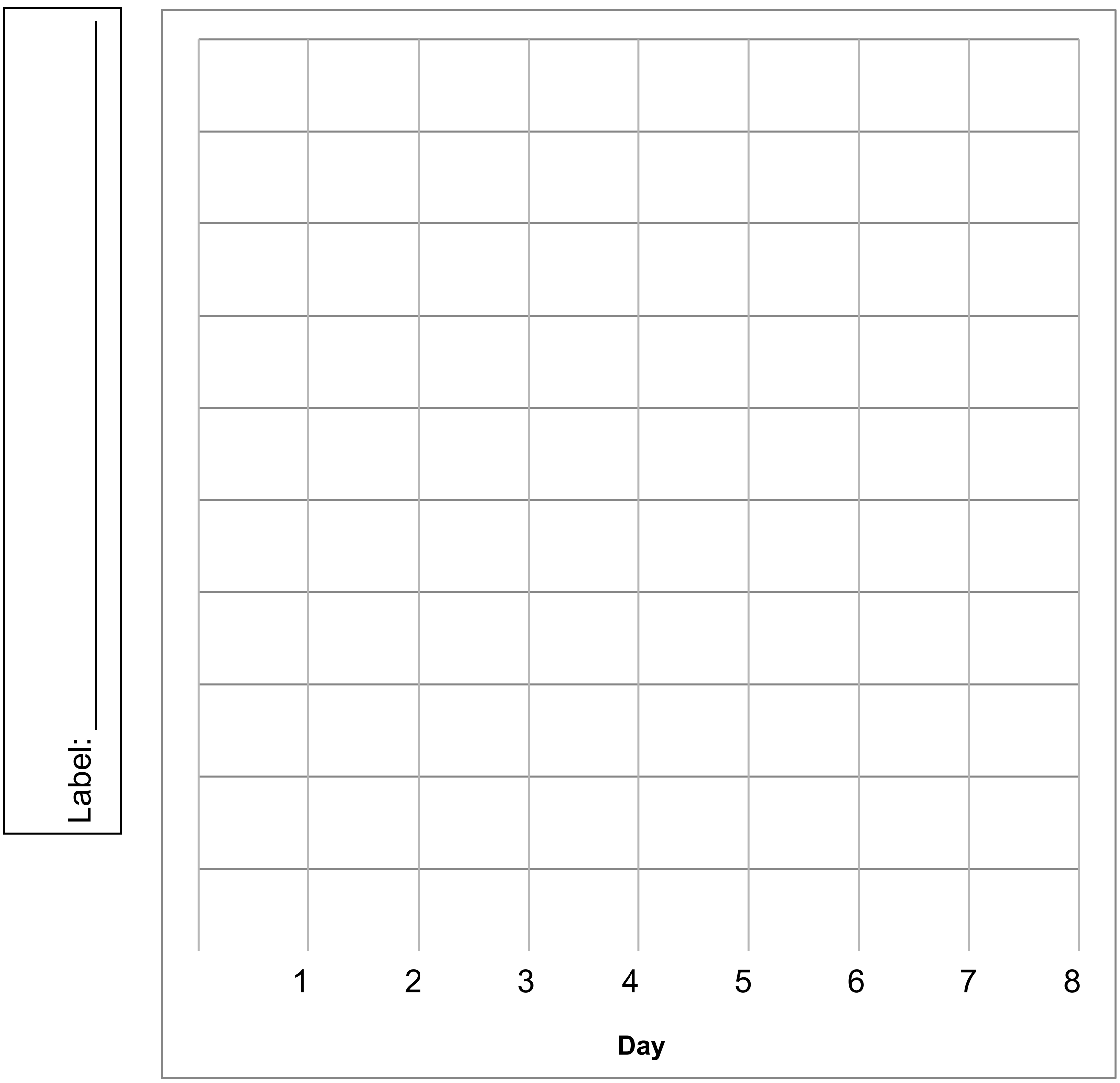
6. Optional student practice

Exciting Exercise Student Page **[SMJ page 129]** can be used for additional student practice with line graphs. Teachers may choose to send it home as homework, use it as an optional activity when students finish their work, or assign it as morning work. For additional numbers and operation practice, there is *Student Mathematicians Practice* Student Page **[SMJ page 133]** that is optional.

Name: ______________________________ Date: ________________

Did It Work?
Line Graph

Title: ____



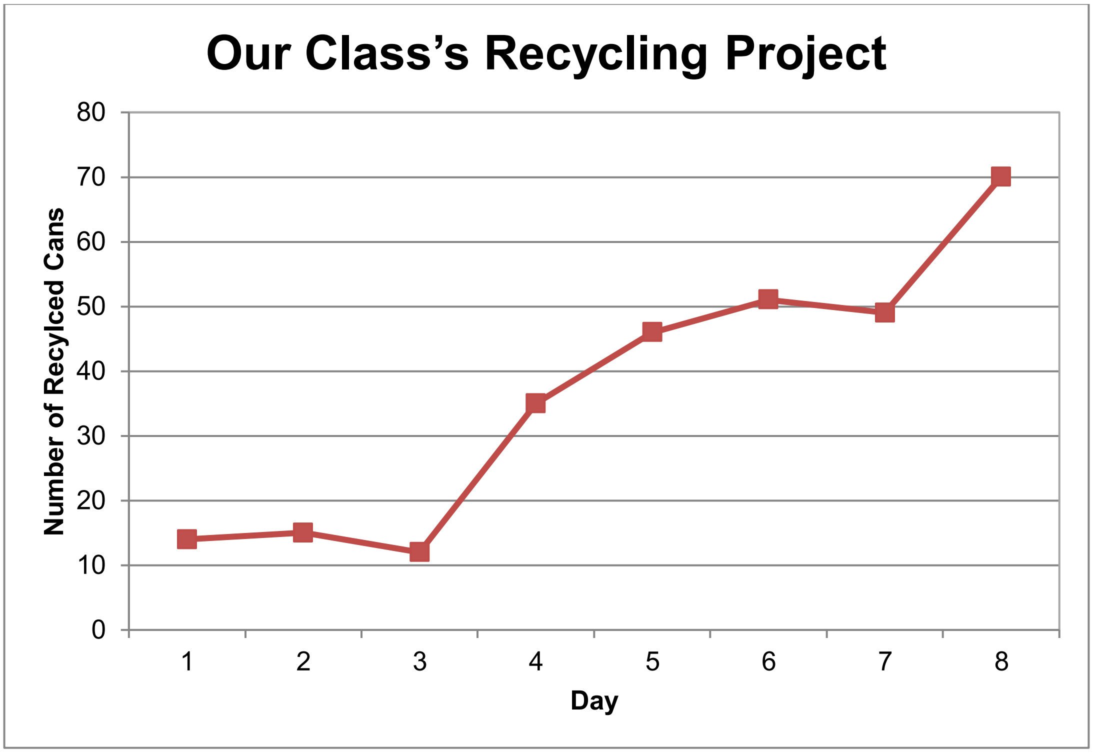
SMJ page 121

Did It Work?

Line Graph

ANSWER KEY

(Answers will vary.)



Name: ________________________________ Date: ________________

Student Mathematicians Think Back: Analyzing Our Graph

1. How can we tell from the line graph whether or not our intervention worked?

____

____

____

____

2. How would you change the intervention if you were to do it again?

____

____

____

____

____

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____

SMJ page 123

Student Mathematicians Think Back: Analyzing Our Graph
ANSWER KEY

Possible Student Responses

1. How can we tell from the line graph whether or not the intervention worked?

   If the line on the graph increases, it shows that more items were being recycled. If the line shows a decreasing trend, the intervention may not have been successful. If the data points after the baseline are higher, then the intervention may have been successful even if it decreases somewhat at certain points.

2. How would you change the intervention if you were to do it again?

   Answers will vary, but may include: make signs larger and more visible or involve more students in the intervention.

Name: ______________________________ Date: ________________

Understanding Line Graphs

1. Below is the average height of boys from birth to age 8. Make a line graph to display the data. Be sure to fill in the missing title and label.

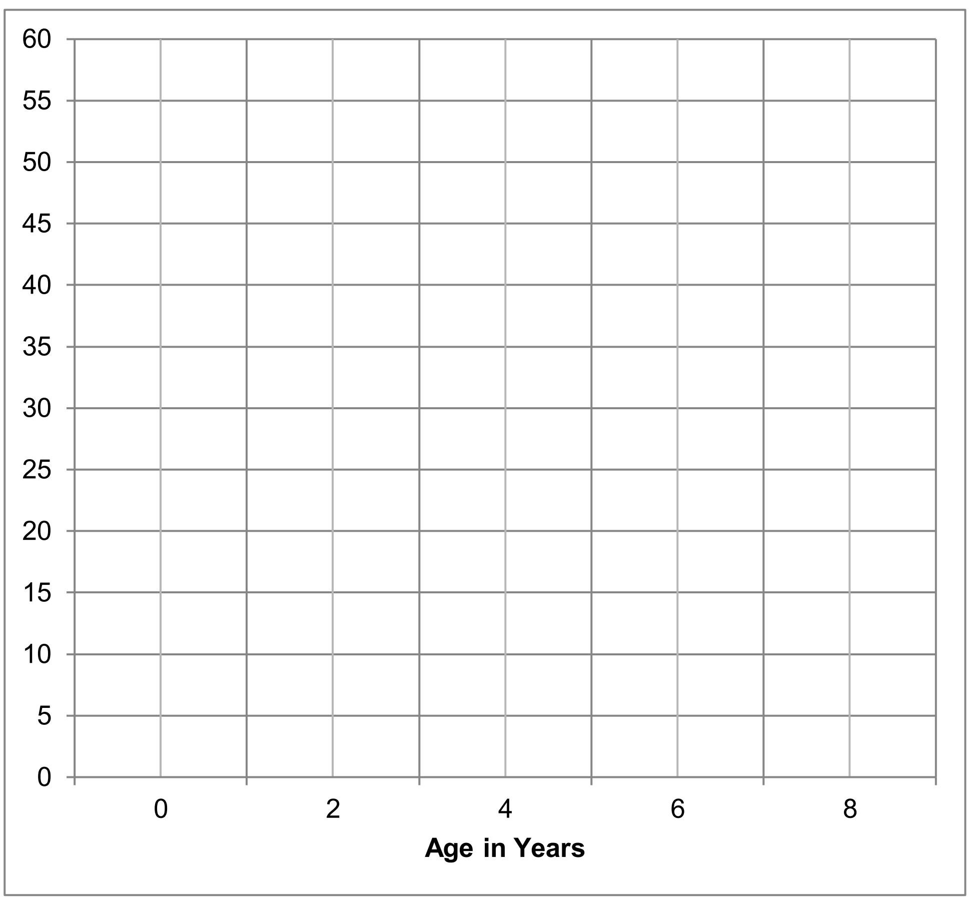
Average Height of Boys

Age (years)	Height (inches)
Birth (0)	30
2	36
4	42
6	47
8	51

http://www.babybag.com/articles/htwt_av.htm

Title: ____

Label: ____________



SMJ page 125

2. What happens to the average height of boys over time? Would it continue to increase if the graph went to age 39?

____

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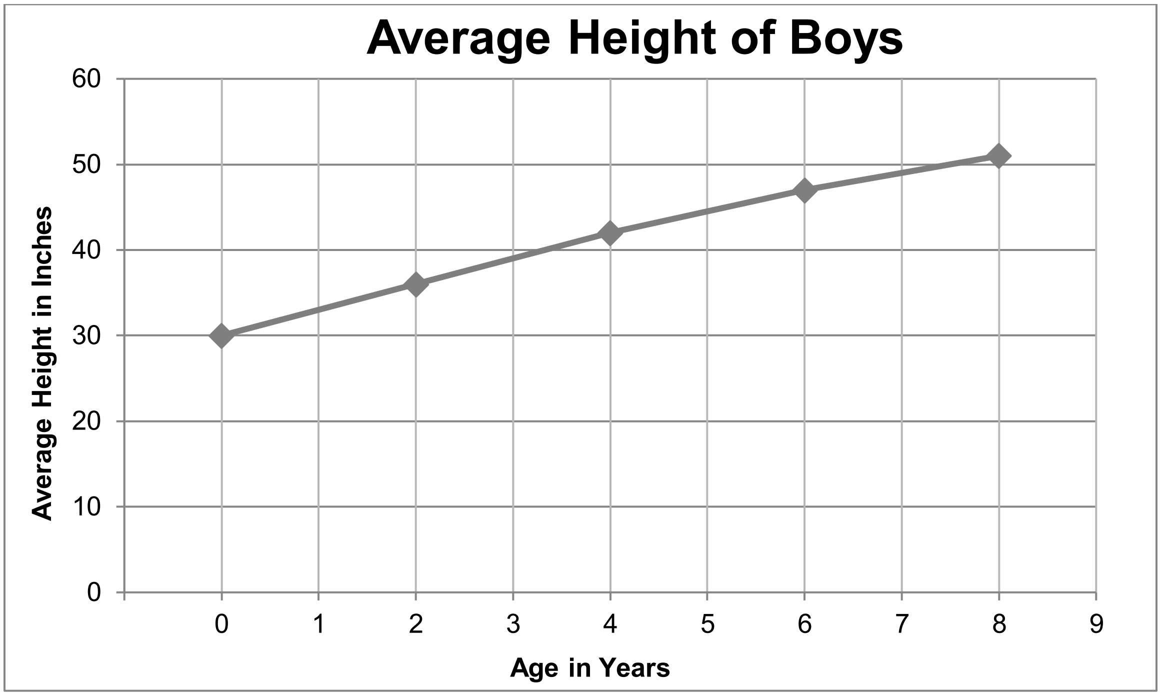
SMJ page 126

Understanding Line Graphs
ANSWER KEY

1.



2. What happens to the average height of boys over time? Would it continue to increase if the graph went to age 39?

   The average height increases over time. The boys are growing up and getting taller. Eventually the line would not continue to increase because most adults are not still growing.

Name: ________________________________ Date: ________________

Read the following story. Create a line graph to show how the height of Nate, the gnome, changes throughout the story.

A Gnome's Wish

Nate, the gnome, was playing all alone on a tree stump outside his house. He watched all of his gnome friends as they climbed the bigger trees. You see, Nate was really short, even for a gnome. He was only **10 inches** tall! As Nate was playing, he noticed a brightly colored mushroom. He remembered a story his grandfather had told him. His grandfather had told him that if he found such a mushroom, he could make a wish.

Nate decided to wish that he were as tall as his gnome friends. Just like that, Nate grew to be **24 inches** tall. Nate was happy. He went and played in the trees with his friends. When he got home for dinner that night, his mom was really angry. She asked, "What happened to my little gnome?" Nate explained the story about the mushroom to Mother Gnome. She told him to return to his old height at once.

Nate returned to the mushroom the next morning. He hoped that he could make another wish. Nate said, "Oh, please magic mushroom, make me short once again. My mom is really angry." In a flash, Nate shrunk to **4 inches.** Nate looked around. The grass seemed a lot taller than before. "Oh no," sobbed Nate, "this is not my right height."

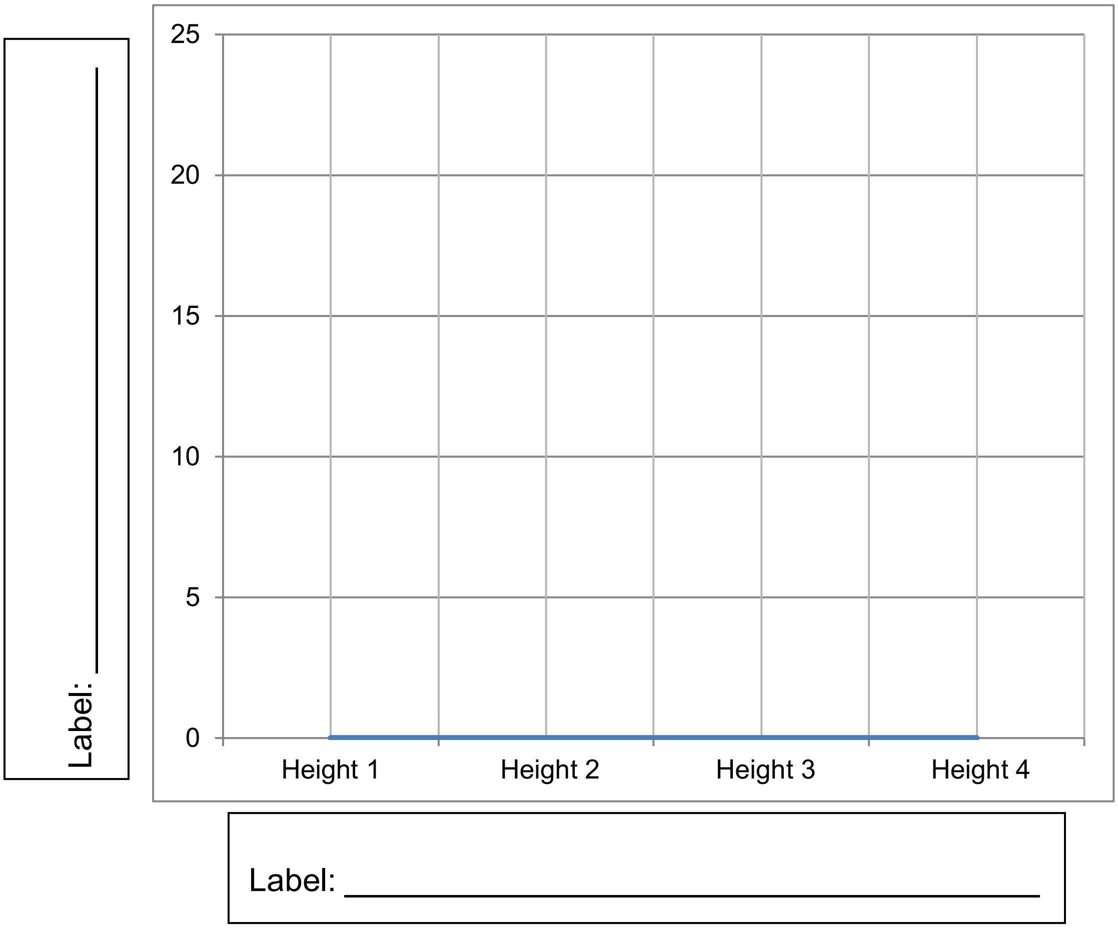
Nate thought for a while. Then, he said, "Magic mushroom, please make me the same height I was yesterday morning." Instantly, Nate grew to be **10 inches** once again. He returned home to a smiling Mother Gnome and a plate of cookies!

SMJ page 127

Name: ________________________________ Date: ________________

A Gnome's Wish: Line Graph

Title: ____



When did Nate grow? __________

How can you tell by looking at the graph?

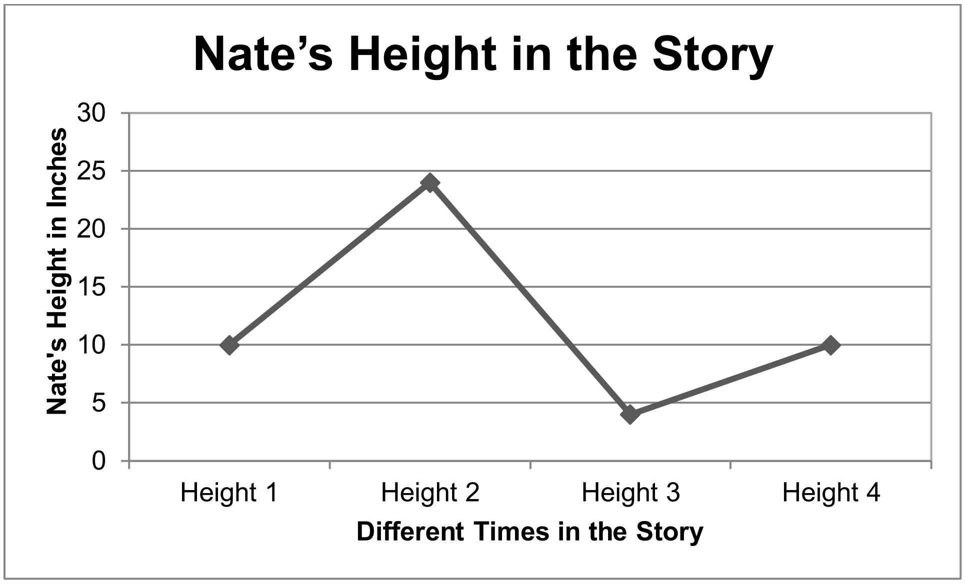
____

____

____

SMJ page 128

A Gnome's Wish: Line Graph
ANSWER KEY



When did Nate grow? How can you tell by looking at the graph?

Sample Response: You can tell Nate is getting taller because the line is going upward, or increasing (see Height 1 to Height 2)

Name________________________________ Date:________________________

Exciting Exercise (Optional)

Dylan and Olivia wanted to see the effect of jumping jacks on their heart rate. They took their resting heart rate for their baseline. A resting heart rate can be taken when you are sitting down and relaxed. Then they did 60 jumping jacks and immediately took their heart rate again. After 5 minutes, they took their heart rate again.

Collect your own data and add to the graph. (Hint: You could count how many times your heart beats in 30 seconds and then double it.)

Baseline: _____ beats per minute

Right After Jumping Jacks: _____ beats per minute

Five Minutes After:_____ beats per minute

Whose heart rate increased the most right after the jumping jacks?
How do you know?

____

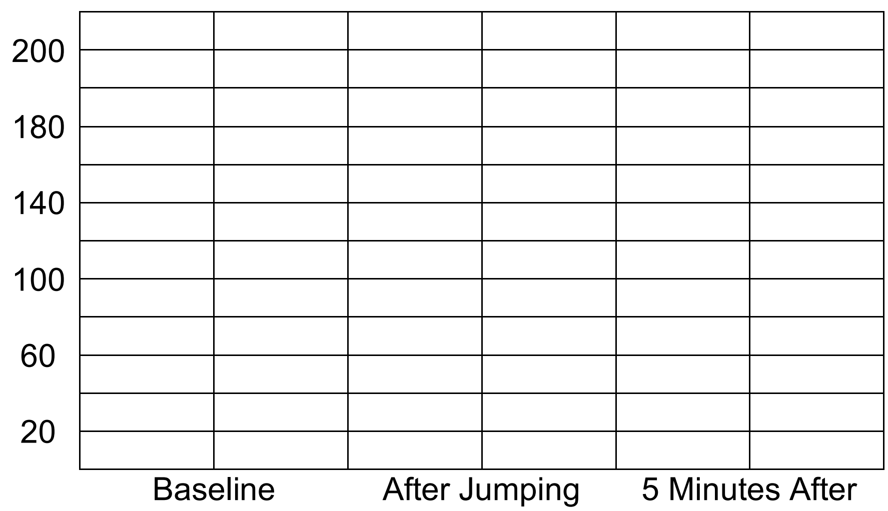
____

____

SMJ page 129

Compare your data with Olivia and Dylan's. Discuss all of the data points as well as the overall trends.

Jelly, the alien, was very out of shape. What would a graph of his heart rate look like? (Make a hypothesis.) Explain your thinking.



SMJ page 131

Exciting Exercise (Optional)
ANSWER KEY

The students will have their own data for the first part.

Whose heart rate increased the most right after the jumping jacks?
How do you know?

Answers will vary. Sample answer: Dylan's heart rate was the highest right after the jumping jacks. Mine was in the middle, and Olivia's was the lowest. I looked at the graph and knew the highest point was the highest heart rate.

Compare your data with Olivia and Dylan's. Discuss all of the data points as well as the overall trends.

Answers will vary. Sample answer: My baseline was lower than Olivia and Dylan's, but then right after the jumping jacks, my heart rate was faster than Olivia's and lower than Dylan's. 5 minutes after the jumping jacks, my heart rate was still faster than both Dylan and Olivia's.

Jelly, the alien, was very out of shape. What would a graph of his heart rate look like? (Make a hypothesis.) Explain your thinking.

Answers will vary. The graph should be at the highest point right after jumping. If he is really out of shape, it should be relatively higher than the students. Unless of course, being an alien causes him to have opposite heart behavior. Then the graph would be opposite.

Name: ______________________________ Date: ________________

Student Mathematicians Practice (Optional)

1. Which amount of money is the least in value?

   A. 1 quarter B. 2 dimes

   C. 6 nickels D. 32 pennies

2. What is the best estimate for the height of a door?

   A. 3 meters B. 7 feet

   C. 300 inches D. 24 centimeters

3. Kari and Luke were making cupcakes for their classes. Kari made 28 cupcakes, and Luke made 34 cupcakes. How many more cupcakes did Luke make than Kari?

   A. 6 B. 14

   C. 29 D. 62

4. $$\begin{array}{r} 786 \\ \underline{+73} \end{array}$$

SMJ page 133

Student Mathematicians Practice (Optional)
ANSWER KEY

1. *B. 2 dimes*
2. *B. 7 feet*
3. *A. 6*
4. *859*

LESSON 12

LINE PLOTS—

DISCOVERING LINE PLOTS

Big Mathematical Ideas

Researchers use a variety of graphical forms to display data and to communicate their findings in an interpretable way. Line plots are used to present numerical data by illustrating the distribution of the data set between the minimum and maximum.

Lesson Objectives	• Students will physically create line plots. • Students will interpret a line plot. They will determine the mode and median from a data set. • Students will describe the purposes of a line plot.
Materials	• Student Page—*Reusing in the News* [SMJ page 135] • Student Page—*Our Class's Reuse Invention Convention* [SMJ page 137] • Student Page—*Our Class's Reuse Invention Convention Line Plot* [SMJ page 139] • Student Page—*Miniature Golf Adventure on Hole 17 (Babbage, Galileo, and Falconer)* [SMJ pages 141, 143, & 145] • Blank paper
Mathematical Language	• **Reuse:** To use again, sometimes for a different purpose. • **Line Plot:** A graph that shows the frequency of data on a number line. It is sometimes called a pictograph. • **Mode**: The number that appears most frequently in a set of numbers. • **Median**: The middle value in a data set when the data is presented in numerical order. • **Maximum:** The largest value in a data set. • **Minimum:** The smallest value in a data set.

Lesson Preview

Students create their own physical line plot. They examine the mode and median of their own plot. They discuss the purpose of a line plot. Then the students can read and analyze an article about reusing or participate in a reuse invention convention and graph their own data. Students brainstorm reusing ideas and then use a line plot to show how many ideas were developed.

Initiate

1. Get on your feet

Write the numbers 0-10 spaced evenly across the entire board. Tell the students that they are going to physically make a line plot. Ask the students to think about the number of letters in their first name. Instruct the students to record the number of letters on a piece of paper or a note card. For example, Elizabeth would have 9 letters and would write "9" on her card. Then instruct the students to create a line in front of their number on the board.

Have the students stay in their place, but instruct them to look around. Ask them what they notice. Are the names in this class typically long or short? How can they tell? What is the most popular number of letters in the students' first names?

Introduce **mode**. Tell the students that mode is the number that is represented the most in the data set. The mode occurs the most in the data. You may want to emphasize mode and most are words that both start with "mo" as a way to remember what the mode is. Ask the students to explain what the mode is in this data set.

Introduce **maximum**. Ask the students what is the longest first name in the class. That is the maximum number of letters in this data set.

Introduce **minimum**. Ask the students what is the shortest first name in the class. That is the minimum number of letters in the data set.

The line plot should begin with the minimum and end with the maximum. All numbers between the minimum and maximum should be on the number line.

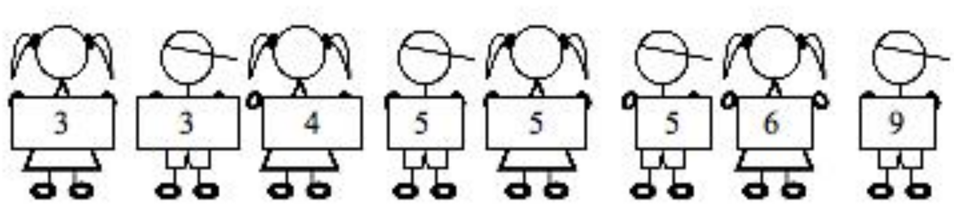
On a piece of chart paper, draw the graph. The line plot may look like this:

Our Class's First Name Letters

		X				
X		X				
X	X	X	X			X
3	4	5	6	7	8	9

Number of Letters in First Name

Introduce **median**. Tell students that median is the number in the middle. To find the median in this data set, it is hard to just look and know what the middle number is, so have your students create a line while holding their number cards.



Then remind them the median is the number in the middle. Ask them to brainstorm how to find the number in the middle.

Finding the Median with an Odd Number of Data Points: With an odd number of students, it is simple to have one student from each side take a step backward until only the middle student is left.

Finding the Median with an Even Number of Data Points: With an even number of students, one student from each side can take a step backward, but in the end, there will be 2 students in the middle. Students should understand finding the middle point between the two numbers. You may want to practice this by giving students two numbers and asking them to find the middle.

If you want to practice the odd and even number of data points, you could join the line or step out of the line to change the number of data points.

In the example above there is an even number of students, so the exact middle is between the fourth and fifth student. Because both the fourth and fifth students have 5 letters, the median is 5. With an even number of data points, average the two numbers in the middle to find the median. If it were between 6 and 7, the median would be 6.5.

Ask the students, who is exactly in the middle of the line? How do you know?

The students may return to their seats. Instruct the students to look at the line plot. Ask the students how they could find the mode and median on the line plot without getting out of their seats. Give them a few minutes to think. Encourage them to think of more than one way.

The mode is very straightforward. You can tell by which column is the highest. The median could be found by striking through 1 X off both sides until only 1 or 2 X's remain. Here is an example:

		~~X~~				
~~X~~		X				
~~X~~	~~X~~	X	~~X~~			~~X~~
3	4	5	6	7	8	9

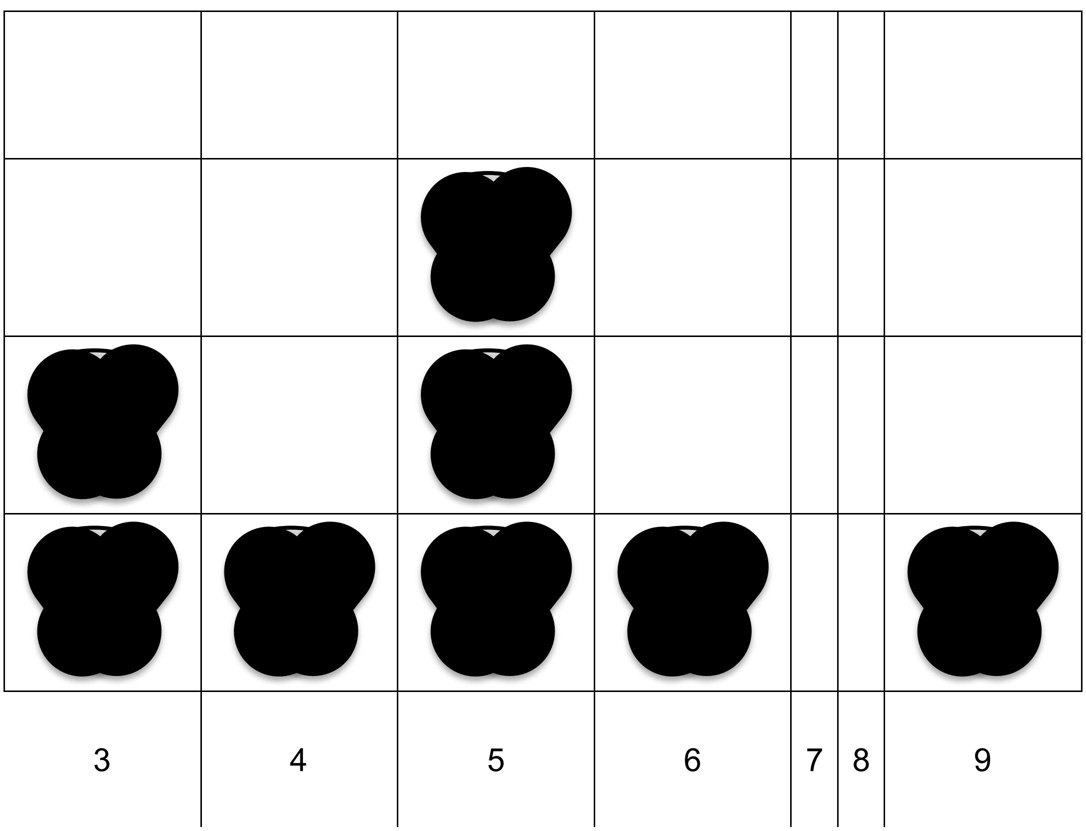
Another method students may come up with is writing the numbers out in a line just as they themselves had stood in a line. Here is an example:

~~3 3 4~~ 5 5 ~~5 6 9~~

Then they can find the middle number for the median. Have the students explain their strategies. Introduce the two strategies above if they are not mentioned.

Ask the students what the X stands for in the line plot. Ask them how they would use the graph to find out how many students are in the class. Then ask them how they could figure out how many total letters are in the entire class's first names.

Students seem to struggle with this idea: The X represents a person. If the students need extra scaffolding, you can have them draw a circle on top of the X, which then resembles a person. Here is an example:



The students may enjoy decorating their “X” with hair or shoes. Once the students grasp that the X represents a person, it becomes much easier for them to figure out how many students are in the class.

Optional: If you would like to make a few more physical line plots, you could ask about the number of family members at home or the number of pets.

Investigate

2.

Discuss the purpose

Ask the students the following questions:

1. What is the different between a line plot and a bar graph*? Students should realize that there are distinct marks for each data point rather than a filled in bar. They should also note that the categories for the line plot are numbers and not words.*
2. What is the difference between a line plot and a line graph? *Students tend to get these two confused because they both have “line” in their names. Line graphs use lines to graph the trends over time. Line plots use a number line to start with and the plot the data according to where it falls on the number line. Whenever the students see “ line plot,” they could add the word “number” to make it a “number line plot.”*
3. What is the purpose of a line plot? You may want to make this more concrete by asking the students to describe the purpose of the line plot they created. *Line plots help you to see how many items are in a data set. For example, the line plot we just created looked at how many letters were in each of our first names. It is easy to look at a line plot and see the mode of the data.*

3.

Your choice

There are two options for this part of the lesson. You should choose at least one of them, but you could do both. The first option is the analysis of an article with an embedded line plot and the second option is the creation of a class line plot. The article could also be used for homework.

Discuss the article

Direct students to the *Reusing in the News* Student Page **[SMJ page 135]** and read aloud with the class. You may require that they record their answers to the questions at the end of the article or use the questions to spark discussion about the graph.

Discuss questions 1-4 with students, making sure students understand how to read the line plot. Ask the students to explain the maximum and minimum in this graph. What are the actual numbers, and what do they tell you?

Encourage students to share their answers to question 5. Record student responses on the board or other display.

Check responses to make sure they include the following information:

- The item was used before.
- Only part of the item may have been changed, not the entire item. If the entire item has been changed, it is probably an example of recycling.
- The item may be used for something different than before.

Reuse Invention Convention

After discussing the article, hold up a can (a soda can or a vegetable can). Then explain to the students that they are going to have a mini-Reuse Invention Convention. They have 5 minutes to think as many ways possible to reuse this can. They may record their ideas on the *Our Class's Reuse Invention Convention* Student Page **[SMJ page 137]**.

After they have had 5 minutes to come up with their ideas, allow the students to share their favorite idea with a partner or the whole class. Then have the students complete the *Our Class's Reuse Invention Convention* Student Page **[SMJ page 137]**. Ask the students to make a note of how many different ideas they had and raise their hands when you say their total number of ideas. Have the students complete the line plot in the *Our Class's Reuse Invention Convention* Student Page **[SMJ page 137]** as you complete it on the board. Ask the students to describe the data. What is the maximum? What is the minimum? If the teacher would have given everyone 10 minutes instead of 5, how would the minimum and maximum change? Ask some comprehension questions to ensure students understand how to read the line plot. For example, how many students had 14 ideas?

Conclude

4. Question discussion

In both the *Reusing in the News* Student Page **[SMJ page 135]** and the *Our Class's Reuse Invention Convention Line Plot* Student Page **[SMJ page 139]**, there are two questions that can be used to guide a discussion on the line plot. Students are asked to create and answer their own question using the line plot. The students should understand what kind of questions could be asked and answered through the use of a line plot. You may want to have the students ask their questions and give the other students a chance to answer the question by looking at the line plot.

Look Ahead

5. **Preview tomorrow's lesson**
Challenge students to think of a way to determine how much reusing goes on in their school.

Assess

6. ***Miniature Golf assessment***
There are several opportunities for assessment within this lesson. The *Miniature Golf Adventure on Hole 17 (Babbage, Galileo, and Falconer)* Student Pages **[SMJ pages 141, 143, & 145]** could serve as a more formal method of assessment.

	Babbage Group	**Galileo Group**	**Falconer Group**
How student scored on #3 on the pretest	Scored 0 or 1 point on item	Scored 2 or 3 points on item	Scored 4 points on item
Student Pages for Lesson	*Miniature Golf Adventure on Hole 17—*Babbage	*Miniature Golf Adventure on Hole 17—*Galileo	*Miniature Golf Adventure on Hole 17—*Falconer

Students could work on this in school or at home. These pages are differentiated after the first four questions.

You could also informally assess students through their work on the *Our Class's Reuse Invention Convention* Student Page **[SMJ page 137]** or the *Our Class's Reuse Invention Convention Line Plot* Student Page **[SMJ page 139]**.

Reusing in the News

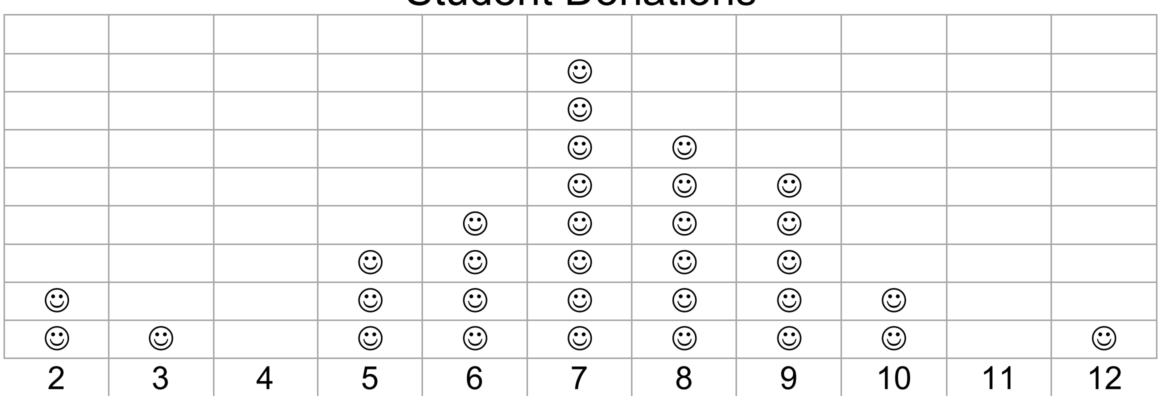
Reuse Invention Convention

Columbia Elementary held its first ever Reuse Invention Convention on November 3rd.

The event was held to give students an opportunity to be creative and to save the environment. The students first collected 2-liter pop bottles. Then they were given supplies such as construction paper, glue, pipe cleaners, and paper clips. Each student had a half hour to come up with the most creative invention that reused the soda bottle. A group of students from all different grade levels at the school organized the drive with their teacher, Mrs. Jones.

According to students, the Reuse Invention Convention was a complete success. Jade, one of the third graders involved in the convention, was very pleased with the turnout. She said, “We had many students and families donate pop bottles and supplies. It was so much fun to think of the different ways to reuse!”

Student Donations



Number of 2-Liter Soda Bottles

The graph is called a **line plot.** It shows the number of soda bottles that each student in the class gave to this drive. **Each smiley face is one student**, so, for example, two students gave two bottles and one student gave three bottles.

SMJ page 135

Students are not only helping their fellow town members but are learning about ways to reduce at the same time. Mrs. Jones said, “The students are really learning a valuable lesson in reusing materials. Too often, bottles are just thrown away, which helps to contribute to the amount of waste in our town. This is a great solution that will help reduce waste and increase creativity.”

Discussion Questions:

1. How many students gave five bottles?

    ____

2. Exactly six students gave the same number of bottles. How many did they give?

    ____

3. What is the mode in this line plot?

    ____

4. What is the median in this line plot?

    ____

5. What do you think it means to reuse?

    ____

6. Write a question that you could answer using this graph.

    ____

7. What is the answer to your question?

    ____

Advanced (Optional):

8. How many bottles were donated all together?

    ____

SMJ page 136

Reusing in the News

Reuse Invention Convention

ANSWER KEY

1. How many students gave five bottles?

   3 students gave 5 bottles.

2. Exactly six students gave the same number of bottles. How many did they give?

   They gave 8 bottles.

3. What is the mode in this line plot?

   They mode is 7.

4. What is the median in this line plot?

   The median is 7.

5. What do you think it means to reuse?

   Answers will vary, but should indicate that the student understands that to reuse something, he or she uses it again in the same or very similar form. For example: I think it means that you use it again like it is.

6. Write a question that you could answer using this graph.

   Answers will vary.

7. What is the answer to your question?

   Answers will vary.

Advanced (Optional):

8. How many bottles were donated all together?

   227 bottles were donated.

Name_______________________________ Date:_______________________

Our Class's Reuse Invention Convention

Think of as many different ways as you can to reuse the can.
You may draw them or write them out.

1.	2.	3.	4.	5.	6.
7.	8.	9.	10.	11.	12.
13.	14.	15.	16.	17.	18.
19.	20.	21.	22.	23.	24.

How many reuse ideas did you have? ________
Circle the idea you like the most.

SMJ page 137

Our Class's Reuse Invention Convention
ANSWER KEY
(Answers will vary.)

Think of as many different ways as you can to reuse the can.
You may draw them or write them out.

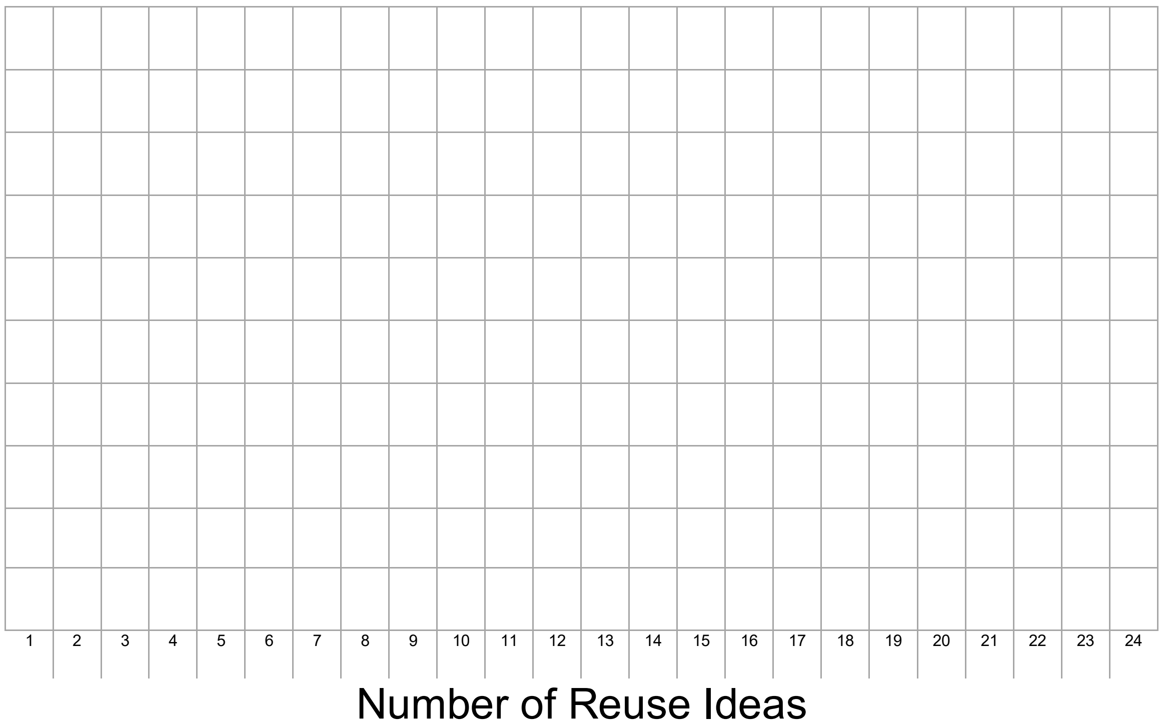
1. *Hat*	2. *Flower Pot*	3. *Pencil Holder*	4. *Cup*	5. *Toy Phone*	6. *Tower*
7.	8.	9.	10.	11.	12.
13.	14.	15.	16.	17.	18.
19.	20.	21.	22.	23.	24.

How many reuse ideas did you have? __*6*_____
Circle the idea you like the most.

Name________________________________ Date:________________________

Our Class's Reuse Invention Convention Line Plot

Let one X stand for one student.



1. What is the mode of these data? ________________________________
2. What is the median of these data? ________________________________
3. Write a question that you could answer using this graph.

   ____
4. What is the answer to your question?

   ____
5. Describe how the line plot would change if students were given 10 minutes instead of 5.

   ____

   ____

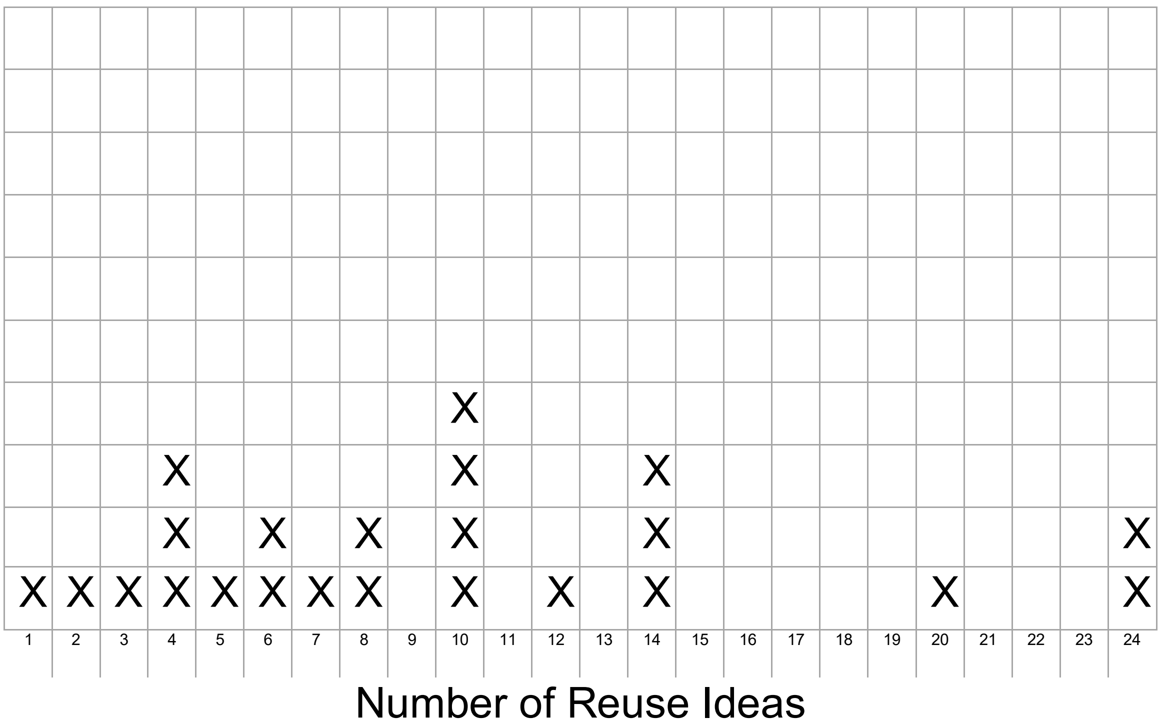
SMJ page 139

Our Class's Reuse Invention Convention Line Plot

ANSWER KEY

(Answers will vary.)

Let one X stand for one student.



1. What is the mode of these data? *10*

2. What is the median of these data? *8*

3. Write a question that you could answer using this graph.

   Answers will vary. A sample question may be, "How many students came up with 14 reuse ideas?"

4. What is the answer to your question?

   Answers will vary. 3 students came up with 14 reuse ideas.

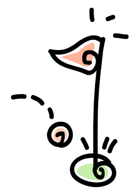
5. Describe how the line plot would change if students were given 10 minutes instead of 5.

   Answers will vary. I think more students would have been able to think of more ideas, so there would be more X's closer to 24.

Babbage

Name_______________________________ Date:_______________________

Miniature Golf Adventure on Hole 17



Mrs. Juno's class went golfing on the last day of school. Mrs. Juno wanted to know how well her students did on Hole 17, so she created this line plot. Each X stands for one student.

					X	
		X		X	X	
		X		X	X	
	X	X		X	X	
	X	X	X	X	X	
X	X	X	X	X	X	X
1	2	3	4	5	6	7

Number of Strokes Taken

1. What is the mode of this data? _______________________________________
2. What is the median of this data? _____________________________________
3. What is the minimum number of strokes taken? ________________________
4. What is the maximum number of strokes taken? ________________________
5. How many students took 4 strokes? ___________________________________
6. How many total students are in the class? ____________________________

SMJ page 141

Babbage

Miniature Golf Adventure on Hole 17
ANSWER KEY

Mrs. Juno's class went golfing on the last day of school. Mrs. Juno wanted to know how well her students did on hole 17, so she created this line plot. Each X stands for one student.

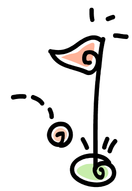
					X	
		X		X	X	
		X		X	X	
	X	X		X	X	
	X	X	X	X	X	
X	X	X	X	X	X	X
1	2	3	4	5	6	7

Number of Strokes Taken

1. What is the mode of this data? *6 strokes*
2. What is the median of this data? *5 strokes*
3. What is the minimum number of strokes taken? *1 stroke*
4. What is the maximum number of strokes taken? *7 strokes*
5. How many students took 4 strokes? *2 students*
6. How many total students are in the class? *23 students*

Name______________________________ Date:______________________

Miniature Golf Adventure on Hole 17



Mrs. Juno's class went golfing on the last day of school. Mrs. Juno wanted to know how well her students did on hole 17, so she created this line plot. Each X stands for one student.

					X	
		X		X	X	
		X		X	X	
	X	X		X	X	
	X	X	X	X	X	
X	X	X	X	X	X	X
1	2	3	4	5	6	7

Number of Strokes Taken

1. What is the mode of this data? ____________________________________
2. What is the median of this data? ___________________________________
3. What is the minimum number of strokes taken? ______________________
4. What is the maximum number of strokes taken? ______________________
5. How many students took 4 or more strokes? _________________________
6. How many more students took 6 strokes than took 2 strokes? ___________

SMJ page 143

Miniature Golf Adventure on Hole 17
ANSWER KEY

Mrs. Juno's class went golfing on the last day of school. Mrs. Juno wanted to know how well her students did on hole 17, so she created this line plot. Each X stands for one student.

					X	
		X		X	X	
		X		X	X	
	X	X		X	X	
	X	X	X	X	X	
X	X	X	X	X	X	X
1	2	3	4	5	6	7

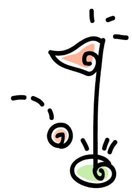
Number of Strokes Taken

1. What is the mode of this data? *6 strokes*
2. What is the median of this data? *5 strokes*
3. What is the minimum number of strokes taken? *1 stroke*
4. What is the maximum number of strokes taken? *7 strokes*
5. How many students took 4 or more strokes? *14 students*
6. How many more students took 6 strokes than took 2 strokes? *3 students*

Falconer

Name______________________________ Date:______________________

Miniature Golf Adventure on Hole 17



Mrs. Juno's class went golfing on the last day of school. Mrs. Juno wanted to know how well her students did on hole 17, so she created this line plot. Each X stands for one student.

					X	
		X		X	X	
		X		X	X	
	X	X		X	X	
	X	X	X	X	X	
X	X	X	X	X	X	X
1	2	3	4	5	6	7

Number of Strokes Taken

1. What is the mode of this data? ___________________________________
2. What is the median of this data? _________________________________
3. What is the minimum number of strokes taken? _____________________
4. What is the maximum number of strokes taken? _____________________
5. How many strokes did the whole class take? _______________________
6. Kiesha took 2 strokes on all 18 holes. What was her final score? __________

SMJ page 145

Falconer

Miniature Golf Adventure on Hole 17
ANSWER KEY

Mrs. Juno's class went golfing on the last day of school. Mrs. Juno wanted to know how well her students did on hole 17, so she created this line plot. Each X stands for one student.

					X	
		X		X	X	
		X		X	X	
	X	X		X	X	
	X	X	X	X	X	
X	X	X	X	X	X	X
1	2	3	4	5	6	7

Number of Strokes Taken

1. What is the mode of this data? *6 strokes*

2. What is the median of this data? *5 strokes*

3. What is the minimum number of strokes taken? *1 stroke*

4. What is the maximum number of strokes taken? *7 strokes*

5. How many strokes did the whole class take? *98 strokes*

6. Kiesha took 2 strokes on all 18 holes. What was her final score? *36 strokes*

LESSON 13

LINE PLOTS—

LEARNING ABOUT SURVEYS

Big Mathematical Ideas

Information can be gathered in multiple ways such as in observations, interviews, and surveys. Surveys enable researchers to collect information from a large group of people. Researchers can condense the responses into graphs and analyze the patterns in peoples' responses to draw conclusions.

Lesson Objectives	• Students will explain what a survey is and how a survey is used.
Materials	• Student Page—*Ways of Knowing Survey* [SMJ page 147] • Student Page—*Greening Up Survey* [SMJ page 149] • Student Page—*Administering Our Survey* [SMJ page 150] • Student Page—*Ways of Knowing Cube: Think Deeply 1* [SMJ page 151] • Student Page—*Designing Surveys: Think Deeply 2* [SMJ page 153] • Chart paper
Mathematical Language	• **Survey:** A list of questions presented to people to gather information from them.

Lesson Preview

Students explore what a survey is by taking one, and then select 9 questions to include on the *Greening Up Survey* Student Page **[SMJ page 149]**.

Initiate

1. **Brainstorm items for survey**
Set up three stations around the room. Put 2 pieces of chart paper at each station and label each station as follows: recycle, reduce, and reuse. Tell

the students that they have 3 minutes at each station to brainstorm as many ways to recycle, reuse, or reduce and record their thoughts on the chart paper. They can carry their own writing utensil from station to station, and they are not allowed to write something down if it is already written.

Some suggestions for reusing might include:

- Use cloth napkins or towels instead of paper.
- Donate old magazines or books.
- Reuse boxes for storage or classroom activities.
- Reuse empty jars for storage or classroom activities.
- Use refillable pens and pencils.
- Use the back of printer paper for scratch paper.

Some suggestions for recycling might include:

- Place paper in the recycle bin.
- Place cardboard in the recycle bin. (Cans and glass bottles can also go in the recycle bins.)
- Recycle paper yourself by using the blender and a paper screen.
- Recycle crayons by melting them down into new shapes or crayons.

Some suggestions for reducing might include:

- Use cloth bags at the grocery store (reducing the number of plastic bags).
- Buy items in bulk to reduce packaging.
- Buy fresh fruits and vegetables rather than the packaged fruits/vegetables.
- Donate clothes to a thrift shop.

Following the brainstorming session, lead the students in a discussion. You may want to review the ways that they have already changed the school's practices.

- Which of these ways do you think the people in our school use?
  Answers may vary. Guide students so that they understand that we currently do not know for sure which ways people use.
- How might we gather that information?
  Answers will vary. Guide the students so that they mention talking to people (interviewing), watching people (observing), or asking many people to respond to a list of questions (surveying).
- Why is this information useful?
  Answers will vary. This information will show how much recycling, reusing, and reducing people are currently doing. That can be used to see what more can be done.

Investigate

2. Complete *Ways of Knowing Survey*

Tell students that when researchers want to find out something, they can use one of the three ways: interviewing, observing, or surveying. Tell them you have prepared a list of questions to find out which ways they think are best. Direct students to the *Ways of Knowing Survey* Student Page **[SMJ page 147]**. Read the directions with the students, then guide students as they answer the first question. Students should check the column “yes” if they think they should use that method to answer the question about favorite ice cream.

Encourage students to answer each question <u>and think about why</u> they selected “yes” or “no.” Students should be able to justify their answers. Allow students to complete the survey independently or in pairs.

3. Discuss *Ways of Knowing Survey*

Review the survey items and have students raise their hands to indicate whether they responded “yes” or “no.” Discuss pros and cons of each item:

1. **Ask a teacher.**
   Pros: *Many different foods are possible; you are not limited to a list of choices.*
   Cons: *The teacher may not know; it might be difficult to narrow it down to five favorite foods; asking a teacher may not be representative of all students.*

2. **Ask four of your friends.**
   Pros: *Many different foods are possible; you are not limited to a list of choices.*
   Cons: *Friends may not know; it might be difficult to narrow it down to five favorite foods; asking a few people may not be representative of all students.*

3. **Watch a class eat in the cafeteria.**
   Pros: *Many different foods are possible; you are not limited to a list of choices.*
   Cons*: It may be difficult to narrow it down to five favorite foods; it may be time-consuming.*

4. **Ask all students to write their five favorite foods.**
   Pros: *Many different foods are possible; you are not limited to a list of choices.*
   Cons: *It may be difficult to narrow it down to five favorite foods; a teacher may not eat the same foods as a student; it may be hard to add up.*

5. **Watch your best friend eat.**
   Pros: *Many different foods are possible; you are not limited to a list of choices.*
   Cons: *Asking a friend may not be representative of all students.*

6. **Ask the custodian.**
   Pros: *Many different foods are possible; you are not limited to a list of choices.*
   Cons: *The custodian may not know; it may be difficult to narrow it down to five favorite foods; asking a custodian may not be representative of all students.*

7. **Give students a list of foods and ask them to circle their five favorite foods.**
   Pros: *It is possible to narrow it down to five favorite foods; it would be easy to tally.*
   Cons: *You cannot put all possible foods on the list.*

Ask students if they know what the *Ways of Knowing* list of questions is called (**a survey**). Help students understand when you have one or more specific questions to ask many people, a survey is a good way to collect information.

Write the definition of "survey" on the board:

> **Survey**: A list of questions presented to people to gather information from them.

Tell them that surveys are designed to answer one or more questions, and that our question will be, "What are the ways students can help save the environment in our school?"

4. Create the *Greening Up Survey*

Explain to students that they will be selecting the 9 most common ways of recycling/reusing/reducing to include in their *Greening Up Survey*. Point to the brainstormed lists they created earlier. Remind students that sometimes these categories can get a little blurry. You can reduce waste by reusing. Recycling reduces the waste in the landfills. The goal is to choose 3 from each category to include on the survey. The best way to accomplish this would be to allow the students to look at each category and pick the 3 most common ways. If the students are having a hard time deciding, you could have a class vote. You could have them pick out the 5 most common and then vote on those. Ask students to agree or disagree with the choices before the final survey is completed. An additional option would be to have the teacher choose the most common ways. This is not optimal because the students lose some ownership of the project.

Once the nine ways have been selected, direct students to the *Greening Up Survey* Student Page **[SMJ page 149]**. Have students copy the list into their *Greening Up Survey* in the order they appear on the board or chart paper. Depending upon whom you choose to interview, you may want to have a copy of the survey for the students to write the questions on outside of the journal, so they can easily take the paper home. Another time-saving option is to write the list down yourself or have a student write it and then make photocopies for each student.

Conclude

5.

Discuss data collection procedures

Describe data collection procedures to students. You first need to decide to whom your class is going to administer the surveys. You could allow the students to take the survey home and pick one family member or another student on the bus. This would bring some variation to the data. You could also schedule recess with another class and allow the students to administer the survey then. You could set up a visit to another classroom or allow the students to ask the questions to other students during lunch.

Tell the students who their population is and how the data will be collected. Review the *Administering Our Survey* Student Page **[SMJ page 150]**, describing to students how they should record the responses on the survey. Ask the students why this discussion is important. If there is time, you may want to discuss how the person giving the survey could influence the respondent.

Offer an additional challenge

There are two *Think Deeply* options. The students could choose which interests them the most. The students may want to complete the *Ways of Knowing Cube* Student Page **[SMJ page 151]** or *Designing Surveys* Student Page **[SMJ page 153]**. The *Ways of Knowing Cube* Student Page **[SMJ page 151]** instructs students to decorate, cut out, and assemble their cube for display in the room. The *Designing Surveys* Student Page **[SMJ page 153]** instructs the students to take on the role of various people and think about what kind of surveys they may use. You could tell the students only to pick one type of person, which would make the assignment differentiated by interest. This paper could also be done during social studies.

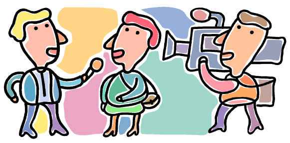
Look Ahead

6. Explain to students that in the next lesson they will be learning more about surveys, and then the following lesson they will be analyzing the data they collected using their *Greening Up Surveys*. Remember to specify how their data is to be collected and when it is to be done.

Assess

7. Informally assess students' understanding of the concept of what a survey is through the classroom discussions and peer interactions.

Ways of Knowing Survey



Interviewing

Observing

Surveying

If you wanted to know students' FIVE favorite foods in our cafeteria, what "Ways of Knowing" would you use? Put a check mark under "YES" or "NO."

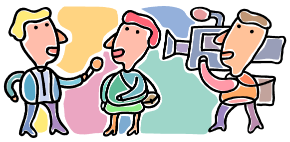
Would you…	Yes (Pros)	No (Cons)
1. Ask a teacher?		
2. Ask four of your friends?		
3. Watch a class eat in the cafeteria?		
4. Ask each student to write his or her five favorite foods?		
5. Watch your best friend eat?		
6. Ask the custodian?		
7. Give students a list of foods and ask them to circle their five favorite foods?		

SMJ page 147

Ways of Knowing Survey

ANSWER KEY

(Answers will vary.)



Interviewing

Observing

Surveying

If you wanted to know students' FIVE favorite foods in our cafeteria, what "Ways of Knowing" would you use? Put a check mark under "YES" or "NO."

Would you…	**Yes (Pros)**	**No (Cons)**
Ask a teacher?	*Many different foods are possible; you are not limited to a list of choices. They take the lunch count and have lots of experience with kids so they may be able to see many students preference at once.*	*The teacher may not know; it might be difficult to narrow it down to five favorite foods; asking a teacher may not be representative of all students.*
Ask four of your friends?	*Many different foods are possible; you are not limited to a list of choices. Because you know them, they may feel comfortable saying something unusual.*	*Friends may not know; it might be difficult to narrow it down to five favorite foods; asking a few people may not be representative of all students. Friends may have similar tastes or be influenced by each other.*
Watch a class eat in the cafeteria?	*Many different foods are possible; you are not limited to a list of choices. You get to see a lot of people at once.*	*It may be difficult to narrow it down to five favorite foods; it may be time-consuming. Their favorite food may not be an option for lunch.*

Would you…	**Yes (Pros)**	**No (Cons)**
Ask each student to write his or her five favorite foods?	*Many different foods are possible; you are not limited to a list of choices. The students have lots of options and are not limited to the lunch choices of the day.*	*It may be difficult to narrow it down to five favorite foods; teacher may not eat the same foods as a student; it may be hard to add up.*
Watch your best friend eat?	*Many different foods are possible; you are not limited to a list of choices.*	*Asking a friend may not be representative of all students. Your best friend may not eat his/her favorite foods for lunch.*
Ask the custodian?	*Many different foods are possible; you are not limited to a list of choices. The custodian gets to see the food that is thrown away, so he/she may be able to tell when lots of people like something the cafeteria serves because less of it will be thrown away.*	*The custodian may not know; it may be difficult to narrow it down to five favorite foods; asking a custodian may not be representative of all students.*
Give students a list of foods and ask them to circle their five favorite?	*It is possible to narrow it down to five favorite foods; it would be easy to tally.*	*You cannot put all possible foods on the list.*

Name: ________________________________ Date: ________________

Greening Up Survey

Person Surveyed:______________________

Greening Practice: Do you…?	**Yes**	**No**
Recycling		
1.		
2.		
3.		
Reducing		
4.		
5.		
6.		
Reusing		
7.		
8.		
9.		
Totals		

SMJ page 149

Name: ________________________________ Date: _________________

Administering Our Survey

1. Make sure the person wants to participate. Ask:

   "Could I ask you 9 questions about saving the planet?"

2. If the person says "Yes," read the following:

   "I will read an example of a way to help save Earth, and then you say whether or not you do it. You need to answer 'yes' or 'no' to each question."

3. Ask, "Do you (read the first item)?" Ask if their answer is "yes" or "no."

4. Check the column with answer on the survey page.

5. Do this for each item.

6. Thank the student and his or her teacher for their time and participation. ☺

SMJ page 150

Greening Up Survey
ANSWER KEY

(Note: Reusing and reducing are very similar concepts and may overlap because when you reuse you reduce waste.)

Person Surveyed: ***Bob the Bus Driver***

Greening Practice: Do you…?	Yes	No
Recycling		
1. Recycle cans?	*X*	
2. Recycle paper?	*X*	
3. Recycle plastic items?		*X*
Reducing		
4. Use the back of papers?	*X*	
5. Give away clothes?	*X*	
6. Buy fresh fruits instead of canned fruits? (reduces packaging waste)		*X*
Reusing		
7. Reuse cans for cups?		*X*
8. Use cloth napkins instead of paper?		*X*
9. Use cloth grocery bags?	*X*	
Totals	*5*	*4*

Ways of Knowing Cube
Think Deeply 1

2. Think of two questions you would write on your survey. Write them here:

____
____
____
____
____

1. What topic would you like to learn more about through a survey?

____
____
____
____
____

Ways-of-Knowing Cube by:

____

5. Draw a picture of what you would look like giving your survey:

3. Who would take your survey?

____
____
____
____
____
____

4. What would you do with what you found out?

____
____
____
____
____
____
____

SMJ page 151

Ways of Knowing Cube
Think Deeply 1
ANSWER KEY
(Answers will vary.)

2. Think of two questions you would write on your survey. Write them here:

What is your favorite subject in school?
What do you like about it?

1. What topic would you like to learn more about through a survey?

I would like to learn about students' favorite subjects in school.

Ways-of-Knowing Cube
by:
Sophie

5. Draw a picture of what you would look like giving your survey:

3. Who would take your survey?

The people in my class would take the survey.

4. What would you do with what you found out?

I would tell the teacher so he could spend a little extra time on that subject or create projects that interest specific students.

Name: ________________________________ Date: ________________

Designing Surveys
Think Deeply 2

Many different types of people design surveys. Your job is to think about what kinds of questions these people may ask and why.

Type of Person	Give 2 questions for this person's survey.	Whom would he or she ask?	What would he or she do with the information?
President 			
Scientist 			
TV Producer 			

SMJ page 153

Designing Surveys
Think Deeply 2
ANSWER KEY
(Answers will vary.)

Many different types of people design surveys. Your job is to think about what kinds of questions these people may ask and why.

Type of Person	Give 2 questions for this person's survey.	Whom would he or she ask?	What would he or she do with the information?
President 	***What are your concerns for the nation?*** ***What is the biggest thing I should work to change?***	***Congress***	***He/she could prioritize what he/she works on to make sure it is important to Congress people as well as the voters.***
Scientist 	***What chemicals do you use?*** ***What equipment do you need?***	***Other lab members***	***He/she could use the information to order the needed supplies for the lab.***
TV Producer 	***What is your favorite television show?*** ***What time do you watch television?***	***Random people who watch television***	***He/she would create a show that is similar to the show that everyone likes and play it at a time when people watch television.***

LINE PLOTS—

MY VERY OWN SURVEY

LESSON 14

Big Mathematical Ideas

Graphs give us a general idea of the trends in data and help us generate summary information. Line plots are used to present numerical data by illustrating the distribution of the data set between the minimum and maximum.

Lesson Objectives	• Students will design their own surveys. • Students will administer their surveys to group members. • Students will locate the minimum and maximum values in their survey. • Students will create a line plot for their surveys.
Materials	• Student Pages—*My Very Own Survey (Babbage, Galileo, and Falconer)* [SMJ pages 155-159, 161-165, & 167-171] • Student Pages—*You Decide (Babbage & Galileo and Falconer)* (Optional) [SMJ pages 173 & 175]
Mathematical Language	• **Frequency:** The number of times a particular item appears in a set of data. • **Frequency Table:** A table that is used to count and total data for different categories. • **Horizontal Axis (*x*-axis):** The line on a graph that runs from left to right. • **Line plot:** A graph that shows the frequency of data on a number line. It is sometimes called a pictograph. • **Minimum:** The smallest value in a data set. • **Maximum:** The largest value in a data set. • **Range:** The minimum to maximum values in a data set (ex: 2 to 6). • **Data:** Information such as numbers that researchers gather during an experiment. • **Tally:** To count using some type of mark. • **Unit:** One of an item. • **Outlier:** An extreme value in a data set.

Lesson Preview

Students create a survey and administer it to their group's members and then create a line plot of the data.

Initiate

1. Think of a question

Separate students into groups: Babbage, Galileo, and Falconer. Base your grouping for this lesson on your knowledge of the students and how they scored on the pretest, as follows:

	Babbage Group	**Galileo Group**	**Falconer Group**
How student scored on #3 on the pretest	Scored 0 or 1 points on item	Scored 2 or 3 points on item	Scored 4 points on item
Student Pages for Lesson	My Very Own Survey—Babbage	My Very Own Survey—Galileo	My Very Own Survey—Falconer

Direct students to the *My Very Own Survey* Student Pages **[SMJ pages 155-159, 161-165, or 167-171]**.

Explain that to learn more about surveys, students will be creating their own survey on a topic of their choice. They will use these to gather data from their group members. Remind students that they will be returning to the *Greening Up Survey* in the next lesson, applying what they have learned.

Ask students to think of a question they would like answered. Their question should begin with the words "How many…."

Brainstorm with students a list of questions for surveys. Sample questions might include:

- How many pets do the classmates in my group have?
- How many brothers and sisters do the classmates in my group have?
- How many television shows do the classmates in my group watch every week?

Instruct students to write their question on the *My Very Own Survey* Student Pages **[SMJ pages 155, 161, or 167]** under the "My Very Own Question" section.

You may want to model the whole process as they are going through it by selecting your own question and collecting class data to show on the board.

Investigate

2. **Collect data on *My Very Own Survey***

Tell students that we need a way to record answers to the survey questions and that we can do it with a frequency table. Tell students that this type of table shows the frequency of students who answered their questions with the number on the row.

Instruct students to take turns asking students in their groups their questions. They should tally the responses in the Student Count column on their *My Very Own Survey* Student Pages **[SMJ pages 155, 161, or 167]**. Show students how to tally responses by placing a check mark (✓) in the appropriate box for each student's answer. They should then count the check marks and enter the number in the Number of Students column.

For example, if three students answer "5," and five students answer "6," then the Frequency Table would look like this:

Frequency Table:

Answer	Students Count	Number of Students
5	✓✓✓	3
6	✓✓✓✓✓	5

3. **Examine the frequency table**

The three groups have different frequency tables. The Babbage table has numbers 0-15, the Galileo table has ranges, and the Falconer table has no numbers at all. Guide the Falconer group through thinking about the range of answers their question will receive. They could ask a question that has answers like 60-75.

After students collected their data, pose the following questions.

Ask:

- What is the smallest answer on your survey?
  Answers will vary depending on the class data. Make sure students understand that zero is an amount.
- What is the largest number someone could respond?
  Answers will vary, depending on the data.

Tell students that they have identified the minimum and maximum numbers. Write these definitions on the board:

- **Minimum**: The smallest value in a data set.
- **Maximum**: The largest value in data set.

Direct students to record their maximum and minimum numbers on their Student Pages.

Introduce the range. The range is the minimum number to the maximum number. Add the definition to the board.

Ask:

- How is the frequency table easier to understand than the individual survey sheets?
  It helps us to see that there are many different answers.
- What is still difficult about the Frequency Table?
  Because there are only numbers, it is not always easy to see when the numbers are alike and when they are different. A picture would help us to understand this type of information better.

4. Create a line plot

Review with students that a graph is a way to use a picture to show patterns in numbers. Earlier we saw a line plot using smiley faces or X's as units to show numbers of students. A unit is one of something that can be shown using a picture, such as one smiley face or one "X." Now we will be using X's as units to show numbers of students. A line plot is a graph that shows the frequency of data on a number line by stacking these units on top of each other.

Ask:

- Do you see a number line on the page?
  No, we need to create one at the bottom of the page. Note: Galileo and Falconer Group students' number lines may be written in ranges: 0-2, 3-4, etc., with each range occurring in a separate cell.
- Do we really need the numbers smaller than our minimum or larger than our maximum?
  No, we really don't, since no students used those amounts.
- What number should our line plot start with, then?
  It should start with the minimum.
- What number should our line plot end with, then?
  It should end with the maximum.
- Do you have to include every number in between the minimum and maximum number? Why?
  Yes, because it would break up the number "line" if we didn't include all the in-between numbers. It also wouldn't show the relationship among the numbers.

Direct students' attention to the solid line below the graph. Tell students that the dark line at the bottom of a graph that runs from left to right (horizontal) is called an *x*-axis and that in a graph, numbers or words are written along the *x*-axis. For line plots, we write numbers.

Guide students to write the numbers along the *x*-axis, one number (or range) per box, starting with their minimum number and ending with their maximum number.

It is not necessary to use all of the columns.

Example for Babbage Groups:

		X					
X		X		X			
X		X		X	X	X	X
2	3	4	5	6	7	8	9

Example for Galileo Groups:

	X			
X	X	X	X	
X	X	X	X	X
0-2	3-4	5-6	7-8	9-10

Example for Falconer Groups:

	X			
X	X	X	X	
X	X	X	X	X
91	92	93	94	95

Using the overhead or board, model for students how to complete the line plot for the first number. For example, if three students responded with the number "4," have the students make an "X" in three of the boxes above the number "4," (or in the range 3-4) as shown above. Note that some

Concept Check Up

Students may believe they must begin their number scale at 1 or 0 instead of the minimum value. Students may believe they can skip numbers between the minimum and maximum values, or that they can count by 2s. For line plots, counting by a number other than 1 might cause students to lose some of their data.

numbers have no X's above them, but because they occur between the minimum and maximum numbers, they must be included.

Instruct students to complete the line plots on their worksheets for their own survey data, filling in Xs above each number. Circulate among students, checking that they are completing the line plots correctly.

5. **Title the line plot**

Ask:

- What could make our plot easier to understand?
  A title would help the reader to know more about the graph.
- What could we call our line plot? *Answers will vary, but should relate to the data.*
  For example, if students collected information on the numbers of pets in their group, an appropriate title might be "Number of Pets Owned by Students."

Instruct students to write their titles at the top of their pages.

Conclude

6. Introduce the idea of an outlier. Outliers are extreme values in the data set—atypical responses. Ask students if they have any outliers in their data. Why or why not?

Give the students a chance to think through the questions on the *My Very Own Survey* Student Pages **[SMJ pages 159, 165, or 171]**. Then ask them to share their conclusions with a partner. Ask them to describe their ranges (minimum - maximum) and what those ranges say about their data.

You may want to discuss what the students learned from their surveys and what they would want to do next with their survey.

Assess

7. **Lesson conclusions**

Review Student Pages for understanding.

8. **Optional student practice**

You Decide Student Pages **[SMJ pages 173 & 175]** can be used for additional student practice with line plots. Teachers may choose to send it home as homework, use it as an optional activity when students finish their work, or assign it as morning work. Further instructions and differentiation options can be found in the teacher's manual on page 401.

My Very Own Survey

My Name: ______________________________

My Very Own Question:

____

____

Frequency Table:

Answer	Student Count	Number of Students
0		
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		
13		
14		
15		
More than 15		

SMJ page 155

Babbage

My Very Own Survey
ANSWER KEY
(Answers will vary.)

My Very Own Question:

Answers will vary. How many dolls do you have? How many pets do you have?

Frequency Table:

Answer	Student Count	Number of Students
0	✓ ✓ ✓	*3*
1		*0*
2	✓	*1*
3		*0*
4	✓	*1*
5		*0*
6		*0*
7	✓ ✓ ✓ ✓	*4*
8		*0*
9		*0*
10		*0*
11	✓ ✓	*2*
12		*0*
13		*0*
14	✓	*1*
15		*0*
More than 15		*0*

Babbage

My Very Own Survey—Line Plot

Minimum: ____________________ Maximum: ____________________

My Line Plot

Title: ____

Label: ____

SMJ page 157

Babbage

My Very Own Survey—Line Plot
ANSWER KEY
(Answers will vary.)

Minimum:0 Maximum: 14

My Line Plot

Title: *Students' Doll Collections*

							X									
X							X									
X							X				X					
X		X		X			X				X			X		
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14		

Number of Dolls Owned

Babbage

My Very Own Survey—Reflection

1. Mode: ________

2. Median: _______

3. What does your data tell you? Did most people respond in the same way?

____

____

4. Were there any outliers? Why or why not?

____

____

5. What do you think would happen if you used the same question and surveyed the whole school?

____

____

SMJ page 159

Babbage

My Very Own Survey—Reflection
ANSWER KEY
(Answers will vary.)

1. Mode: _7____

2. Median: _7____

3. What does your data tell you? Did most people respond in the same way?

   Most people did not respond in the same way. These data show that some people in our class have lots of dolls and some don't.

4. Were there any outliers? Why or why not?

   Yes. There was only one person who had 14 dolls, and the next closest person had 11.

5. What do you think would happen if you used the same question and surveyed the whole school?

   There would be a lot more X's. Also, the younger students may have more dolls, so some may have more than 14.

Galileo

My Very Own Survey

My Name: ____

My Very Own Question:

____

____

Frequency Table:

Answer	Student Count	Number of Students
0 - 2		
3 - 5		
6 - 8		
9 - 11		
12 - 14		
15 - 17		
18 - 20		
More than 20		

SMJ page 161

Galileo

My Very Own Survey
ANSWER KEY
(Answers will vary.)

My Very Own Question:

Answers will vary. How many marshmallows can you fit in your mouth? How many pairs of shoes do you own?

Frequency Table:

Answer	Student Count	Number of Students
0 - 2	✓ ✓	*2*
3 - 5	✓ ✓ ✓ ✓ ✓	*5*
6 - 8	✓	*1*
9 - 11	✓ ✓ ✓ ✓ ✓	*5*
12 - 14	✓ ✓ ✓	*3*
15 - 17	✓ ✓ ✓ ✓ ✓ ✓	*6*
18 - 20		*0*
More than 20		*0*

Galileo

My Very Own Survey—Line Plot

Range That Contains the Minimum: ______________________________

Range That Contains the Maximum: _____________________________

My Line Plot

Title: ____

Label: ____

SMJ page 163

My Very Own Survey—Line Plot

ANSWER KEY

(Answers will vary.)

Range That Contains the Minimum: *0-2*

Range That Contains the Maximum: *15-17*

My Line Plot

Title: *Marshmallow Mania*

					X	
	X		X		X	
	X		X		X	
	X		X	X	X	
X	X		X	X	X	
X	X	X	X	X	X	
0-2	3-5	6-8	9-11	12-14	15-17	

Number of Marshmallows

Galileo

My Very Own Survey—Reflection

1. What was the most popular answer to your survey question? Explain how you can tell.

2. What does your data tell you? Did most people respond in the same way?

3. Were there any outliers?

4. What do you think would happen if you used the same question and surveyed the whole school?

SMJ page 165

My Very Own Survey—Reflection

ANSWER KEY

(Answers will vary.)

1. What was the most popular answer to your survey question? Explain how you can tell.

   Answers will vary.

2. What does your data tell you? Did most people respond in the same way?

   Many people responded differently. It shows that there is a large range of mouth sizes and marshmallow holding ability.

3. Were there any outliers?

   No. There weren't any outliers.

4. What do you think would happen if you used the same question and surveyed the whole school?

   The range of answers may have gotten bigger because younger students could fit fewer marshmallows while older students would be able to hold lots more.

When discussing these questions, ask students to think about when ranges (instead of individual numbers) would be helpful. When are they not helpful?

Falconer

My Very Own Survey

My Name: ____

My Very Own Question:

____

____

Frequency Table:

Answer	Student Count	Number of Students

SMJ page 167

Falconer

My Very Own Survey
ANSWER KEY
(Answers will vary.)

My Very Own Question:

Students could use ranges if they wanted.

How many points out of 100 did you get on your last spelling test? How many hours in a week do you watch television?

Frequency Table:

Answer	Student Count	Number of Students
0-10	✓	*1*
11-20		*0*
21-30		*0*
31-40		*0*
41-50	✓	*1*
51-60	✓ ✓	*2*
61-70	✓ ✓ ✓	*3*
71-80	✓ ✓ ✓ ✓ ✓	*5*
81-90	✓ ✓ ✓ ✓ ✓ ✓	*6*
91-100	✓ ✓ ✓ ✓	*4*

Falconer

My Very Own Survey—Line Plot

My Line Plot

Title: ____

Label: __________________________________

SMJ page 169

My Very Own Survey—Line Plot
ANSWER KEY

(Answers will vary.)

My Line Plot

Title: *Spelling Test Scores*

								X	
							X	X	
							X	X	X
						X	X	X	X
					X	X	X	X	X
X				X	X	X	X	X	X
0-10	11-20	21-30	31-40	41-50	51-60	61-70	71-80	81-90	91-100

Test Scores

Falconer

My Very Own Survey—Reflection

1. What does your data tell you? Did most people respond in the same way?

____

____

2. Were there any outliers?

____

____

3. What do you think would happen if you used the same question and surveyed the whole school?

____

____

SMJ page 171

Falconer

My Very Own Survey—Reflection

ANSWER KEY

(Answers will vary.)

1. What does your data tell you? Did most people respond in the same way?

   The data show that many students did very well on the test. Most students scored 71 or better. Some students probably need to study a little bit more.

2. Were there any outliers?

   One person got between 0-10. The next lowest score was between 41 and 50. That one person is an outlier.

3. What do you think would happen if you used the same question and surveyed the whole school?

   It would probably stay pretty similar to the graph except there would be a lot more X's. It seems in every class someone will have forgotten to study, but lots will pass.

Discuss why they picked the numbers or ranges they did to answer their questions. How would changing the ranges affect the line plot?

Babbage & Galileo

Name: ___________________________________ Date: __________________

You Decide (Optional)

____

				X			
	X			X			
	X		X	X			
	X	X	X	X			
	X	X	X	X	X		
X	X	X	X	X	X		X
3	4	5	6	7	8	9	10

____

Think of a story to go with the graph. Label the graph.

Explain this graph. Be sure to talk about the median and the mode.

____

____

____

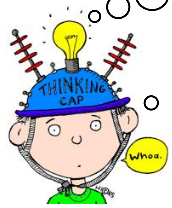
____

____

____

Number of pizza toppings?

What about the number of doors each person has in their home?



What about the number of pets students have?

SMJ page 173

Falconer

Name: ______________________________ Date: ________________

You Decide (Optional)

Your job is to create a line plot that illustrates that information below. You will need to make up the data. There is not one right answer. Think about what the mode is. Then illustrate that. Then think about what the median is and change the graph to fit the median.

Mode: 4 Median: 7 Range: 3-10

3	4	5	6	7	8	9	10

Label: ______________________________

Think of a story to go with the graph. Label the graph.

Explain your graph.

____

____

____

____

SMJ page 175

You Decide
DISCUSSION GUIDE AND ANSWER KEY

Differentiation Options: *The Falconer version of this assignment is much more challenging than the Babbage/Galileo version. The students working on the Falconer version must create their own graph using only the mode, median, and range. There are many ways to accomplish this, but they have to be willing to try something without knowing for sure that is the answer. The Babbage/Galileo version also forces students to take a risk, but it is a little less of a risk. They need to develop a story to go with a given graph.*

Discussion Questions: *After the students share a little about their stories and struggles, ask the students all to think about the Falconer version. Have a student explain how he/she created the graph. Then ask the other students to answer the following questions. What would a line plot look like if you knew the mode was 3 and the range was 1-5? Have a student draw it on the board. Then add in the challenge that the median is 4. How does that change the graph? What happens when the median and mode are the same? Does that make it easier or harder to visualize?*

Babbage and Galileo Key

Sample Answer
My graph explains how many books the people in my class read in the last month. The median is 6 and the mode is 7. The range is 3 to 10. Most people in class read between 4 and 7 books. Everyone in class read at least 3 books, but nobody in class read more than 10 books.

Falconer Key

Sample Answer

	x						
	x			X	x	x	
	x	x	x	X	x	x	x
x	x	x	x	X	x	x	x
3	4	5	6	7	8	9	10

Number of Visits to the Library

This graph shows how many times a month students in class visit the school library. Most students in the class go to the library 4 times a month or more. One student went only 3 times. The most anyone went was 10 times. Three students went to the library 7, 8, or 9 times, and two students went to the library 5, 6, or 10 times. There were no outliers. Everyone in class goes to the school library.

LINE PLOTS—

USING FREQUENCY TABLES

LESSON 15

Big Mathematical Ideas

After collecting data, it is important that the information is represented properly. A line plot is a good way to show data that involve counting or tallies. All line plots should include a label for the horizontal axis, as well as a title. The spacing of the data points also should be aligned accurately.

Lesson Objectives	• Students will construct accurate, labeled line plots from their *Greening Up* survey data.
Materials	• Student Page—*Greening Up Frequency Table* [SMJ page 177] • Student Page—*Greening Up Line Plot* [SMJ page 179] • Student Page—*Greening Up Conclusions (Babbage, Galileo, and Falconer)* [SMJ pages 181, 183, & 185] • Student Page—*Miss Numbers' Cookie Party* (Optional) [SMJ page 187] • Student Page—*Crazy Circus Hats* (Optional) [SMJ page 189] • Completed survey from SMJ page 149 • Blank paper

Lesson Preview

Students use the survey data they collected to make a frequency table and then a line plot.

Initiate

1. Data collection and debriefing

If you are collecting data as a class, circulate during the collection process. If there are missing answers, try to ask students about them during the data collection experience. After the experience, ask students the following questions, and use their responses to gauge understanding:

- Did you find it easy or hard to give your survey to other people? What made it easy? What made it difficult? *Answers will vary.*
- Would you do anything differently if you were to do this again? If so, what would you change and why? *Answers will vary.*
- What questions will this data help us answer?

Investigate

2. Total data using a frequency table

Direct students to student surveys and the *Greening Up Frequency Table* Student Page **[SMJ page 177]**. Review with students the purpose of a frequency table—to help count the number of students who answered in a particular way. This is only one example of a question that could be answered through this survey. This answers how many students practice a certain number of ways of saving the earth.

Have students refer to SMJ page 149:

	Students Who Used This Number of Greening Practices
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

Tell students:

- Please count the number of "YES" answers on your survey. Think of that number in your head. Raise your hand when I come to your number.
- How many of you counted zero "YES" answers? Please raise your hand. *Count the number of hands raised and record that number in the appropriate box under "0."* Direct students to enter the number in the same box on their Frequency Tables.
- How many of you counted one YES answer? Please raise your hand. *Count hands and record the total in the appropriate box under "1."* Direct students to enter the number in the same box on their Frequency Tables.
- Repeat for numbers 2-9.

3. Examine the frequency table

Ask:

- How many people used 4 greening practices in our survey?
  Answers will vary.
- What is the minimum (smallest) number of greening practices someone reported using?
  Answers will vary depending on the class data. Make sure students understand that zero is an amount.
- What is the maximum (largest) number of greening practices someone reported using?
  Answers will vary depending on the class data, but the number cannot be greater than 9 since only 9 ways were presented.
- Are you surprised by these data?
  Answers will vary.

4. Complete the *Greening Up Line Plot*

Instruct students to refer to the *Greening Up Line Plot* Student Page **[SMJ page 179]**. Tell students that they will be completing a line plot for the data. Remind students that a line plot is a graph that shows the frequency of answers on a number line. Also, remind students that their line plots should start at the minimum number on the frequency table and end at the maximum number and should include all the numbers in between.

Call students' attention to the solid line below the table. Remind students that this is called an *x*-axis. Guide students as they write the numbers along the *x*-axis, one number per box, starting with their minimum number and ending with their maximum number.

Ask:

- How could we use the numbers from the *Frequency Table* to make a picture on the *Greening Up Line Plot*?
  We could create a line plot by drawing X's above the numbers to show how many people reuse each number of ways. For example, if five people said they use zero ways, we could fill in five boxes above the number 0.

Using the board, model for students how to complete the line plot for the first number. Instruct students to draw the appropriate number of X's above each number. Check to see that students are completing the line plot correctly.

Conclude

5. Ask:

- What could make our plot easier to understand?
  A title would help the reader to know more about the graph. Instruct students to write their titles at the top of their pages.
- What could we call our line plot?
  Answers will vary, but should relate to the data. For example, an appropriate title might be “Number of Ways Students Recycle, Reuse, and Reduce.”
- What is the mode and median of our data? What do they mean?
  Specific answers will vary. These should be recorded on the Greening Up Conclusions *Student Pages* **[SMJ pages 181, 183, or 185]**. *The mode of the data set will tell you the most common number of greening practices the people taking the survey uses. The median tells you the middle of the data set, so half the people in the data use that many practices or they use more and half use that many or less.*

Based on their performance in this line plot section, place students in appropriate groups (Babbage, Galileo, or Falconer). They may work independently in their groups on the hypothetical situations presented in questions 3 and 4 at the bottom of their *Greening Up Conclusions* Student Pages **[SMJ pages 181, 183, or 185]**. After the groups have taken a few minutes to think through their questions, have a class discussion examining what factors influence survey results.

Miss Numbers’ Cookie Party Student Page (Optional) **[SMJ page 187]** may be used for a quick assessment of students’ understanding of several data analysis terms.

Look Ahead

6. Make sure the students keep track of the survey data they collected. In the next lesson, they will be using the data to create a different type of graph to answer different questions.

Assess

7. **Optional student practice**

Crazy Circus Hats Student Page (Optional) **[SMJ page 189]** can be used for additional student practice with tables and line plots. Teachers may choose to send it home as homework, use it as an optional activity when students finish their work, or assign it as morning work.

Name: ____

Greening Up Frequency Table

	People Who Used This Number of Greening Practices
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

SMJ page 177

Greening Up Frequency Table

ANSWER KEY

(Numbers will vary.)

	People Who Used This Number of Greening Practices
0	*6*
1	*14*
2	*0*
3	*3*
4	*7*
5	*3*
6	*3*
7	*3*
8	*11*
9	*5*

Name: ____

Greening Up Line Plot

Directions: Create a line plot using the data in your table.

Title: ____

SMJ page 179

Greening Up Line Plot
ANSWER KEY
(Numbers will vary.)

Directions: Create a line plot using the data in your table.

Title: *Number of Ways People Report Using to Save the Earth*

16											
15											
14		X									
13		X									
12		X									
11		X							X		
10		X							X		
9		X							X		
8		X							X		
7		X			X				X		
6	X	X			X				X		
5	X	X			X				X	X	
4	X	X			X				X	X	
3	X	X		X	X	X	X	X	X	X	
2	X	X		X	X	X	X	X	X	X	
1	X	X		X	X	X	X	X	X	X	
	0	1	2	3	4	5	6	7	8	9	

Number of Greening Up Practices

Babbage

Name: ________________________________ Date: ________________

Greening Up Conclusions

1. Use the data from the class survey to find the mode, range, and median.

   Mode: _________ Range: _________ Median: ________

2. Think about the number of different "greening up" practices on the survey. How many "greening up" practices were on the survey?

   ____

   ____

3. What if we only asked about 3 "greening up" practices? What might happen to the results?

   ____

   ____

4. How does the number of "greening up" practices on the survey affect the line plot?

   ____

   ____

SMJ page 181

Babbage

Greening Up Conclusions
ANSWER KEY

1. Use the data from the class survey to find the mode, range, and median.

   The mode, median, and range will depend on your data.

2. Think about the number of different "greening up" practices on the survey. How many "greening up" practices were on the survey?

   9

3. What if we only asked about 3 "greening up" practices? What might happen to the results?

   We would not be able to see the other ways the students help save the Earth. We would not have collected as much information.

4. How does the number of "greening up" practices on the survey affect the line plot?

   Answers will vary. More questions may discourage people from participating. Fewer questions may not give you as much information.

Galileo

Name: ________________________________ Date: ________________

Greening Up Conclusions

1. Use the data from the class survey to find the mode, range, and median.

   Mode: _________ Range: _________ Median: ________

2. Think about the "greening up" practices on the survey. Were they the most common "greening up" practices?

   ____

   ____

3. How would the data change if you only used the unpopular "greening up" practices, like reusing a lampshade as a hat?

   ____

   ____

4. How does the popularity of the "greening up" practice affect the line plot?

   ____

   ____

SMJ page 183

Greening Up Conclusions
ANSWER KEY

1. Use the data from the class survey to find the mode, range, and median.

   The mode, median, and range will depend on your data.

2. Think about the "greening up" practices on the survey. Were they the most common "greening up" practices?

   Yes. They were the practices we thought people would actually do.

3. How would the data change if you only used the unpopular "greening up" practices like reusing a lampshade as a hat?

   You would have more people saying they don't use that recycling practice. There would be fewer people saying they use many of the practices.

4. How does the popularity of the "greening up" practice affect the line plot?

   The more popular the practice, the more likely people are to say that they do it. Then there are more people represented close to the 7-9 range.

Falconer

Name: ________________________________ Date: ________________

Greening Up Conclusions

1. Use the data from the class survey to find the mode, range, and median.

   Mode: _________ Range: _________ Median: ________

2. Think about the types of people the class surveyed. Who participated in the survey?

   ____

   ____

3. How would the data change if you only surveyed people who lived in the same house?

   ____

   ____

4. How does the type of people you survey affect the line plot?

   ____

   ____

SMJ page 185

Falconer

Greening Up Conclusions
ANSWER KEY

1. Use the data from the class survey to find the mode, range, and median.

   The mode, median, and range will depend on your data.

2. Think about the types of people the class surveyed. Who participated in the survey?

   Answers will vary.

3. How would the data change if you only surveyed people who lived in the same house?

   There would be fewer differences in the data because people who live in the same house tend to use similar greening up practices.

4. How does the type of people you survey affect the line plot?

   If the people have a lot in common, the line plot will show how they are all clustered around one area. If the people are randomly selected and quite different, the line plot will show a variety of responses.

Name: ____

Miss Numbers' Cookie Party (Optional)

Miss Numbers loves numbers! She loves numbers so much that she wants to give a party and invite all of the numbers she knows. She wants to invite her friends **2, 4, 5, 1, and 9.**

However, she has to plan carefully to have enough food. Each number will eat the same number of cookies as it stands for, and so number **1** will eat **one** cookie, number **2** will eat **two** cookies, and so on. Help Miss Numbers plan her party by answering the questions below (Do your work on a separate piece of paper).

1. **HOW MANY** cookies in total will Miss Numbers need to buy to feed her guests? ______________________________________

2. What is the **MEDIAN** of all the cookies eaten? _______________

3. What is the **MAXIMUM** number of cookies a guest will eat? _____

4. What is the **MINIMUM** number of cookies a guest will eat? _____

5. Is there a guest number that could be considered an **OUTLIER**? If so, which guest?__________________________________

Now suppose the number 2 has a twin sister, and we'll call her 2b. She also eats 2 cookies.

6. What is the **MODE** of all the cookies eaten? _________________

Miss Numbers thanks you! Mmmm….cookies!

SMJ page 187

Miss Numbers' Cookie Party (Optional)
ANSWER KEY

Miss Numbers loves numbers! She loves numbers so much that she wants to give a party and invite all of the numbers she knows. She wants to invite her friends **2, 4, 5, 1, and 9.**

However, she has to plan carefully to have enough food. Each number will eat the same number of cookies as it stands for, and so number **1** will eat **one** cookie, number **2** will eat **two** cookies, and so on. Help Miss Numbers plan her party by answering the questions below (Do your work on a separate piece of paper).

1. **HOW MANY** cookies in total will Miss Numbers need to buy to feed her guests? *21*

2. What is the **MEDIAN** of all the cookies eaten? *4*

3. What is the **MAXIMUM** number of cookies a guest will eat? *9*

4. What is the **MINIMUM** number of cookies a guest will eat? *1*

5. Is there a guest number that could be considered an **OUTLIER**? If so, which guest? *9*

Now suppose the number 2 has a twin sister, and we'll call her 2b. She also eats 2 cookies.

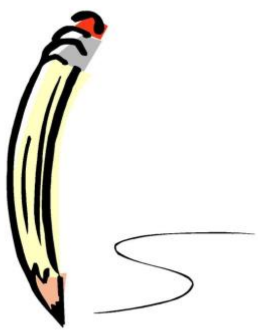
6. What is the **MODE** of all the cookies eaten? *2*

Miss Numbers thanks you! Mmmm….cookies! 

Name: ________________________________ Date: ________________

Crazy Circus Hats (Optional)

Clara went to the circus and kept track of how many hats each person or animal was wearing. Help her out by completing the table with the totals.



	People/Animals Wearing this Number of Hats	Total
0	~~IIII~~	
1	IIII	
2	III	
3	~~IIII~~ III	
4		
5	I	

Create a line plot using the information in the table.

0	1	2	3	4	5

1. What is the mode? _____ What is the median? _____

SMJ page 189

2. What does it mean when the mode and the median are different?

___

___

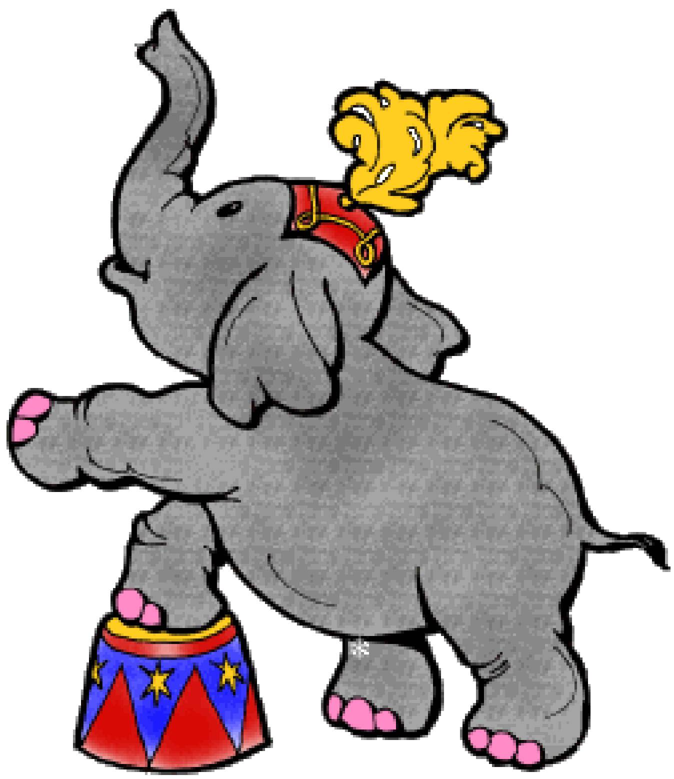
3. Jane wanted to join the circus. How many hats should she wear? Use the data from the line plot to explain your thoughts.

___

___

___

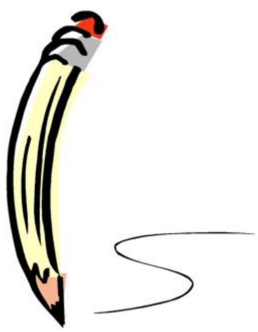
___



SMJ page 191

Crazy Circus Hats (Optional)
ANSWER KEY

Clara went to the circus and kept track of how many hats each person or animal was wearing. Help her out by completing the table with the totals.



<table>
<tr><th></th><th>People/Animals Wearing this Number of Hats</th><th>Total</th></tr>
<tr><td>0</td><td>𝍸</td><td>5</td></tr>
<tr><td>1</td><td>||||</td><td>4</td></tr>
<tr><td>2</td><td>|||</td><td>3</td></tr>
<tr><td>3</td><td>𝍸 |||</td><td>8</td></tr>
<tr><td>4</td><td></td><td>0</td></tr>
<tr><td>5</td><td>|</td><td>1</td></tr>
</table>

Even if no one wears 4 hats, it is still on the line plot. Ask students, "Why is that?"

Create a line plot using the information in the table.

			X		
			X		
			X		
X			X		
X	X		X		
X	X	X	X		
X	X	X	X		
X	X	X	X		X
0	1	2	3	4	5

1. What is the mode? _*3*____ What is the median? _*2*____

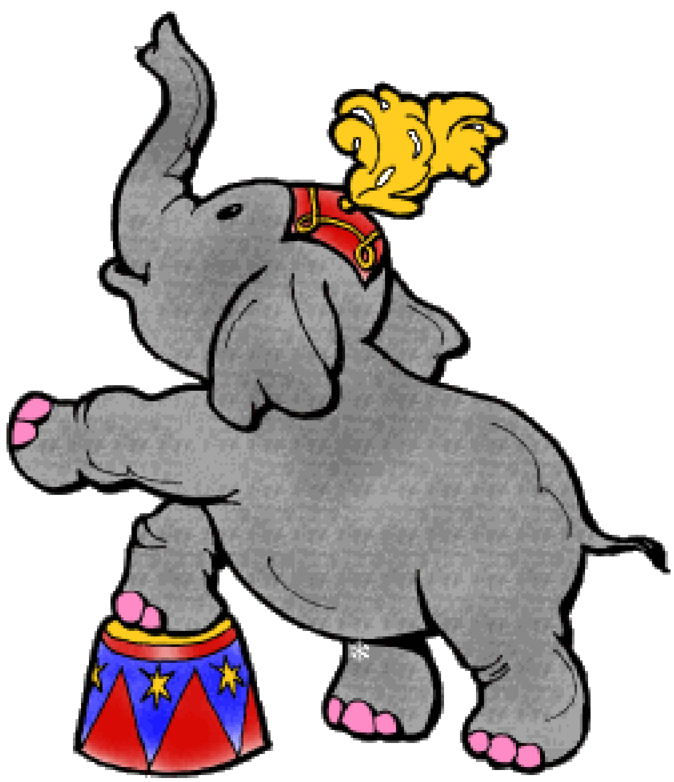
Do more people wear more than 2 hats or less than 2 hats? If students understand 2 is the median, they will realize they have to be equal.

2. What does it mean when the mode and the median are different?

The mode is the most common occurrence and the median is the exact middle of the data set. When they are different, it means that the most common response is not in the middle. The data is not normally distributed. (Students only have to say that most frequent is not in the middle, or they could use a graph specific explanation. 3 hats were the most frequently seen number of hats, but 2 hats is the exact middle of the data set.)

3. Jane wanted to join the circus. How many hats should she wear? Use the data from the line plot to explain your thoughts.

Answers will vary. She should wear 4 hats because no one else is wearing 4 hats. Or, she could wear 3 hats to fit in with the most of the circus members. There are many possibilities.



LINE PLOTS AND BAR GRAPHS—

REUSING OUR DATA

LESSON 16

Big Mathematical Ideas

Data can be represented in different graphs for different purposes. Bar graphs can be used to compare categorical data, and line plots illustrate the distribution of the data over a number line. Surveys can be analyzed using both types of graph.

Lesson Objectives	• Students will use a bar graph to analyze the survey data in a different way. • Students will compare the line plot and bar graph. • Students will describe trends in their data.
Materials	• Student Page—*Curious Questions* [SMJ page 193] • Student Page—*Greening Up Data Table 2* [SMJ page 195] • Student Page—*A New Greening Up Graph* [SMJ page 197] • Student Page—*Ice Cream Party* (Optional) [SMJ page 199] • Check Up #3 [SMJ page 203] • Completed survey from SMJ page 149

Lesson Preview

In this lesson, the students will think about other questions that could be answered using the data collected from the survey. Then, they will complete a data table and graph to answer their questions.

Initiate

1. Examination of survey data

Ask the students to think about the surveys. Ask the students to remember what question the line plot answered. Then, ask them to think about other questions they might like to know about. You may want them to find a partner to discuss possible questions. They can record their questions on the *Curious Questions* Student Page **[SMJ page 193]**. They may be curious about which practice is the most popular. Which one is done the least? How many people practice number 9? Is there a difference between the amount of people who are recycling and reusing?

Have the students regroup and discuss the questions. Ask them how they might find answers to those questions. Then, pick a few questions that could be answered by calculating how many people said yes to each item. (All the sample questions could be answered by doing that.) Ask the students to make their hypotheses on which was the most frequently done greening practice. Which do they think was the least?

Investigate

2. Creation of *Greening Up Data Table 2*

Ask the students to look at their original survey **[SMJ page 149]**. Have the students raise their hands when the person they interviewed said "yes" and complete the *Greening Up Data Table 2* Student Page **[SMJ page 195]**. The table has room for tallies and totals. Feel free to use both or only the totals.

For example:
How many people interviewed a person who said that he or she did our greening practice number 1? Raise your hand if your person said, "yes."

Continue through all nine questions. Examine the data together. Ask the students if their hypotheses were correct. Are they surprised by any of the numbers?

3. Create *A New Greening Up Graph*

Explain that this information is different from the information graphed for the last lesson. For the line plots, the students reported the total number of yeses the person reported. In the bar graph for the *A New Greening Up Graph* Student Page **[SMJ page 197]**, the students are reporting the response to individual questions. The questions are different categories, so bar graphs are appropriate for this data set.

Because the students have had a lot of practice with bar graphs already, allow them to create their graphs on their own.

Conclude

4. Share findings

Discuss the final graphs. Ask the students to compare the line plot with the bar graph.

- Which graph do you think is the most useful? Why?

- Is there a time when the other kind of graph would be helpful? When?
- Are you able to get the same information from both graphs?

Look Ahead

5. In the next lesson, the students will pull all the pieces and data together from the entire unit to create a unit project. You may ask the students to reflect upon what they have learned throughout the unit and how they would want to share their knowledge with others.

Assess

6. **Lesson conclusions**
The bar graphs could be done independently and used as an assessment. *Check Up #3* Student Page **[SMJ page 203]** is an assessment tool for the line plot. There are also some number and operation questions on that assessment.

7. **Optional student practice**
Ice Cream Party Student Page **[SMJ page 199]** can be used for additional student practice with comparing line plots and bar graphs. Teachers may choose to send it home as homework, use it as an optional activity when students finish their work, or assign it as morning work.

Name: ______________________________ Date: ________________

Curious Questions

Think about the survey data you collected. What other questions could you answer using these data?

____

____

____

____

____

____

____

____

____

SMJ page 193

Name: ____

Greening Up Data Table 2

Question #	People Who Said "Yes"	Total
1		
2		
3		
4		
5		
6		
7		
8		
9		

SMJ page 195

Greening Up Data Table 2

ANSWER KEY

(Answers will vary.)

Question #	People Who Said "Yes"	Total
1	~~||||~~ ~~||||~~ ~~||||~~ ||||	***19***
2	~~||||~~ ||||	***9***
3	~~||||~~ ~~||||~~ ||||	***14***
4	~~||||~~ ~~||||~~ ~~||||~~ ||	***17***
5	~~||||~~ ~~||||~~	***10***
6	~~||||~~ ||||	***9***
7	~~||||~~ ||	***7***
8	~~||||~~ ~~||||~~ ~~||||~~	***15***
9	~~||||~~ ~~||||~~ ~~||||~~ ||	***17***

Name: ______________________________ Date: ________________

A New Greening Up Graph

Title: ____

Label: ____________________

Label: ______________________________

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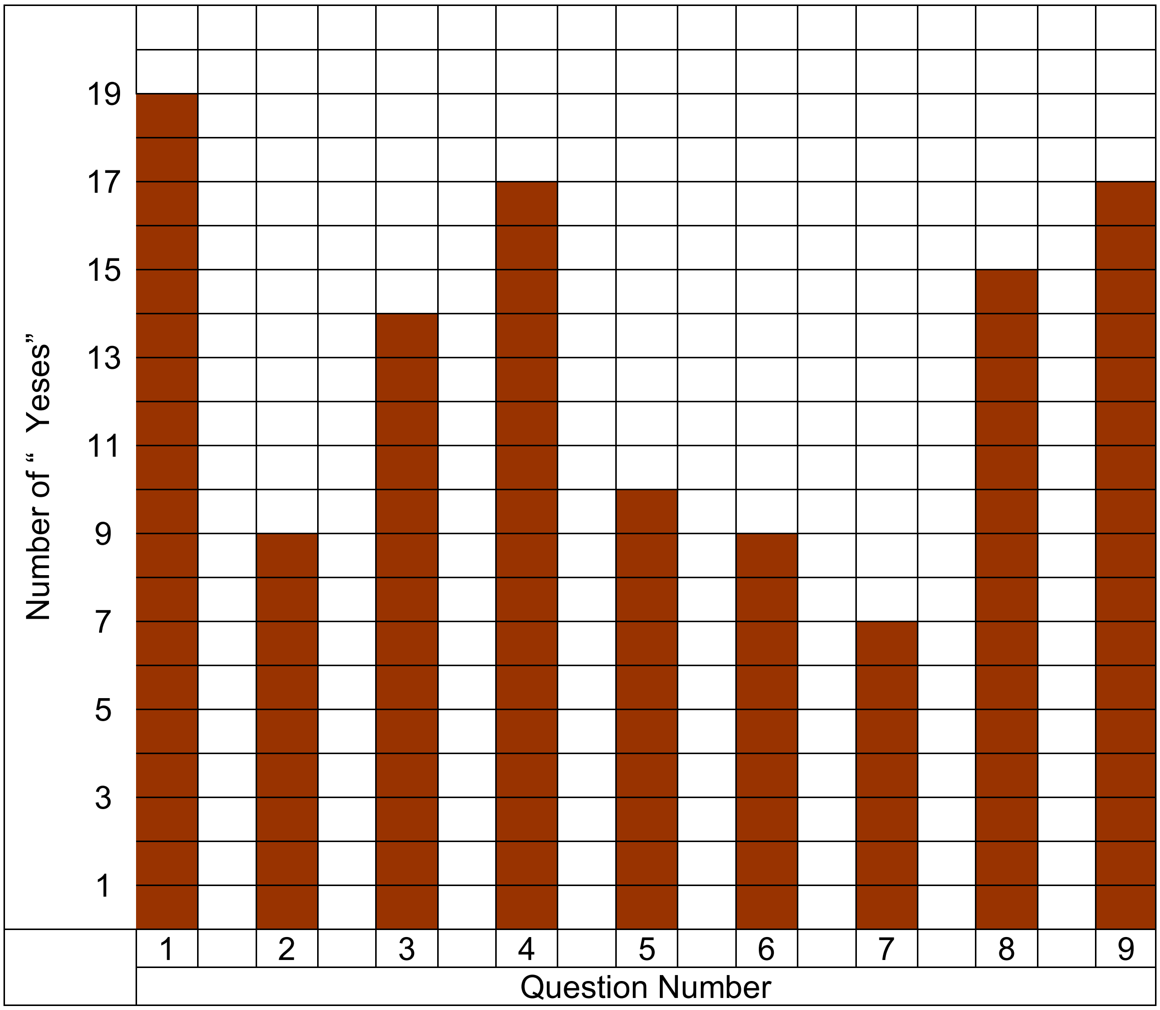
A New Greening Up Graph

ANSWER KEY

(Answers will vary.)

Title: Greening Up Survey Question Results



Name: ________________________________ Date: ________________

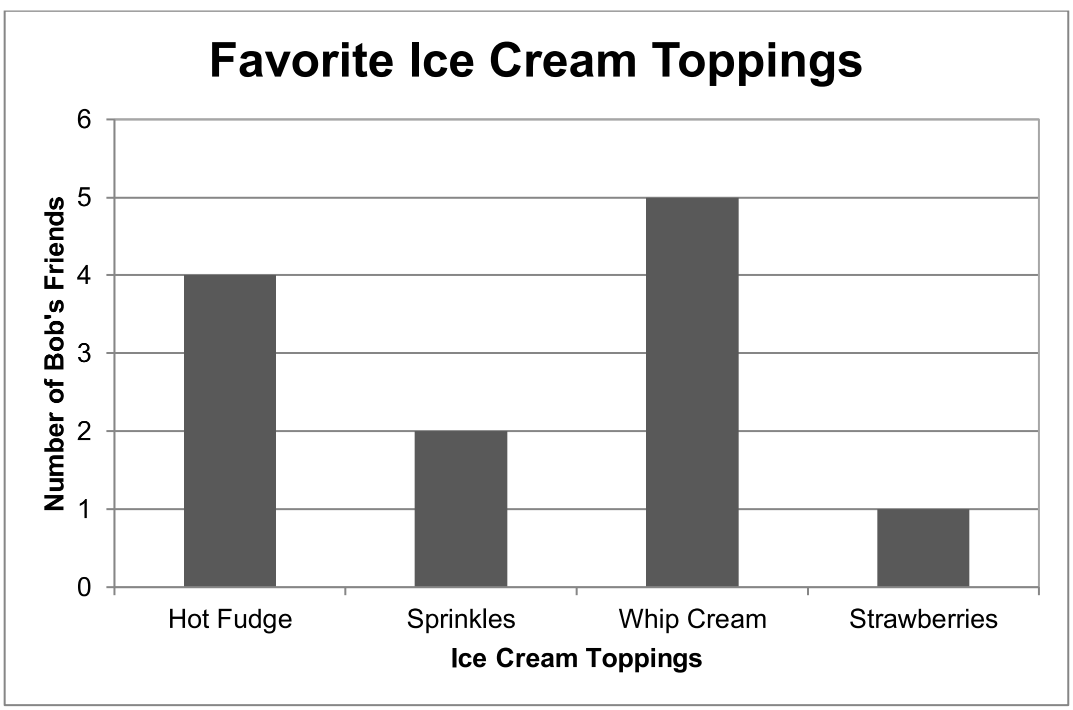
Ice Cream Party (Optional)

Bob had an ice cream party to celebrate his birthday. The first game at Bob's party was to see who could eat their ice cream in the fewest bites. Because he loves graphing so much, he also had all of his friends record their favorite toppings.

Total Bites to Eat a Cup of Ice Cream

		X		
	X	X		
X	X	X	X	
X	X	X	X	X
18	19	20	21	22

Number of Bites



SMJ page 199

1. How many students came to Bob's party? Explain how you can tell from the line plot and the bar graph.

____

____

____

____

2. Who won the ice cream contest?

____

____

3. What is different about the two graphs?

____

____

4. Bob decided next year he would see how many bites it took to eat a piece of pizza. What would the line plot of that look like?

5. Compare your pizza line plot to the ice cream line plot.

____

____

____

____

Optional: How many ice cream bites did all the friends take? __________

SMJ page 201

Ice Cream Party (Optional)
DISCUSSION GUIDE AND ANSWER KEY

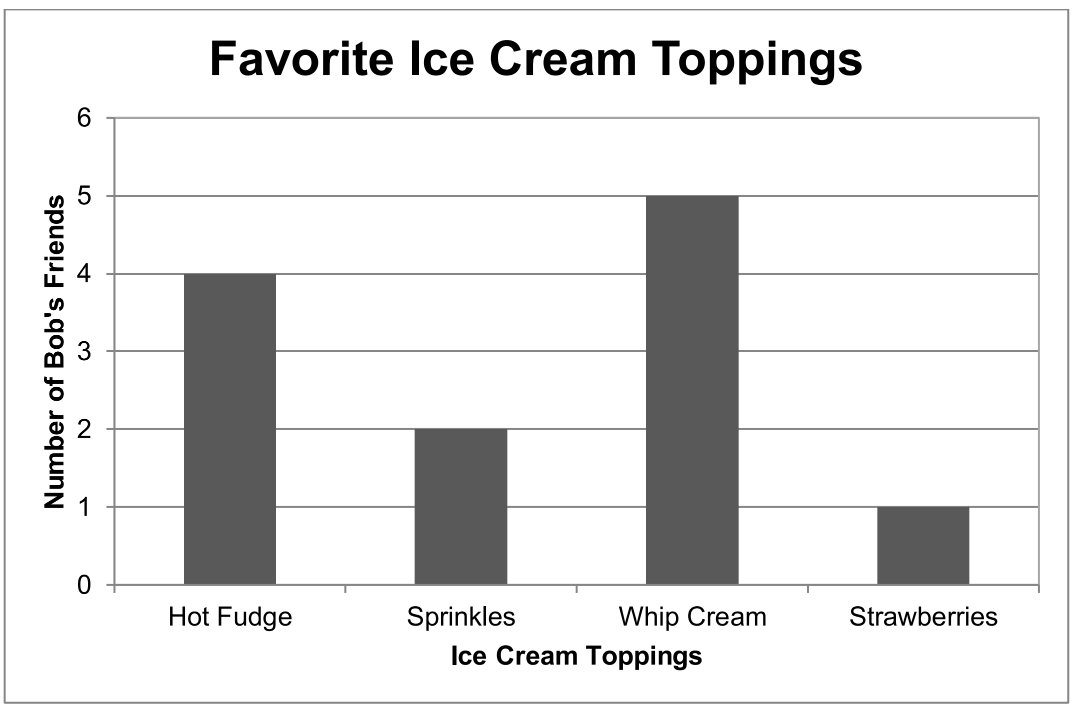
Discussion Questions: *Ask the students to think of a question that could be answered by looking at only the bar graph. Then ask them to think of a question for only the line plot. Ask which graph do they find for interesting. More useful?*

Bob had an ice cream party to celebrate his birthday. The first game at Bob's party was to see who could eat their ice cream in the fewest bites. Because he loves graphing so much, he also had all of his friends record their favorite toppings.

Total Bites to Eat a Cup of Ice Cream

		X		
	X	X		
X	X	X	X	
X	X	X	X	X
18	19	20	21	22

Number of Bites



1. How many students came to Bob's party? Explain how you can tell from the line plot and the bar graph.

   12 students came to the party. Using the line plot, you can count each X as it stands for one student. Using the bar graph, you have to add the total of each topping to get the total of students.

   Talk about using raw data to find the names rather than a graph.

2. Who won the ice cream contest?

   Two students tied with 18 bites each. You can not tell from the graphs the names of these students.

3. What is different about the two graphs?

   These two graphs answer two different questions. One uses a line plot to address how many bites everyone took, and the other uses a bar graph to examine what toppings are preferred.

4. Bob decided next year he would see how many bites it took to eat a piece of pizza. What would the line plot of that look like?

   Answers will vary, but it should take students less bites to eat a piece of pizza than to eat a cup of ice cream.

5. Compare your pizza line plot to the ice cream line plot.

   Answers will vary. Students may want to compare the mode, median or scales.

Optional: How many ice cream bites did all the friends take? *237*

Name: ______________________________ Date: ________________

Check Up #3

1. Mr. Robinson’s math class surveyed 10 students in their school. They asked them how many books they read last month. Here are their answers:

   7 2 6 8 7 5 7 6 5 7

   Make a line plot that shows these data. (Don’t forget to label!)

2	3	4	5	6	7	8

2. What is the mode of these data? ______________________________

3. What is the median? ______________________________

4. What is the maximum? ______________________________

5. What is the minimum? ______________________________

6. The principal wants to know if most students are reading at least 6 books. What would you tell her? Use data from the line plot.

   ____

   ____

   ____

   ____

SMJ page 203

7. The closest estimate for $4.78 + $1.13 is ___.

   A. $4
   B. $5
   C. $6
   D. $7

8. 75 - 28 =

9. $\begin{array}{r} 696 \\ \underline{+\ 302} \end{array}$

10. 6,086 - 45 =

SMJ page 205

Check Up #3
ANSWER KEY

1.

Books Read Last Month

					X	
					X	
			X	X	X	
X			X	X	X	X
2	3	4	5	6	7	8

Number of Books

Possible Student Difficulties

- Students commonly forget to put the values on a line plot that are not represented in the data set. Students must be sure to include numbers 3 and 4 even if there are no data to count for those values.

2. What is the mode of these data? *7*

3. What is the median? *6.5*

4. What is the maximum? *8*

5. What is the minimum? *2*

6. The principal wants to know if most students are reading at least 6 books. What would you tell her? Use data from the line plot.

   Answers will vary. From the students surveyed, most students are reading at least 6 books. Out of ten students, only 3 read less than 6. It would be helpful to survey a larger section of the school to make sure these results are typical for all students.

7. *C. $6*

8. *47*

9. *998*

10. *6,041*

Appendix A:
Written Communication in Mathematics

Young students need guidance when communicating mathematically in writing. Writing is very important in helping student think deeply about a problem to solidify their understanding. The following steps for how to write a good response in math are important to help students develop strong mathematics communication skills.

The following writing process should be displayed in the classroom for continued reference throughout all three units:

How to write a good response in math:

1. **Understand the Question**
   - Read the question. Make sure you know what it is asking you to do.
2. **Make a Plan**
   - Think about how you will solve the problem.
   - Write down you ideas.
   - Think about <u>why</u> you are solving the problem that way.
3. **Come up with an Answer**
   - Use your plan to solve the problem
4. **Write a Response**
   - Write your answer down
   - Write down step-by-step how you solved the problem.
   - If you have to explain, write down why you are solving the problem that way.
5. **Reflect and Review**
   - Read your response to yourself. Be sure it makes sense.
   - Sometimes have someone else read your response.
6. **Revise**
   - Make changes to your response if you need to.

Because this strategy can be used with any mathematical problem, the example presented here is not specific to the content of the "What Works" units. Rather, it is intended to demonstrate to students how to break down any mathematical problem in order to write about it effectively. Discuss each step with your students.

Provide each student with one of the following worksheets based on his or her need for more or less support in responding to the prompts. The first worksheet has less support and the second has more.

Student Mathematicians Write About It:

The Problem: Carlos is making trail mix. He buys 14 ounces of nuts, 8 ounces of dried fruit, and 12 ounces of chocolate chips. He and his brother eat 3 ounces of the chocolate chips out of the bag. Carlos mixes all the remaining ingredients to make his trail mix. How many ounces of trail mix does Carlos make?

Understand the Question

____

____

Make a Plan

____

____

____

Come Up With an Answer

____

____

Write a Response

____

____

Reflect and Review

Revise (if necessary). Rewrite or add to your response. Use the back if necessary.

Student Mathematicians Write About It:

The Problem: Carlos is making trail mix. He buys 14 ounces of nuts, 8 ounces of dried fruit, and 12 ounces of chocolate chips. He and his brother eat 3 ounces of the chocolate chips out of the bag. Carlos mixes all the remaining ingredients to make his trail mix. How many ounces of trail mix does Carlos make?

Understand the Question
The question is asking me to

____

____

Make a Plan
I should start solving the problem by

____

____

Come Up With an Answer
I discovered that the answer to the problem is

____

____

Write a Response
The answer to the problem is ___________. I found this by

____

____

____

Reflect and Review

Revise (if necessary). Rewrite or add to your response. Use the back if necessary.

Appendix B: Talk Moves

Another strategy to extend student's discussion of mathematical problems is **Talk Moves**. This strategy involves asking students to restate and agree or disagree with what another student has said. A video clip of one teacher's use of this strategy can be found on the Awesome Algebra DVD you received with your supplies for the Algebra unit.

A class discussion using **Talk Moves** might proceed like this:

Teacher: How did you find your answer, Sarah?
Sarah: Well, I knew that the two friends wanted to share the 5 cookies, but when they each got two, there was still one cookie left. So the only fair way to split the last cookie was to break it in half. So then each friend got two and a half cookies.
Teacher: That sounds like a good strategy, Sarah. *Paul, could you please restate how Sarah got her answer.*
Paul: Yeah. She said that the only fair way to share the 5 cookies was to divide the remainder into two halves and give an extra half to each friend.
Teacher: I like the way you used the word "remainder" like a mathematician, Paul. *Now, does anyone disagree with how Sarah and Paul solved the problem? Can anyone find a different way to solve the problem?*

Verbal Communication
Discussion is very important in nudging student thinking forward. This is in line with the social constructivist theory of Lev Vygotsky whose research gives credence to the idea that students can be guided to better mathematical understandings as they analyze complex skills and concepts together (Biehler & Snowman, 1993). Vygotsky's research showed that actions involving complex knowledge could be internalized more quickly with the help of guiding questions and discussion (Vygotsky, 1978). One way to conceptualize this type of communication is to compare it to the writing process (Cazden, 2001). Students engaged in this type of communication, known as exploratory talk, are manipulating their ideas much like in the beginning phases of the writing process. Students participating in more elaborate talk, the predominant form of communication in more traditional mathematics classes, tend to express refined ideas that are like those found in the final draft of a written work. Students should have ample opportunities to engage in exploratory talk to help them develop more elaborate ideas.

Students who have had practice in talking at length with their peers and teacher about solving mathematics problems tend to persist longer in trying to understand a new problem. As they get used to the process of explaining their thinking and revising their thinking in light of others' comments, they come to understand that it takes time to think through a problem. As the class becomes more practiced at communicating mathematically, students are motivated to organize, consolidate, and clarify their own thinking to be able to participate with their peers. Students learn to view problems from different perspectives and to appreciate a variety of thinking and problem-solving styles as they listen to their peers' methods of solving problems.

Teachers can utilize particular strategies to foster this type of communication in their classrooms. Chapin, O'Connor, and Anderson refer to such strategies as "talk moves" in their book, *Classroom Discussions: Using Math Talk to Help Students Learn* (2003). In this unit, we use five talk moves:

1. The first move is *revoicing* in which the teacher restates a student's idea and then verifies whether it was accurate. For example, a teacher might ask, "You said that the area is 24 square inches?" This move can be used when a student's response is unclear. It also allows students to clarify their thinking.

2. A second move is *repeat/rephrase* and is similar to revoicing. Instead, the teacher asks other students to restate an idea by asking questions such as, "Can you repeat what she said in your own words?" It is important to follow up with the student contributing the original thought to ensure that the idea was heard as intended. This not only validates the idea but also gives the class another version of the idea, allows time to process it, and makes certain that students are following the conversation.

3. The third move, *agree/disagree and why*, is used after the teacher makes sure that students have heard and have had time to process the thought. By posing questions like, "Do you agree or disagree with that idea? Why?" teachers can draw out student thinking by having students apply their own understanding to someone else's thoughts. It is essential that teachers do not offer their positions at this juncture, allowing students to grapple with their own thoughts. Teachers can help students focus on the correct concepts after they have had the opportunity to develop their reasoning.

4. Teachers can prompt students to participate further by using the fourth move, *adding on*. Questions like, "Who would like to add to their ideas?" serve to advance the discussion. Students benefit from this move because original ideas become more comprehensive as more perspectives are considered.

5. Finally, a fifth move, *wait time*, allows students to organize their ideas and serves to encourage all students to contribute, not just those who process their thoughts quickly. **Since students need time to process their responses to complicated questions, teachers should not only wait to call on a student after posing a question but also should wait for a student who has been called on to share his or her idea.** Comments like, "We'll wait for your idea," serve this purpose. If necessary, teachers can ask apprehensive students questions such as, "Should we come back to you later?" It is important to follow up with these students later in the discussion and continue to work with them to encourage them to be more active in the discussions. It is necessary for *all* students to participate in discussions to benefit.

References

Biehler, R. F., & Snowman, J. (1993). *Psychology applied to teaching* (7th ed.). Boston, MA: Houghton Mifflin.

Cazden, C. B. (2001). *Classroom discourse: The language of teaching and learning.* Portsmouth, NH: Heinemann.

Chapin, S. H., O’Connor, C., & Anderson, N. C. (2003). *Classroom discussions: Using math talk to help students learn.* Sausalito, CA: Math Solutions Publications.

Vygotsky, L. S. (1978). *Mind in society*. Cambridge, MA: Harvard University Press.

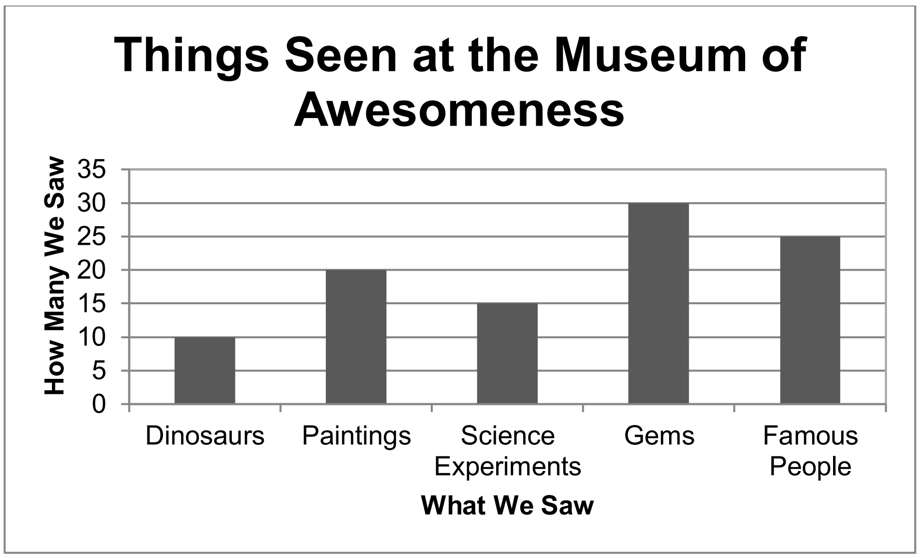
Appendix C: Pictograph Challenge

Name: ______________________________ Date: _______________

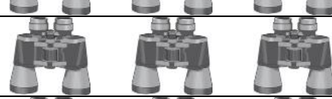
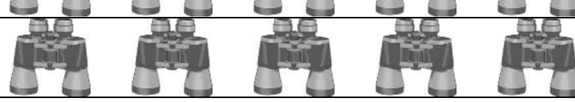
Museum Madness (Optional)

All the Sunny Elementary students went to the Museum of Awesomeness. Caleb and Kobe decided to create graphs of their trip. They both saw the same things at the museum.

Caleb made this bar graph.



Kobe made this pictograph.

Things Seen at the Museum of Awesomeness	
What We Saw	How Many We Saw
Dinosaurs	
Paintings	
Science Experiments	
Gems	
Famous People	

Kobe forgot to include a key for his pictograph. Create a key to go with Kobe's pictograph.

 = _______

SMJ page 207

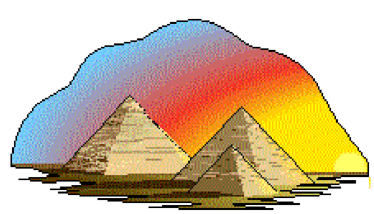
Sara wanted to make her own pictograph of her trip to the museum. Here is a table of what she saw.

Sara's Trip to the Museum of Awesomeness	
What She Saw	How Many She Saw
Pyramids	8
Dinosaurs	24
Dresses	12

She does not want to use <u>part of</u> a picture to represent what she saw. Could this be her key? Explain.

 = 3 Items

Give Sara two suggestions for her key. Explain why your ideas would keep her from having to use part of a picture.



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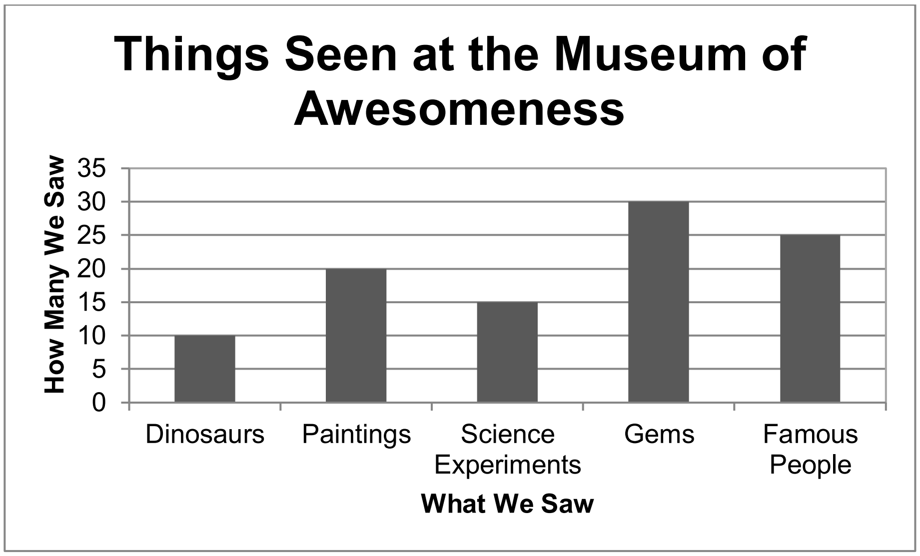
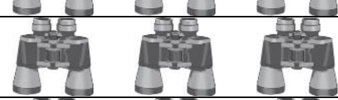
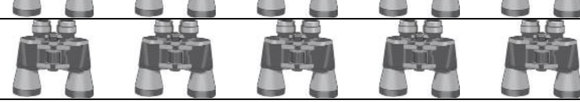
Museum Madness (Optional)
ANSWER KEY

All the Sunny Elementary students went to the Museum of Awesomeness. Caleb and Kobe decided to create graphs of their trip. They both saw the same things at the museum.

Caleb made this bar graph.



Kobe made this pictograph.

Things Seen at the Museum of Awesomeness	
What We Saw	How Many We Saw
Dinosaurs	
Paintings	
Science Experiments	
Gems	
Famous People	

Kobe forgot to include a key for his pictograph. Create a key to go with Kobe's pictograph.

 = 5

Sara wanted to make her own pictograph of her trip to the museum. Here is a table of what she saw.

Sara's Trip to the Museum of Awesomeness	
What She Saw	How Many She Saw
Pyramids	8
Dinosaurs	24
Dresses	12

She does not want to use <u>part of</u> a picture to represent what she saw. Could this be her key? Explain.

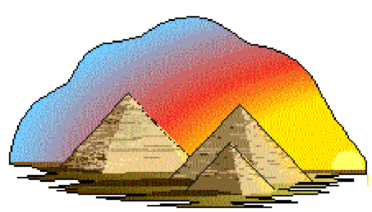
 = 3 Items

This could not be a key because when she shows how many pyramids she saw she would have to use 2 binoculars and a part of one. She doesn't want to use part of a picture, so this will not work.

This would be a good place to bring in the concept of fair sharing.

Give Sara two suggestions for her key. Explain why your ideas would keep her from having to use part of a picture.

Sara could let her symbol stand for 1, 2, or 4. Students could then say how many symbols would be needed for each sight. They should explain how there would be no remainders.



GREENING UP WITH GRAPHING MATHEMATICAL LANGUAGE

Bar Graph: A graph that uses bars to display quantities of categorical data.

Baseline: The data collected before the intervention.

Categorical Data: Data that can be organized in groups (ex: types of food, eye color, book genre).

Category: A set of things grouped together because they share a common trait.

Column: The vertical, or up and down, display on a graph that represents numbers.

Conclusion: A supported answer to a question in an experiment.

Data: Information such as numbers that researchers gather during an experiment.

Data Collection: The process of collecting information and writing it down.

Data Set: Information that is organized to answer a research question.

Data Table: A way for researchers to organize their data.

Experiment: A test, trial, or procedure to discover something new or test an idea.

Fair Share: Distributing a group of objects so that each person receives an equal number.

Frequency: The number of times a particular item appears in a set of data.

Frequency Table: A table that is used to count and total data for different categories.

Horizontal Axis (*x*-axis): The line on a graph that runs from left to right.

Hypothesis: What you think will happen in an experiment, based on facts and your ideas.

Intervention: The process of changing something to determine if you can affect the outcome.

Line Graph: A graph that shows how data changes over time.

Line Plot: A graph that shows the frequency of data on a number line.

Maximum: The largest value in a data set.

Median: The middle value in a data set when data is presented in numerical order.

Minimum: The smallest value in a data set.

Mode: The number that appears most frequently in a set of numbers.

Outlier: An extreme value in a data set.

Question: What you want to know in an experiment.

Range: The minimum to maximum values in a data set (ex: 2 to 6).

Recycle: To create new products from waste materials.

Reduce: To decrease the amount of waste that you produce.

Reuse: To use again, sometimes for a different purpose.

Scale: A series of marks along the axes to determine unit lengths; a scale could vary by 1 unit, 2 units, 10 units, ½ units, and so forth.

Sort: To arrange objects by something they have in common.

Spreadsheet: A table used in computer programs to organize data in rows and columns.

Survey: A list of questions presented to people to gather information from them.

Tally: To count using some type of mark.

Unit: One of an item.

Vertical Axis (*y*-axis): The line on a graph that runs vertically up and down.

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